Math 21B-B - Homework Set 4

Section 5.6:

1. Evaluate the following integrals.

   (a) $\displaystyle\int_0^{\frac{\pi}{4}} \tan x \sec^2 x \, dx$

   (b) $\displaystyle\int_{-\frac{\pi}{4}}^0 \tan x \sec^2 x \, dx$

2. Evaluate the following integrals.

   (a) $\displaystyle\int_0^{\sqrt{7}} t \left(t^2 + 1\right)^{1/3} dt$

   (b) $\displaystyle\int_{-\sqrt{7}}^0 t \left(t^2 + 1\right)^{1/3} dt$

3. Evaluate the following integrals.

   (a) $\displaystyle\int_0^1 \frac{x^3}{\sqrt{x^4 + 9}} \, dx$

   (b) $\displaystyle\int_{-1}^0 \frac{x^3}{\sqrt{x^4 + 9}} \, dx$

4. Evaluate the following integrals.

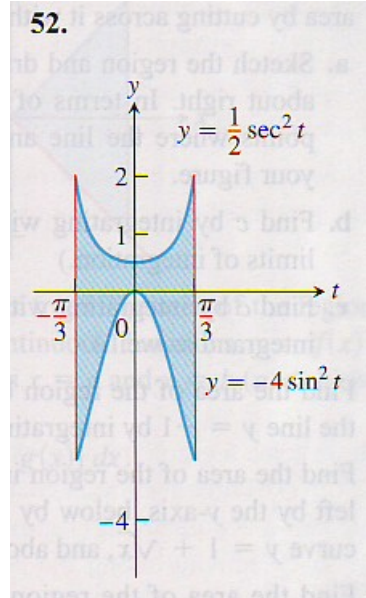   (a) $\displaystyle\int_0^{\sqrt[3]{\pi^2}} \sqrt{\theta} \, \cos^2\left(\theta^{3/2}\right) d\theta$

   (b) $\displaystyle\int_{-1}^{-1/2} t^{-2} \sin^2\left(1 + \frac{1}{t}\right) dt$

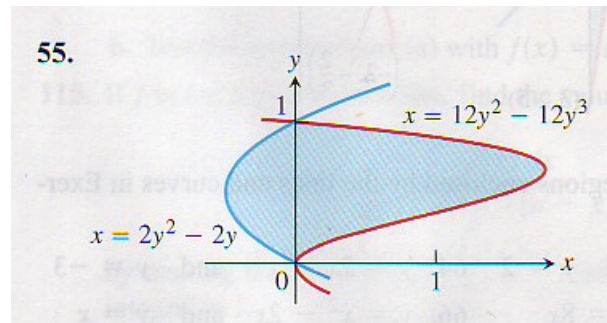5. Find $\displaystyle\int_2^4 \frac{dx}{x \ln x}$.

6. Find $\displaystyle\int_0^{\ln \sqrt{3}} \frac{e^x \, dx}{1 + e^{2x}}$.

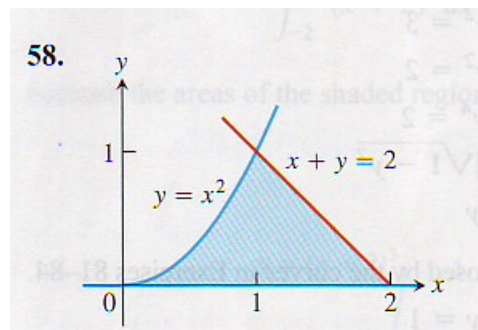7. Find the area between the graph of $y = x \sqrt{4 - x^2}$ and the x-axis.

8. Find the total area of the shaded region:

52.

$$y = \frac{1}{2}\sec^2 t$$

$$y = -4\sin^2 t$$

9. Find the total area of the shaded region:

55.

$$x = 12y^2 - 12y^3$$

$$x = 2y^2 - 2y$$

10. Find the total area of the shaded region:

58.

$$x + y = 2$$

$$y = x^2$$

2

11. Find the area between the graphs of $y = 2x - x^2$ and $y = -3$.

12. Find the area between the graphs of $y = \sqrt{|x|}$ and $5y = x + 6$ (How many intersection points are there?).

13. Find the area between the graphs of $y = 3 - x^2$ and $y = -1$.

14. Suppose that $F(x)$ is an antiderivative of $f(x) = (\sin x)/x$, $x > 0$. Express
$$\int_1^3 \frac{\sin(2x)}{x} \, dx$$
in terms of F.

15. (a) Show that if f is odd on $[-a, a]$ then $\int_{-a}^{a} f(x) \, dx = 0$.

    (b) Test the result in part (a) with $f(x) = \sin x$ and $a = \pi/2$.

16. A basic property of definite integrals is their invariance under translation, as expressed by the equation
$$\int_a^b f(x) \, dx = \int_{a-c}^{b-c} f(x + c) \, dx. \tag{1}$$

Use a substitution to verify Equation (1).