Abstracts

Distinguishing Physical and Mathematical Knots and Links

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(joint work with Alexander Coward)

The theory of knots and links studies one-dimensional submanifolds of $\mathbb{R}^3$. These are often described as loops of string, or rope, with their ends glued together. Real ropes however are not one-dimensional, but have a positive thickness and a finite length. Indeed, most physical applications of knot theory are related more closely to the theory of knots of fixed thickness and length than to classical knot theory. For example, biologists are interested in curves of fixed thickness and length as a model for DNA and protein molecules. In these applications the thickness of the curve modeling the molecule plays an essential role in determining the possible configurations.

The two most fundamental problems concerning physical knots and links are to show the existence of a *Gordian Unknot* and a *Gordian Split Link*. A Gordian Unknot is a loop of fixed thickness and length whose core is unknotted, but which cannot be deformed to a round circle by an isotopy fixing its length and thickness. A Gordian Split Link is a pair of loops of fixed thickness whose core curves can be split, or isotoped so that its two components are separated by a plane, but cannot be split by an isotopy fixing each component's length and thickness.

In recent joint work with Alexander Coward, we established the existence of a Gordian Split Link. We thus showed for the first time that the theory of physically realistic curves of fixed thickness and length is distinct from the classical theory of knots and links.

Theorem 1. *A Gordian Split Link exists.*

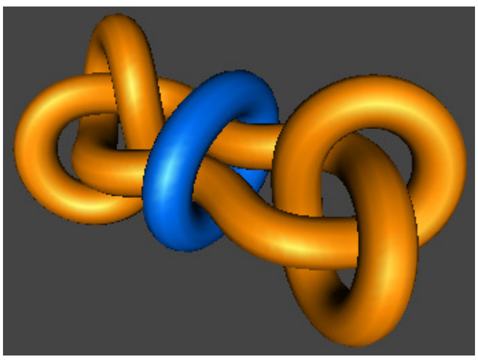

Figure 1. A Gordian Split Link.

The proof of Theorem 1 is by a construction of a link, illustrated in Figure 1, that can be topologically but not physically split. While this property is intuitively quite clear, it's proof is not so simple, and involves new arguments involving isoperimetric inequalities for families of curves.

The existence of a Gordian Unknot remains open.

Theorem 1 is a consequence of the following result, which gives an explicit lower bound on the length required for L_1, the unknotted component of L, under the assumption that L can be split by a physical isotopy.

Theorem 2. *If there is a physical isotopy of $L = L_1 \sqcup L_2$ that splits its two components, then the length of L_1 must be at least $4\pi + 6 \approx 18.566$.*

Since the link L can be constructed with the length of the unknotted component L_1 equal to $4\pi + 4$, this result implies Theorem 1. Details are available at [3].

References

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