Math 16B **Midterm 1** **Fall 2010**

NAME(print in CAPITAL letters, first name first): KEY

NAME(sign): _____

ID#: _____

Instructions: There are nine problems. Some questions are easier than others so you are encouraged to read the entire exam before beginning your work. Make sure that you have all 9 problems.

1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

$$\begin{aligned}
\sin A \sin B &= \frac{1}{2}(\cos(A-B) - \cos(A+B)) \\
\sin A \cos B &= \frac{1}{2}(\sin(A-B) + \sin(A+B)) \\
\cos A \cos B &= \frac{1}{2}(\cos(A-B) + \cos(A+B))
\end{aligned}$$

$$\sin^2 A = \frac{1}{2}(1 - \cos(2A)), \quad \cos^2 A = \frac{1}{2}(1 + \cos(2A))$$

1. (10 points.) Find $\frac{d}{dx}(e^x + e^{-x})^2$.

$$\frac{d}{dx}(e^x+e^{-x})^2 =$$

$$2(e^x + e^{-x})\frac{d}{dx}(e^x+e^{-x}) =$$

$$2(e^x+e^{-x})(e^x-e^{-x}) =$$

$$2(e^{2x}-e^{-2x})$$

2. (10 points.) Find $\frac{d}{dx}\ln\left(\frac{2x}{\sqrt{x^2-1}}\right)$

$$\frac{d}{dx}\ln\left(\frac{2x}{\sqrt{x^2-1}}\right) =$$

$$\frac{d}{dx}\left(\ln 2 + \ln x - \frac{1}{2}\ln(x^2-1)\right) =$$

$$0 + \frac{1}{x} - \frac{1}{2}\cdot\frac{1}{x^2-1}\cdot 2x =$$

$$\frac{1}{x} - \frac{x}{x^2-1}$$

3. (20 points.) Bacteria in a dish is growing exponentially. After 1 day there are 60 cells. After 4 days there are 480 cells. How many cells were there initially? Simplify your answer as much as you can.

$$y = Ce^{kt}$$

$$60 = Ce^{k}$$

$$480 = Ce^{4k}$$

$$\Rightarrow \quad 8 = e^{3k}$$

$$\ln 8 = 3k$$

$$k = \frac{\ln 8}{3}$$

$$60 = Ce^{\ln 8/3}$$

$$= C\left(e^{\ln 8}\right)^{1/3}$$

$$= C\,8^{1/3}$$

$$= 2C$$

$$\boxed{C = 30}$$

4. (10 points.) Find $\int \frac{x}{\sqrt{1-x^2}}\,dx$.

$$\begin{bmatrix} u = 1-x^2 \\ du = -2x\,dx, \quad x\,dx = \frac{-1}{2}\,du \end{bmatrix}$$

$$\int \frac{x\,dx}{\sqrt{1-x^2}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}}$$

$$= -\sqrt{u} + C$$

$$= -\sqrt{1-x^2} + C$$

5. (10 points.) Find $\int 2e^{2-x}\,dx$.

$$\begin{bmatrix} u = 2-x \\ du = -dx \end{bmatrix}$$

$$\int 2e^{2-x}\,dx = -2\int e^u\,du$$

$$= -2e^u + C$$

$$= -2e^{2-x} + C$$

6. (10 points.) Find $\int_0^1 \frac{x-\sqrt{x}}{2}\,dx$.

$$\int_0^1 \frac{x-\sqrt{x}}{2}\,dx =$$

$$\int_0^1 \frac{x}{2} - \frac{\sqrt{x}}{2}\,dx =$$

$$\left[\frac{1}{4}x^2 - \frac{1}{3}x^{3/2}\right]_0^1 =$$

$$\frac{1}{4} - \frac{1}{3} = \frac{-1}{12}$$

7. (10 points.) Find $\int_{-2}^{2} |x|\,dx$.

$$\int_{-2}^{2} |x|\,dx =$$

$$2\int_0^2 |x|\,dx \quad \text{because even}$$

$$= 2\int_0^2 x\,dx$$

$$= x^2\Big]_0^2 = 4$$

8. (10 points.) Find $\frac{d}{dx}x^x$.

$$\frac{d}{dx}x^x =$$

$$\frac{d}{dx}e^{x\ln x} =$$

$$e^{x\ln x}\frac{d}{dx}(x\ln x) =$$

$$e^{x\ln x}\left(\ln x + \frac{1}{x}x\right) =$$

$$e^{x\ln x}(\ln x + 1)$$

9. (10 points.) Find $\int \frac{1}{1+e^x}\,dx$.

$$\int \frac{e^{-x}}{e^{-x}}\frac{1}{1+e^x}\,dx =$$

$$\int \frac{e^{-x}}{e^{-x}+1}\,dx$$

$$\begin{bmatrix} u = e^{-x}+1 \\ du = -e^{-x}dx \end{bmatrix}$$

$$= \int \frac{-du}{u} = -\ln|u| + C$$

$$= -\ln|e^{-x}+1| + C$$