Math 21B, Winter 2022.
March. 14, 2022.

FINAL EXAM

NAME(print in CAPITAL letters, *first name first*): ______________________________

NAME(sign): ______________________________

ID#: ______________________

Instructions: Each of the 8 problems has equal worth. Read each question carefully and answer it in the space provided. *You must show all your work for full credit.* Clarity of your solutions may be a factor when determining credit. Calculators, books or notes are not allowed. The proctor has been directed not to answer any interpretation questions.

Make sure that you have a total of 10 pages (including this one) with 8 problems.

1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

distance between (x_1, y_1) and $Ax + By + C = 0 : \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

$\sin A \sin B = \frac{1}{2}(\cos(A - B) - \cos(A + B)), \quad \sin A \cos B = \frac{1}{2}(\sin(A - B) + \sin(A + B)),$

$\cos A \cos B = \frac{1}{2}(\cos(A - B) + \cos(A + B)),$

$\sin^2 A = \frac{1}{2}(1 - \cos(2A)), \quad \cos^2 A = \frac{1}{2}(1 + \cos(2A)).$

1. Let $a_n = \sum_{i=1}^{n} \frac{2n+i}{n(3n+i)}$.

(a) Compute: $a = \lim_{n \to \infty} a_n$.

(b) Is a_5 larger or smaller than a? Explain fully.

2. Compute the following indefinite integrals.

(a) $\displaystyle\int \frac{3x-1}{(x+1)(x^2+1)}\,dx$ =(*)

(b) $\displaystyle\int \frac{1}{\sqrt{x}}\ln(x)\,dx$

3. Determine whether the two integrals below converge or diverge.

(a) $\displaystyle\int_1^\infty \frac{2x + 3x^{1/3} + 4}{\sqrt{x}(x^2 + 2x + 3)}\, dx$

(b) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{\sqrt{\sin x}}\, dx$

4. Consider the functions

$$f(x) = \int_0^x \sqrt{9 + t + t^3}\, dt, \qquad g(x) = \int_0^x t\sqrt{9 + t^3}\, dt.$$

(a) Compute $\lim_{x \to 0} \frac{f(x)}{x}$.

(b) Compute $\lim_{x \to 0} \frac{f(x)^2}{g(x)}$.

5. Consider the curve $r = 1 + \cos(2\theta)$ in polar coordinates.
(a) Sketch the curve.

(b) Compute the area this curve encloses.

6. Consider the curve given parametrically by $x = t^3 + 3t^2 + 9t$ and $y = t^2 + t$, $t \geq 0$.
(a) Find the equation of the tangent to the curve at $t = 1$. (You may leave the equation in the point-slope form.)

(b) Determine where this curve is concave up and where it is concave down. Sketch this curve, indicating clearly any inflection points.

Problem 6, continued.

(c) Let R be the region bounded by the curve, the coordinate axes, and the line $x = 13$. Set up, but do not evaluate, the integral for the area of the region R. (Recall from (a) the point on the curve when $t = 1$.)

(d) Set up, but do not evaluate, the integral for the volume of the solid obtained by rotating the region R (which is the same as in (c)) around the y–axis.

(e) Rotate the the piece of the curve between $t = 2$ and $t = 3$ around the line $y = 3$. Set up, but do not evaluate, the integral for the surface area of the resulting surface. (Explain why the line does not intersect the piece of the curve.)

7. The region R lies in the first quadrant and is bounded by the curves $y = x^2$ and $y = x^4$. Compute the quantities below.

(a) Coordinates of the centroid of R. Express your answer in simple fractions.

(b) The volume of the solid obtained by rotating R around the line $2x - 3y - 2 = 0$.

8. Set up, but do not evaluate, the integrals for the arc length of the following curves. Also sketch roughly each of the curves (no concavity analysis necessary).

(a) $y = x^3$, $0 \le x \le 1$, in Cartesian coordinates.

(b) $r = \tan\theta$, $0 \le \theta \le \pi/4$, in polar coordinates.

(c) $x = 2t - t^2$, $y = t^3 + t$, $0 \le t \le 2$, given parametrically.