

Math 21B-B - Homework Set 3

Section 4.8:

1. Find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation.

- (a) i. $f(x) = 2x^{-3}$
ii. $f(x) = \frac{x^{-3}}{2} + x^2$
iii. $f(x) = -x^{-3} + x - 1$
- (b) i. $f(x) = \frac{1}{2}x^{-1/2}$
ii. $f(x) = -\frac{1}{2}x^{-3/2}$
iii. $f(x) = -\frac{3}{2}x^{-5/2}$
- (c) i. $f(x) = -\pi \sin(\pi x)$
ii. $f(x) = 3 \sin(x)$
iii. $f(x) = \sin(\pi x) - 3 \sin(3x)$
- (d) i. $f(x) = \sec^2(x)$
ii. $f(x) = \frac{2}{3} \sec^2\left(\frac{x}{3}\right)$
iii. $f(x) = -\sec^2\left(\frac{3x}{2}\right)$
- (e) i. $f(x) = \sec(x) \tan(x)$
ii. $f(x) = 4 \sec(3x) \tan(3x)$
iii. $f(x) = \sec\left(\frac{\pi x}{2}\right) \tan\left(\frac{\pi x}{2}\right)$
- (f) i. $f(x) = e^{3x}$
ii. $f(x) = e^{-x}$
iii. $f(x) = e^{x/2}$

2. Find the most general antiderivative or definite integral. Check your answers by differentiation.

- (a) $\int \left(3t^2 + \frac{t}{2}\right) dt$
- (b) $\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right) dx$
- (c) $\int \left(7 \sin\left(\frac{\theta}{3}\right)\right) d\theta$
- (d) $\int (2e^x - 3e^{-2x}) dx$
- (e) $\int (1 + \tan^2 \theta) d\theta$
- (f) $\int (2 + \tan^2 \theta) d\theta$

3. Verify the following formulas by differentiating.

(a) $\int (3x + 5)^{-2} dx = -\frac{(3x+5)^{-1}}{3} + C$

(b) $\int \frac{1}{(x+1)^2} dx = -\frac{1}{x+1} + C$

4. Right or wrong? Say which for each and give a brief reason for each answer.

(a) $\int x \sin x dx = \frac{x^2}{2} \sin x + C$

(b) $\int x \sin x dx = -x \cos x + C$

(c) $\int x \sin x dx = -x \cos x + \sin x + C$

5. Right or wrong? Say which for each and give a brief reason for each answer.

(a) $\int \tan \theta \sec^2 \theta d\theta = \frac{\sec^3 \theta}{3} + C$

(b) $\int \tan \theta \sec^2 \theta d\theta = \frac{1}{2} \tan^2 \theta + C$

(c) $\int \tan \theta \sec^2 \theta d\theta = \frac{1}{2} \sec^2 \theta + C$

6. Suppose that $f(x) = \frac{d}{dx}(1 - \sqrt{x})$ and $g(x) = \frac{d}{dx}(x + 2)$.

Find:

(a) $\int f(x) dx$

(b) $\int g(x) dx$

(c) $\int [-f(x)] dx$

(d) $\int [-g(x)] dx$

(e) $\int [f(x) + g(x)] dx$

(f) $\int [f(x) - g(x)] dx$

Section 5.5:

1. Evaluate the following indefinite integrals by using the given substitutions to reduce the integrals to standard form.

(a) $\int x \sin(2x^2) dx, u = 2x^2$

(b) $\int 28(7x - 2)^{-5} dx, u = 7x - 2$

- (c) $\int \frac{9r^2 dr}{\sqrt{1-r^3}}, \quad u = 1 - r^3$
 (d) $\int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx, \quad u = \frac{1}{x}$
 (e) $\int \sqrt{3-2s} ds, \quad u = 3 - 2s$

2. Evaluate the following integrals.

- (a) $\int \frac{4y dy}{\sqrt{2y^2 + 1}}$
 (b) $\int \tan x dx$
 (c) $\int x^{1/3} \sin(x^{4/3} - 8) dx$
 (d) $\int \sqrt{\frac{x-1}{x^5}} dx$
 (e) $\int (\cos x)e^{\sin x} dx$
 (f) $\int \frac{1}{x^2} e^{1/x} \sec(1 + e^{1/x}) \tan(1 + e^{1/x}) dx$
 (g) $\int \frac{dx}{x \ln x}$
 (h) $\int \frac{1}{\sqrt{e^{2\theta} - 1}} d\theta$
 (i) $\int \frac{18 \tan^2 x \sec^2 x}{(2 + \tan^3 x)^2} dx$

3. Solve the following initial value problems.

- (a) $\frac{ds}{dt} = 12t(3t^2 - 1)^3, \quad s(1) = 3$
 (b) $\frac{dr}{d\theta} = 3 \cos^2\left(\frac{\pi}{4} - \theta\right), \quad r(0) = \frac{\pi}{8}$
 (c) $\frac{d^2s}{dt^2} = -4 \sin\left(2t - \frac{\pi}{2}\right), \quad s'(0) = 100, \quad s(0) = 0$