Math 21B, Winter 2022.
Feb. 25, 2022.

MIDTERM EXAM 2

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): ______________________________

ID#: ______________________

Instructions: Each of the 4 problems has equal worth. Read each question carefully and answer it in the space provided. *You must show all your work for full credit.* Clarity of your solutions may be a factor when determining credit. Calculators, books or notes are not allowed. The proctors have been directed not to answer any interpretation questions.

Make sure that you have a total of 5 pages (including this one) with 4 problems.

1	
2	
3	
4	
TOTAL	

$$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$
$$\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$$
$$\cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$$
$$\sin^2 A = \frac{1}{2}(1 - \cos(2A)), \quad \cos^2 A = \frac{1}{2}(1 + \cos(2A))$$

1. Compute the following two indefinite integrals.

(a) $\int \frac{1}{\sqrt{x+1}+1}\,dx$

$x+1 = u^2 \quad (u \geq 0) \qquad dx = 2u\,du$

$$= \int \frac{2u}{u+1}\,du = 2\int \frac{u+1-1}{u+1}\,du$$

$$= 2\int \left(1 - \frac{1}{u+1}\right) du = 2\left(u - \ln(u+1)\right) + C$$

$$= 2\left(\sqrt{x+1} - \ln\left(\sqrt{x+1}+1\right)\right) + C$$

(b) $\int \frac{1}{x^2+2x+2}\,dx = \int \frac{1}{(x+1)^2+1}\,dx$

$x+1 = u \qquad dx = du$

$$= \int \frac{1}{u^2+1}\,dx = \arctan u + C$$

$$= \arctan(x+1) + C$$

2. Compute the following two definite integrals.

(a) $\displaystyle\int_0^{\pi/4} \cos(3x)\,\cos x\,dx$

$$\cos(3x)\cos x = \frac{1}{2}\left(\cos 4x + \cos 2x\right)$$

$$= \frac{1}{2}\int_0^{\pi/4}\left(\cos 4x + \cos 2x\right)dx$$

$$= \frac{1}{2}\cdot\left(\frac{1}{4}\sin 4x + \sin 2x\right)\Big|_0^{\pi/4} = \frac{1}{2}\cdot\frac{1}{2} = \underline{\underline{\frac{1}{4}}}$$

(b) $\displaystyle\int_1^{2} \frac{2}{x^3+3x^2+2x}\,dx$

$$\frac{2}{x^3+3x^2+2x} = \frac{2}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

$* x,\ x=0:\quad A = 1$

$* (x+1),\ x=-1:\quad B = -2$

$* (x+2),\ x=-2:\quad C = 1$

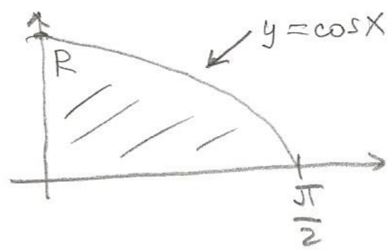
$$= \frac{1}{x} - 2\frac{1}{x+1} + \frac{1}{x+2}$$

$$= \ln|x| - 2\ln|x+1| + \ln|x+2|\Big|_1^2$$

$$= \ln 2 - 2\ln 3 + \ln 4 + 2\ln 2 - \ln 3$$

$$= \underline{\underline{5\ln 2 - 3\ln 3}}$$

3. The region R lies between $x = 0$ and $x = \pi/2$ and is bounded by the graphs of $y = \cos x$ and $y = 0$.
(a) Rotate R around the x-axis and compute the volume of the resulting solid.



$$\text{Volume} = \pi \int_0^{\pi/2} \cos^2 x \, dx$$

$$= \frac{\pi}{2} \int_0^{\pi/2} (1 + \cos 2x) \, dx$$

$$= \frac{\pi}{2} \left[x + \frac{1}{2} \sin 2x \right]_0^{\pi/2} = \frac{\pi^2}{4}$$

(b) Rotate the region around the y-axis and compute the volume of the resulting solid.

$$\text{Volume} = 2\pi \int_0^{\pi/2} x \cos x \, dx$$

$u = x \quad dv = \cos x \, dx$

$du = dx \quad v = \sin x$

$$= 2\pi \left[x \sin x \Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x \, dx \right]$$

$$= 2\pi \left[\frac{\pi}{2} + \cos x \Big|_0^{\pi/2} \right] = 2\pi \left[\frac{\pi}{2} - 1 \right] = \pi^2 - 2\pi$$

4. Consider the curve given as the graph of the function $y = \ln(x+1)$, for $0 \le x \le 1$. Write down, but *do not compute* the integrals for quantities specified below.

(a) The arc length of this curve.

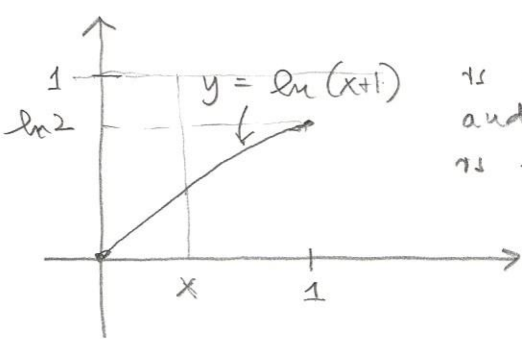
$$y' = \frac{1}{x+1}$$

$$\int_0^1 \sqrt{1 + \left(\frac{1}{x+1}\right)^2}\, dx$$

(b) The surface area of the surface obtained by revolution of this curve around the x axis.

$$2\pi \int_0^1 \ln(x+1) \sqrt{1 + \left(\frac{1}{x+1}\right)^2}\, dx$$

(c) The surface area of the surface obtained by revolution of this curve around the line $y = 1$. (Explain why this line does not intersect the curve!)



$y = \ln(x+1)$ is increasing and its maximum is $\ln 2 < \ln e = 1$.

$$2\pi \int_0^1 (\text{radius of rotation})\, ds$$

$$= 2\pi \int_0^1 (1 - \ln(x+1)) \cdot \sqrt{1 + \left(\frac{1}{x+1}\right)^2}\, dx$$