

Math 21B - Homework Set 1

Section 5.1:

In exercises 1–2, use finite approximations to estimate the area under the graph of the function using

- (a) a lower sum with two rectangles of equal width.
 - (b) a lower sum with four rectangles of equal width.
 - (c) an upper sum with two rectangles of equal width.
 - (d) an upper sum with four rectangles of equal width.
1. $f(x) = x^3$ between $x = 0$ and $x = 1$.
 2. $f(x) = 1/x$ between $x = 1$ and $x = 5$.
 3. Use the midpoint rule to estimate the area under the graph of $f(x) = x^2$, between $x = 0$ and $x = 1$, using first two and then four rectangles.
 4. p. 305, problem 14. In part (a), give the exact value for the velocity.
 5. Use a finite sum to estimate the average value of $f(x) = x^3$ on $[0, 2]$ by partitioning the interval into four subintervals of equal length and evaluating f at the subinterval midpoints.

Section 5.2:

In exercises 1–3, write the sums without sigma notation. Then evaluate them.

1. $\sum_{k=1}^2 \frac{6k}{k+1}$
2. $\sum_{k=1}^3 \frac{k-1}{k}$
3. $\sum_{k=1}^5 \sin(k\pi)$
4. Which of the following express $1 + 2 + 4 + 8 + 16 + 32$ in sigma notation?
 - (a) $\sum_{k=1}^6 2^{k-1}$
 - (b) $\sum_{k=0}^5 2^k$
 - (c) $\sum_{k=-1}^4 2^{k+1}$

5. Express the following sums in sigma notation

(a) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

(b) $2 + 4 + 6 + 8 + 10$

(c) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}$

6. Suppose that $\sum_{k=1}^n a_k = -5$ and $\sum_{k=1}^n b_k = 6$. Find the values of:

(a) $\sum_{k=1}^n 3a_k$

(b) $\sum_{k=1}^n \frac{b_k}{6}$

(c) $\sum_{k=1}^n (a_k + b_k)$

(d) $\sum_{k=1}^n (a_k - b_k)$

(e) $\sum_{k=1}^n (b_k - 2a_k)$

7. Find a formula for the Riemann sum to approximate the area under the graph of $f(x) = 3x + 2x^2$ over the interval $[0, 1]$ using n equal subintervals and using the right-hand endpoint for each c_k . Then take a limit as $n \rightarrow \infty$ to calculate the area under the curve over $[0, 1]$.