Math 16B Midterm 1 Fall ~~2016~~ 2011

NAME(print in CAPITAL letters, first name first): _____

NAME(sign): _____

ID#: _____

Instructions: There are nine problems. Some questions are easier than others so you are encouraged to read the entire exam before beginning your work. Make sure that you have all 9 problems.

Points received:

1 _____

2 _____

3 _____

4 _____

5 _____

6 _____

7 _____

8 _____

9 _____

TOTAL _____

1. (10 points.) Find $\frac{d}{dx}(e^x + \ln x)^2$.

$$= 2(e^x + \ln x)\frac{d}{dx}(e^x + \ln x)$$

$$= 2(e^x + \ln x)(e^x + \frac{1}{x})$$

2. (10 points.) Find $\frac{d}{dx}\ln(xe^x)$

$$= \frac{d}{dx}\ln x + x$$

$$= \frac{1}{x} + 1$$

3. (20 points.) You start with 64 pounds of a radioactive element. After 3 years, 1 pound remains. How many pounds remained after 2 years? Simplify your answer as much as you can.

$$y = 64e^{kt}$$

$$1 = 64e^{3k}$$

$$\frac{1}{64} = e^{3k}$$

$$3k = -\ln(64)$$

$$k = \frac{-\ln(64)}{3}$$

$$y(2) = 64\, e^{\left(\frac{-\ln(64)}{3} \cdot 2\right)}$$

$$= 64\left[e^{\ln(64)}\right]^{-2/3}$$

$$= 64\,(64)^{-2/3}$$

$$= (64)^{1/3} = 4$$

4. (10 points.) Find $\int x\sqrt{1-x^2}\,dx$.

$$\begin{bmatrix} u = 1-x^2 \\ du = -2x\,dx \end{bmatrix}$$

$$= \frac{-1}{2}\int \sqrt{u}\,du$$

$$= \frac{-1}{2}\,\frac{u^{3/2}}{3/2} + C$$

$$= \frac{-1}{3}(1-x^2)^{3/2} + C$$

5. (10 points.) Find $\int xe^{1-x^2}\,dx$.

$$\begin{bmatrix} u = 1-x^2 \\ du = -2x\,dx \end{bmatrix}$$

$$= \frac{-1}{2}\int e^{1-x^2}(-2x)\,dx$$

$$= -\frac{1}{2}\int e^u\,du$$

$$= -\frac{1}{2}e^u + C$$

$$= -\frac{1}{2}e^{1-x^2} + C$$

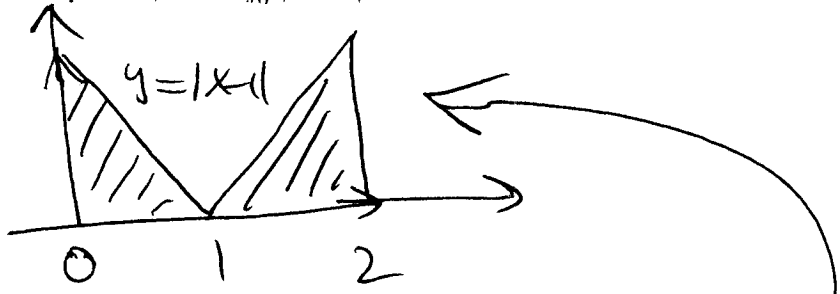
6. (10 points.) Find $\int_0^1 (x+1)^4\,dx$.

$$= \frac{1}{5}(x+1)^5\Big]_0^1$$

$$= \frac{1}{5}2^5 - \frac{1}{5}1^5$$

$$= \frac{32}{5} - \frac{1}{5} = \frac{31}{5}$$

7. (10 points.) Find $\int_0^2 |x-1|\,dx$.



$$\int_0^2 |x-1|\,dx = \text{area}$$

$$= 1$$

8. (10 points.) Find $\frac{d}{dx} x^{\ln x}$.

$$\frac{d}{dx} x^{\ln x} =$$

$$\frac{d}{dx} e^{\ln(x^{\ln x})}$$

$$= \frac{d}{dx} e^{(\ln x)^2}$$

$$= e^{(\ln x)^2} \, 2(\ln x) \frac{1}{x}$$

9. (10 points.) Find $\int \frac{1}{e^{-x}-1} \, dx$.

$$\int \frac{e^x}{e^x} \frac{1}{e^{-x}-1} \, dx$$

$$= \int \frac{e^x}{1-e^x} \, dx = -\int \frac{-e^x}{1-e^x} \, dx$$

$$= -\ln|1-e^x| + C$$