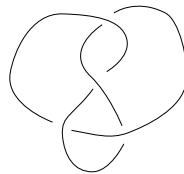
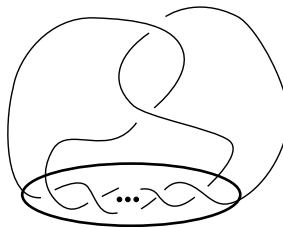


**MSRI SUMMER SCHOOL: 4-MANIFOLD CONSTRUCTIONS PROBLEM  
SESSION 4**

- (1) Using the definition of knot surgery, prove that if  $K$  is the unknot, then there exists a gluing defining knot surgery on  $K$  which is the trivial operation. Varying among all possible gluings, what operations can you realize by knot surgery along the unknot?
- (2) Using the Skein relations, compute the Alexander polynomial for the figure-eight knot as shown below.



- (3) The  $k$ -twist knot is the knot of the form shown below where the circled portion contains  $k$  crossings. Note the figure-eight knot is the 2-twist knot.



- Calculate the Alexander polynomial of the  $k$ -twist knot when  $k$  is even. (You'll need the Skein relation along with an inductive argument.) If you have time, you can also get the formula when  $k$  is odd (the argument is similar).
- (4) Show that you can obtain infinitely many 4-manifolds, which are pairwise non-diffeomorphic but all homeomorphic by performing knot surgery along a fiber of  $E(2)$  with different twist knots.