PREREQUISITE REVIEW 8.5

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–8, evaluate the trigonometric function.

1. $\cos \frac{5\pi}{4}$

2. $\sin \frac{7\pi}{6}$

3. $\sin\left(-\frac{\pi}{3}\right)$

4. $\cos\left(-\frac{\pi}{6}\right)$

5. $\tan \frac{5\pi}{6}$

6. $\cot \frac{5\pi}{3}$

7. $\sec \pi$

8. $\cos \frac{\pi}{2}$

In Exercises 9–16, simplify the expression using the trigonometric identities.

9. $\sin x \sec x$

10. $\csc x \cos x$

11. $\cos^2 x(\sec^2 x - 1)$

12. $\sin^2 x(\csc^2 x - 1)$

13. $\sec x \sin\left(\frac{\pi}{2} - x\right)$

14. $\cot x \cos\left(\frac{\pi}{2} - x\right)$

15. $\cot x \sec x$

16. $\cot x(\sin^2 x)$

In Exercises 17–20, evaluate the definite integral.

17. $\int_0^4 (x^2 + 3x - 4)\, dx$

18. $\int_{-1}^1 (1 - x^2)\, dx$

19. $\int_0^2 x(4 - x^2)\, dx$

20. $\int_0^1 x(9 - x^2)\, dx$

EXERCISES 8.5

In Exercises 1–34, evaluate the integral.

1. $\int (2 \sin x + 3 \cos x)\, dx$

2. $\int (t^2 - \sin t)\, dt$

3. $\int (1 - \csc t \cot t)\, dt$

4. $\int (\theta^2 + \sec^2 \theta)\, d\theta$

5. $\int (\csc^2 \theta - \cos \theta)\, d\theta$

6. $\int (\sec y \tan y - \sec^2 y)\, dy$

7. $\int \sin 2x\, dx$

8. $\int \cos 6x\, dx$

9. $\int x \cos x^2\, dx$

10. $\int x \sin x^2\, dx$

11. $\int \sec^2 \frac{x}{2}\, dx$

12. $\int \csc^2 \frac{x}{2}\, dx$

13. $\int \tan 3x\, dx$

14. $\int \csc 2x \cot 2x\, dx$

15. $\int \tan^3 x \sec^2 x\, dx$

16. $\int \sqrt{\cot x} \csc^2 x\, dx$

17. $\int \cot \pi x\, dx$

18. $\int \tan 5x\, dx$

19. $\int \csc 2x\, dx$

20. $\int \sec \frac{x}{2}\, dx$

21. $\int \frac{\sec^2 2x}{\tan 2x}\, dx$

22. $\int \frac{\sin x}{\cos^2 x}\, dx$

23. $\int \frac{\sec x \tan x}{\sec x - 1}\, dx$

24. $\int \frac{\cos t}{1 + \sin t}\, dt$

25. $\int \frac{\sin x}{1 + \cos x}\, dx$

26. $\int \frac{\sin \sqrt{x}}{\sqrt{x}}\, dx$

27. $\int \frac{\csc^2 x}{\cot^3 x}\, dx$

28. $\int \frac{1 - \cos \theta}{\theta - \sin \theta}\, d\theta$

29. $\int e^x \sin e^x\, dx$

30. $\int e^{-x} \tan e^{-x}\, dx$

31. $\int e^{\sin x} \cos x\, dx$

32. $\int e^{\sec x} \sec x \tan x\, dx$

33. $\int (\sin 2x + \cos 2x)^2\, dx$

34. $\int (\csc 2\theta - \cot 2\theta)^2\, d\theta$

In Exercises 35–38, evaluate the integral.

35. $\int x \cos x\, dx$

36. $\int x \sin x\, dx$

37. $\int x \sec^2 x\, dx$

38. $\int \theta \sec \theta \tan \theta\, d\theta$

In Exercises 39–46, evaluate the definite integral. Use a symbolic integration utility to verify your results.

