Math 236A, Fall 2012.

Homework Assignment 5

Due: Nov. 26, 2012

1. Let an Itô process X_t be given by $dX_t = a\,dt + b\,dB_t$. Show that this decomposition is unique, that is, if also $dX_t = \alpha\,dt + \beta\,dB_t$, then $P(\alpha = a, \beta = b$ for almost all $t)$=1.

2. Assume that $h_1, \ldots, h_n$ are *deterministic* real functions which are orthonormal in $L^2[0,t]$. Find the distribution of the vector $(\int_0^t h_1\,dB_s, \ldots, \int_0^t h_n\,dB_s)$.

3. Find the stochastic integral representation of the following random variables (for a fixed T): $B_{T/2}B_T$, $\left(\int_0^T s\,dB_s\right)^2$, $\int_0^T sB_s\,ds$. You may (but are not required to) use Malliavin calculus.