Math 236A, Fall 2012.

## Homework Assignment 2

**Due:** Oct. 19, 2012

1. On course materials, find scanned pages 46 and 47 from the book "Financial Calculus," by M. Baxter and A. Rennie (Cambridge, 1996). Take a close look at Figure 4.3. Give your opinion on the connection between the figure and preceding text, specifically on whether the scheme illustrated in the figure appears to converge to the Brownian motion.

2. For the standard Brownian motion $B$, compute the following.
(a) $P(B(1) < 0, B(2) > 0)$.
(b) $P(B(1) + 3B(3) > 5B(5) + 7B(7))$.
(c) $P(B(t) = B(t+1)$ for some *integer t*).
(d) $P(B(t) = B(t+1)$ for infinitely many *real t*).

3. Let $\xi_1, \xi_2, \ldots$ be i.i.d., with $E(\xi_1) = 0$, $E(\xi_1^2) = 1$. Let $S_n = \xi_1 + \cdots + \xi_n$, and

$$X_n = \sum_{k=1}^{n} |S_k|.$$

Find a sequence $a_n$ so that $X_n/a_n$ converges in distribution to a nontrivial limit $X$. (*Hint.* $X$ should be a functional of Brownian motion.) Compute $E(X)$. Does $E(X_n)/a_n$ necessarily converge to $E(X)$?