39. $\int_0^{\pi/4} \cos \frac{4x}{3}\, dx$

40. $\int_0^{\pi/2} \sin 2x\, dx$

41. $\int_{\pi/2}^{2\pi/3} \sec^2 \frac{x}{2}\, dx$

42. $\int_0^{\pi/2} (x + \cos x)\, dx$

43. $\int_{\pi/12}^{\pi/4} \csc 2x \cot 2x\, dx$

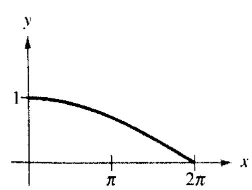
44. $\int_0^{\pi/8} \sin 2x \cos 2x\, dx$

45. $\int_0^1 \tan(1 - x)\, dx$

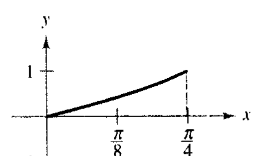
46. $\int_0^{\pi/4} \sec x \tan x\, dx$

In Exercises 47–52, determine the area of the region.

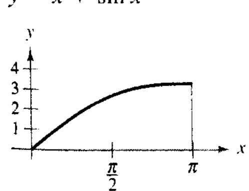
47. $y = \cos \frac{x}{4}$



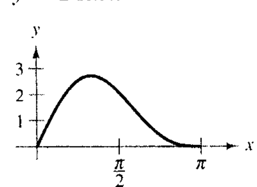
48. $y = \tan x$



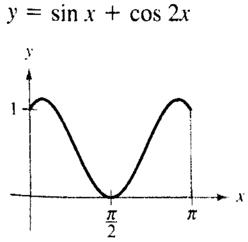
49. $y = x + \sin x$



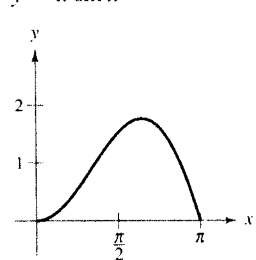
50. $y = 2 \sin x + \sin 2x$



51. $y = \sin x + \cos 2x$



52. $y = x \sin x$



In Exercises 53 and 54, find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the x-axis.

53. $y = \sec x,\ y = 0,\ x = 0,\ x = \frac{\pi}{4}$

54. $y = \csc x,\ y = 0,\ x = \frac{\pi}{6},\ x = \frac{5\pi}{6}$

In Exercises 55 and 56, approximate the definite integral using the Trapezoidal Rule and Simpson's Rule with $n = 4$. Compare these results with the approximation of the integral using a graphing utility.

55. $\int_0^{\pi/2} f(x)\, dx, \qquad f(x) = \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}$

56. $\int_0^1 \cos x^2\, dx$

57. ***Inventory*** The minimum stockpile level of gasoline in the United States can be approximated by the model

$$Q = 217 + 13 \cos \frac{\pi(t - 3)}{6}$$

where Q is measured in millions of barrels of gasoline and t is the time in months, with $t = 1$ corresponding to January. Find the average minimum level given by this model during

(a) the first quarter $(0 \le t \le 3)$

(b) the second quarter $(3 \le t \le 6)$

(c) the entire year $(0 \le t \le 12)$.

58. ***Seasonal Sales*** The sales of a software product are given by the model

$$S = 74.50 + 43.75 \sin \frac{\pi t}{6}$$

where S is measured in thousands of units per month and t is the time in months, with $t = 1$ corresponding to January. Use a graphing utility to estimate average sales during

(a) the first quarter $(0 \le t \le 3)$

(b) the second quarter $(3 \le t \le 6)$

(c) the entire year $(0 \le t \le 12)$.

59. ***Meteorology*** The average monthly precipitation P in inches, including rain, snow, and ice, for Sacramento, California can be modeled by

$$P = 2.34 \sin(0.38t + 1.91) + 2.22$$

where t is the time in months, with $t = 1$ corresponding to January. Find the total annual precipitation for Sacramento. *(Source: U.S. National Oceanic and Atmospheric Administration)*

60. ***Meteorology*** The average monthly precipitation P in inches, including rain, snow, and ice, for Bismarck, North Dakota can be modeled by

$$P = 1.07\sin(0.59t + 3.94) + 1.52$$

where t is the time in months, with $t = 1$ corresponding to January. *(Source: National Oceanic and Atmospheric Administration)*

(a) Find the maximum and minimum precipitation and the month in which each occurs.

