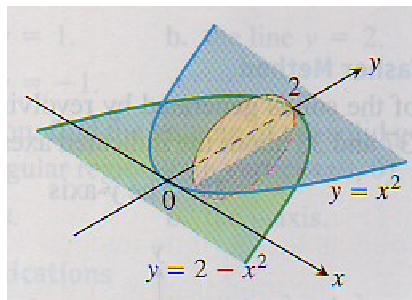
Math 21B-B - Homework Set 5

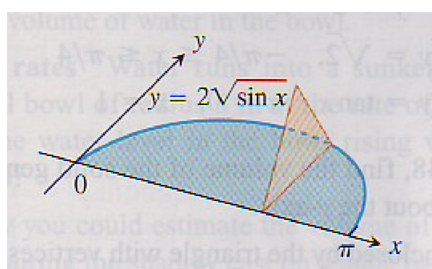
Section 6.1:

1. Find the volume of the solid that lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the x-axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$.



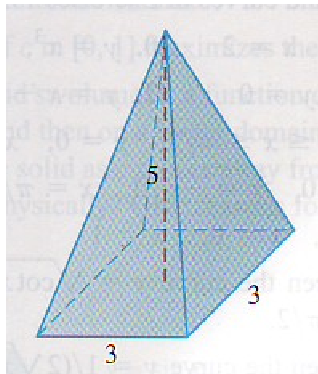
2. Find the volume of the following solid: the base of a solid is the region between the curve $y = 2\sqrt{\sin(x)}$ and the interval $[0, \pi]$ on the x-axis. The cross-sections perpendicular to the x-axis are

   (a) equilateral triangles with bases running from the x-axis to the curve as shown in the accompanying figure.

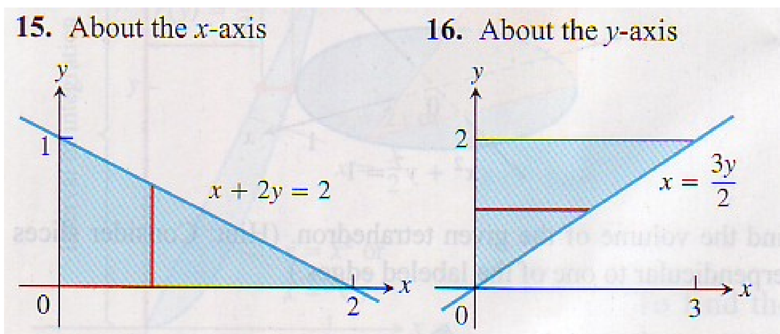


   (b) squares with bases running from the x-axis to the curve.

3. Find the volume of the given pyramid, which has a square base of area 9 and height 5.



4-5. (See Figures 15 and 16 below.) In each case, find the volume of the solid generated by revolving the shaded region about the given axis.



6. Find the volume of the solid generated by revolving the region bounded by $y = x^3$, $y = 0$, and $x = 2$ about the x-axis.

7. Find the volume of the solid generated by revolving the region about the given line where the region is the region in the first quadrant bounded above by the line $y = \sqrt{2}$, below by the curve $y = \sec(x)\tan(x)$, and on the left by the y-axis, about the line $y = \sqrt{2}$.

8. Find the volume of the solid generated by revolving the region bounded by $x = \sqrt{5}y^2$, $x = 0$, $y = -1$, $y = 1$ about the y-axis.

9. Find the volume of the solid generated by revolving the region bounded by $x = \sqrt{2y}/(y^2+1)$, $x = 0$, $y = 1$ about the y-axis.

10. Find the volume of the solid generated by revolving the region bounded by $y = x$, $y = 1$, $x = 0$ about the x-axis.

11. Find the volume of the solid generated by revolving the region bounded by $y = \sec(x)$, $y = \tan(x)$, $x = 0$, $x = 1$ about the x-axis.

