Math 21B-B - Homework Set 2

Section 5.3:

1. Express the following limits as definite integrals:

   (a) $\lim_{\|P\|\to 0} \sum_{k=1}^{n} ({c_k}^2 - 3c_k)\Delta x_k$, where P is a partition of $[-7, 5]$

   (b) $\lim_{\|P\|\to 0} \sum_{k=1}^{n} \sqrt{4 - {c_k}^2}\Delta x_k$, where P is a partition of $[0, 1]$

   (c) $\lim_{\|P\|\to 0} \sum_{k=1}^{n} (\tan c_k)\Delta x_k$, where P is a partition of $[0, \pi/4]$

2. Suppose that f and g are integrable and that:

$$\int_1^2 f(x)\,dx = -4, \qquad \int_1^5 f(x)\,dx = 6, \qquad \int_1^5 g(x) = 8.$$

   Use the rules in Table 5.4 to find:

   (a) $\int_2^2 g(x)\,dx$

   (b) $\int_5^1 g(x)\,dx$

   (c) $\int_1^2 3f(x)\,dx$

   (d) $\int_2^5 f(x)\,dx$

   (e) $\int_1^5 [f(x) - g(x)]\,dx$

   (f) $\int_1^5 [4f(x) - g(x)]\,dx$

3. Suppose that f and h are integrable and that:

$$\int_1^9 f(x)\,dx = -1, \qquad \int_7^9 f(x)\,dx = 5, \qquad \int_7^9 h(x)\,dx = 4.$$

   Use the rules in Table 5.4 to find

   (a) $\int_1^9 -2f(x)\,dx$

   (b) $\int_7^9 [f(x) + h(x)]\,dx$

(c) $\displaystyle\int_7^9 [2f(x) - 3h(x)]\,dx$

(d) $\displaystyle\int_9^1 f(x)\,dx$

(e) $\displaystyle\int_1^7 f(x)\,dx$

(f) $\displaystyle\int_9^7 [h(x) - f(x)]\,dx$

4. Find $\displaystyle\int_{-2}^1 |x|\,dx$.

5. Find $\displaystyle\int_{-1}^1 (1 + \sqrt{1 - x^2})\,dx$.

6. Graph and find the average value of the following functions over the given interval

   (a) $f(x) = x^2 - 1$ on $[0, \sqrt{3}]$.

   (b) $h(x) = -|x|$ on

      i. $[-1, 0]$

      ii. $[0, 1]$

      iii. $[-1, 1]$

7. Use the method of example 4a to evaluate

   (a) $\displaystyle\int_a^b x^2\ dx,\ a < b$

   (b) $\displaystyle\int_{-1}^0 (x - x^2)\ dx$

8. Use the Max-Min Inequality to find upper and lower bounds for the value of

$$\int_0^1 \frac{1}{1 + x^2}\,dx.$$

9. Use the Max-Min Inequality to find upper and lower bounds for the the value of

$$\int_0^{0.5} \frac{1}{1 + x^2}\,dx \qquad \text{and} \qquad \int_{0.5}^1 \frac{1}{1 + x^2}\,dx.$$

10. Use the inequality $\sin x \leq x$, which holds for $x \geq 0$, to find an upper bound for the value of $\int_0^1 \sin x\,dx$.

Section 5.4:

1. Evaluate the following integrals:

   (a) $\displaystyle\int_{-2}^{0} (2x+5)\,dx$

   (b) $\displaystyle\int_{-3}^{4} \left(5-\frac{x}{2}\right)\,dx$

   (c) $\displaystyle\int_{0}^{\pi} \sin x\,dx$

   (d) $\displaystyle\int_{0}^{\pi/3} 2\sec^2 x\,dx$

   (e) $\displaystyle\int_{1}^{-1} (r+1)^2\,dr$

   (f) $\displaystyle\int_{1}^{2} \left(\frac{1}{x}-e^{-x}\right)\,dx$

   (g) $\displaystyle\int_{0}^{1} \frac{4}{1+x^2}\,dx$

   (h) $\displaystyle\int_{2}^{5} \frac{x}{\sqrt{1+x^2}}\,dx$

   (i) $\displaystyle\int_{0}^{1} xe^{x^2}\,dx$

   (j) $\displaystyle\int_{1}^{2} \frac{\ln x}{x}\,dx$

2. Find the derivatives of the following functions (i) by evaluating the integral and differentiating the result, and (ii) by differentiating the integral directly:

   (a) $\frac{d}{dx}\displaystyle\int_{0}^{\sqrt{x}} \cos t\,dt$

   (b) $\frac{d}{d\theta}\displaystyle\int_{0}^{\tan\theta} \sec^2 y\,dy$

3. Find $\frac{dy}{dx}$ if $y=\displaystyle\int_{\sqrt{x}}^{0} \sin\left(t^2\right)\,dt$

4. In the exercises below, find the total area between the region and the x-axis:

   (a) $y=-x^2-2x$, where $-3\le x\le 2$

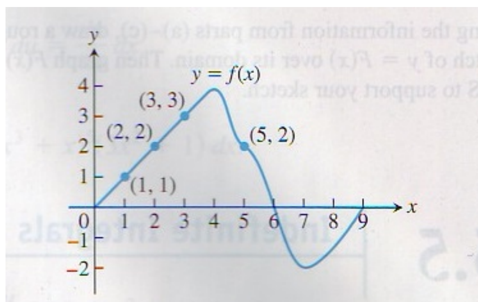
   (b) $y=3x^2-3$, where $-2\le x\le 2$

(c) $y = x^{1/3} - x, \quad$ where $\quad -1 \leq x \leq 8$

5. Suppose that f is the differentiable function shown in the accompanying graph and that the position at time t (sec) of a particle moving along a coordinate axis is

$$s = \int_0^t f(x)dx$$

meters. Use the graph to answer the following questions. Give reasons for your answers.



(a) What is the particle's velocity at time $t = 5$?

(b) Is the acceleration of the particle at time $t = 5$ positive or negative?

(c) What is the particle's position at time $t = 3$?

(d) At what time during the first 9 sec does s have its largest value?

(e) Approximately when is the acceleration zero?

(f) When is the particle moving toward the origin? Away from the origin?

(g) On which side of the origin does the particle lie at time $t = 9$?

6. Find $\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4 + 1}\, dt$.

7. Suppose $f'(x) \geq 0$ for all values of x, and that $f(1) = 0$. Which of the following statements must be true of the function

$$g(x) = \int_0^x f(t)\, dt?$$

Give reasons for your answers.

(a) g is a differentiable function of x.

(b) g is a continuous function of x.

(c) The graph of g has a horizontal tangent at $x = 1$.

(d) g has a local maximum at $x = 1$.

(e) g has a local minimum at $x = 1$.

(f) The graph of g has an inflection point at $x = 1$.

(g) The graph of dg/dx crosses the x-axis at $x = 1$.