(b) Determine the average monthly precipitation for the year.

(c) Find the total annual precipitation for Bismarck.

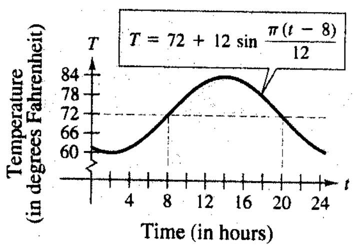
61. ***Cost*** Suppose that the temperature in degrees Fahrenheit is given by

$$T = 72 + 12\sin\frac{\pi(t-8)}{12}$$

where t is the time in hours, with $t = 0$ corresponding to midnight. Furthermore, suppose that it costs \$0.10 to cool a particular house 1° for 1 hour.

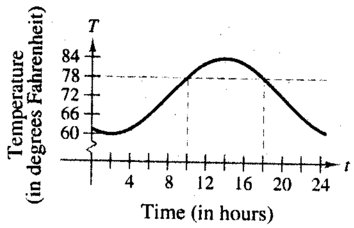
(a) Use the integration capabilities of a graphing utility to find the cost C of cooling this house between 8 A.M. and 8 P.M., if the thermostat is set at 72° (see figure) and the cost is given by

$$C = 0.1\int_8^{20}\left[72 + 12\sin\frac{\pi(t-8)}{12} - 72\right]dt.$$



(b) Use the integration capabilities of a graphing utility to find the savings realized by resetting the thermostat to 78° (see figure) by evaluating the integral

$$C = 0.1\int_{10}^{18}\left[72 + 12\sin\frac{\pi(t-8)}{12} - 78\right]dt.$$



62. ***Health*** For a person at rest, the velocity v (in liters per second) of air flow into and out of the lungs during a respiratory cycle is approximated by

$$v = 0.9\sin\frac{\pi t}{3}$$

where t is the time in seconds. Find the volume in liters of air inhaled during one cycle by integrating this function over the interval $[0, 3]$.

63. ***Health*** After exercising for a few minutes, a person has a respiratory cycle for which the velocity of air flow is approximated by

$$v = 1.75\sin\frac{\pi t}{2}.$$

How much does the lung capacity of a person increase as a result of exercising? Use the results of Exercise 62 to determine how much more air is inhaled during a cycle after exercising than is inhaled during a cycle at rest. (Note that the cycle is shorter and you must integrate over the interval $[0, 2]$.)

64. ***Sales*** In Example 9 in Section 8.4, the sales of a seasonal product were approximated by the model

$$F = 100{,}000\left[1 + \sin\frac{2\pi(t-60)}{365}\right], \quad t \ge 0$$

where F was measured in pounds and t was the time in days, with $t = 1$ corresponding to January 1. The manufacturer of this product wants to set up a manufacturing schedule to produce a uniform amount each day. What should this amount be? (Assume that there are 200 production days during the year.)

In Exercises 65–68, use a graphing utility and Simpson's Rule to approximate the integral.

	Integral	n
65.	$\int_0^{\pi/2}\sqrt{x}\sin x\,dx$	8
66.	$\int_0^{\pi/2}\cos\sqrt{x}\,dx$	8
67.	$\int_0^{\pi}\sqrt{1+\cos^2 x}\,dx$	20
68.	$\int_0^{2}(4 + x + \sin\pi x)\,dx$	20

True or False? In Exercises 69–71, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

69. $\int_a^b \sin x\,dx = \int_a^{b+2\pi}\sin x\,dx$

70. $4\int \sin x\cos x\,dx = -\cos 2x + C$

71. $\int \sin^2 2x\cos 2x\,dx = \frac{1}{3}\sin^3 2x + C$