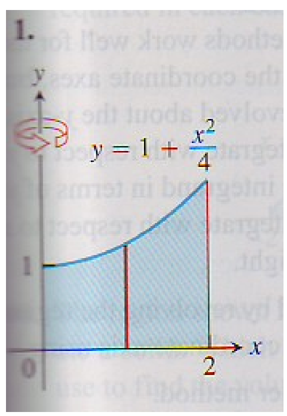
12. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about
    (a) the x-axis.
    (b) the y-axis.
    (c) the line $y = 2$.
    (d) the line $x = 4$.

13. **The volume of a torus** The disk $x^2 + y^2 \leq a^2$ is revolved about the line $x = b$ $(b > a)$ to generate a solid shaped like a doughnut and called a *torus*. Find its volume. (*Hint:* $\int_{-a}^{a} \sqrt{a^2 - y^2} dy = \pi a^2/2$, since it is the area of a semicircle of radius a.)

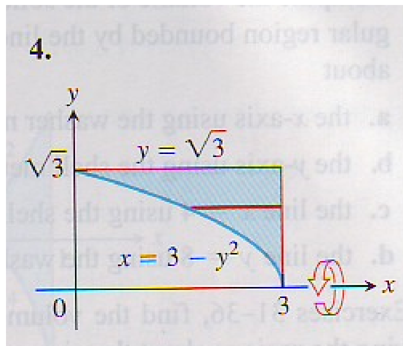
Section 6.2

In exercises 1-2, use the shell method to find the volumes of the solids generated by revolving the shaded region about the indicated axis.

1.



2.



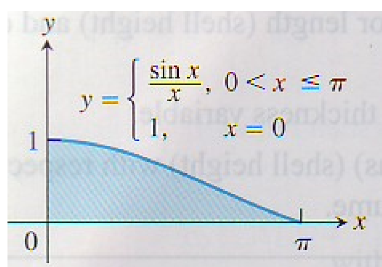
3. Use the shell method to find the volume of the solid generated by revolving the region $y = 2x$, $y = x/2$, $x = 1$ about the y-axis.

4. Let

$$f(x) = \begin{cases} \frac{\sin(x)}{x} & 0 < x \leq \pi \\ 1 & x = 0 \end{cases}$$

   (a) Show that $xf(x) = \sin(x)$, $0 \leq x \leq \pi$.

   (b) Find the volume of the solid generated by revolving the shaded region about the y-axis in the accompanying figure.



For problems 5-6, use the shell method to find the volumes of the solids generated by revolving the regions bounded by the curves and lines about the x-axis.

5. $x = y^2$, $x = -y$, $y = 2$, $y \geq 0$.

6. $y = |x|$, $y = 1$.

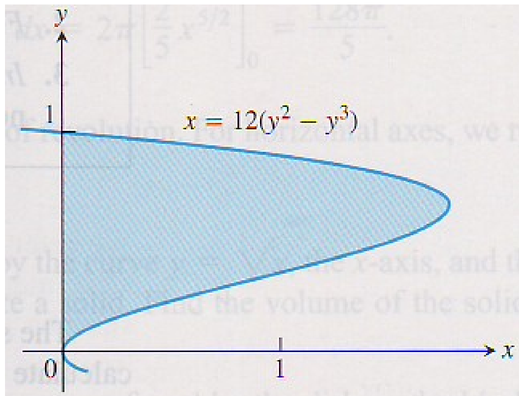
7. Use the shell method to find the volumes of the solids generated by revolving the shaded regions about the indicated axes.

   (a) The x-axis

   (b) The line $y = 1$

(c) The line $y = 8/5$

(d) The line $y = -2/5$



8. Find the volume of the solid generated by revolving the region enclosed by the graphs of $y = e^{x/2}$, $y = 1$, and $x = \ln(3)$ about the x-axis.

Section 6.3

1. Find the length of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$.

2. Find the length of the curve $x = \frac{y^3}{6} + \frac{1}{2y}$ from $y = 2$ to $y = 3$.

3. Find the length of the curve $x = \displaystyle\int_0^y \sqrt{\sec^4 t - 1}\, dt \quad \text{for} \quad -\frac{\pi}{4} \le y \le \frac{\pi}{4}$.