Math 21B-B - Homework Set 5

Section 6.1:

1. Let's consider the area of a cross-section of the solid when $-1 \leq x \leq 1$. The diameter of the circular cross-section at x is given by

$$\text{DIAMETER} = \left(2 - x^2\right) - x^2 = 2 - 2x^2.$$

Thus, the area of the cross-section is given by

$$\text{AREA} = \pi \cdot \left[\tfrac{1}{2}\left(2 - 2x^2\right)\right]^2 = \pi \cdot \left(1 - x^2\right)^2 = \pi \cdot \left(1 - 2x^2 + x^4\right).$$

$$\begin{aligned}
\text{VOLUME} &= \int_{-1}^{1} \pi \cdot \left(1 - 2x^2 + x^4\right)\, dx \\
&= \pi \int_{-1}^{1} \left(1 - 2x^2 + x^4\right)\, dx \\
&= \pi \cdot \left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right)\Bigg|_{-1}^{1} \\
&= \pi \cdot \left(1 - \frac{2}{3} + \frac{1}{5}\right) - \pi \cdot \left(-1 + \frac{2}{3} - \frac{1}{5}\right) \\
&= \pi \cdot \left(2 - \frac{4}{3} + \frac{2}{5}\right) \\
&= \frac{16}{15}\pi
\end{aligned}$$

2. a. Let's consider the area of a cross-section of the solid when $0 \leq x \leq \pi$. The base of the triangular cross-section at x is given by

$$\text{BASE} = 2\sqrt{\sin x}.$$

The height of the triangle is given by

$$\text{HEIGHT} = \sqrt{3} \cdot \sqrt{\sin x}. \quad (\textit{Use properties of equilateral triangles})$$

Therefore, the area of the cross-section is given by

$$\text{AREA} = \left(\tfrac{1}{2}\right)\left(2\sqrt{\sin x}\right)\left(\sqrt{3}\sqrt{\sin x}\right) = \sqrt{3}\sin x.$$

$$
\begin{aligned}
\text{VOLUME} &= \int_0^{\pi} \sqrt{3}\sin x\,dx \\
&= -\sqrt{3}\cos x\Big|_0^{\pi} \\
&= \sqrt{3} - (-\sqrt{3}) \\
&= 2\sqrt{3}
\end{aligned}
$$

b. Let's consider the area of a cross-section of the solid when $0 \le x \le \pi$. The base of the square cross-section at x is given by

$\text{BASE} = 2\sqrt{\sin x}.$

Therefore, the area of the cross-section is given by

$\text{AREA} = (2\sqrt{\sin x})^2 = 4\sin x.$

$$
\begin{aligned}
\text{VOLUME} &= \int_0^{\pi} 4\sin x\,dx \\
&= -4\cos x|_0^{\pi} \\
&= 4 - (-4) \\
&= 8
\end{aligned}
$$

3. We divide the pyramid into cross-sections parallel to the base. The cross-section at distance x from the tip of the pyramid is a square whose sides have length $3x/5$ and area

$$A(x) = \left(\frac{3x}{5}\right)^2 = \frac{9}{25}\,x^2.$$

Hence the volume is

$$
\begin{aligned}
\int_0^5 A(x)\,dx &= \int_0^5 \frac{9}{25}\,x^2\,dx \\
&= \frac{3}{25}x^3\Big]_0^5 \\
&= 15\,.
\end{aligned}
$$

4. We are rotating about the x-axis, so we want to take the integral with respect to x. We want $0 \le x \le 2$. The radius of a cross-section is given by

$\text{RADIUS} = -\frac{1}{2}x + 1$ *(Between graph and x-axis)*

Therefore, the area of the cross-section is given by

$\textsc{Area} = \pi \cdot \left(-\frac{1}{2}x + 1\right)^2 = \pi \cdot \left(\frac{1}{4}x^2 - x + 1\right)$

$$\begin{aligned}\textsc{Volume} &= \int_0^2 \pi \cdot \left(\frac{1}{4}x^2 - x + 1\right)\,dx \\ &= \pi \cdot \left(\frac{1}{12}x^3 - \frac{1}{2}x^2 + x\right)\Big|_0^2 \\ &= \pi \cdot \left(\frac{8}{12} - 2 + 2\right) \\ &= \frac{2\pi}{3}\end{aligned}$$

5. We are rotating about the y-axis, so we want to take the integral with respect to y. We want $0 \le y \le 2$. The radius of the cross-section is given by

   $\textsc{Radius} = \frac{3y}{2}$ *(Between the graph and the y-axis)*

   Therefore, the area of the cross-section is given by

   $\textsc{Area} = \pi \cdot \left(\frac{3y}{2}\right)^2 = \pi \cdot \frac{9y^2}{4}$

$$\begin{aligned}\textsc{Volume} &= \int_0^2 \pi \cdot \frac{9y^2}{4}\,dy \\ &= \pi \cdot \frac{3y^3}{4}\Big|_0^2 \\ &= 6\pi\end{aligned}$$

6. We are rotating about the x-axis, so we want to take the integral with respect to y. We want $0 \le x \le 2$. The radius of the cross-section is given by

   $\textsc{Radius} = x^3$

   Therefore, the area of the cross-section is given by

   $\textsc{Area} = \pi\left(x^3\right)^2 = \pi \cdot x^6$

$$
\begin{aligned}
\text{VOLUME} &= \int_0^2 \pi \cdot x^6 \, dx \\
&= \frac{\pi}{7} x^7 \Big|_0^2 \\
&= \frac{128\pi}{7}
\end{aligned}
$$

7. We are rotating about the line $y = \sqrt{2}$, so we want to take the integral with respect to x. We want $0 \le x \le \sqrt{2}$. The radius of the cross-section is given by

$$\text{RADIUS} = \sqrt{2} - \sec x \tan x \qquad (\textit{Between } y = \sqrt{2} \textit{ and } y = \sec x \tan x)$$

$$\text{AREA} = \pi \cdot (\sqrt{2} - \sec x \tan x)^2 = \pi \cdot \left(2 - 2\sqrt{2} \sec x \tan x + \sec^2 x \tan^2 x\right)$$

$$
\begin{aligned}
\text{VOLUME} &= \int_0^{\pi/4} \pi \cdot \left(2 - 2\sqrt{2} \sec x \tan x + \sec^2 x \tan^2 x\right) \, dx \\
&= \pi \cdot \left(2x - 2\sqrt{2} \sec x + \frac{1}{3} \tan^3 x\right) \Bigg|_0^{\pi/4} \qquad (\textit{Use } u\textit{-substitution}) \\
&= \pi \cdot \left(\frac{\pi}{2} - 2\sqrt{2} \cdot \sqrt{2} + \frac{1}{3}\right) - \pi \cdot \left(-2\sqrt{2}\right) \\
&= \pi \cdot \left(\frac{\pi}{2} - 4 + \frac{1}{3} + 2\sqrt{2}\right) \\
&= \pi \cdot \left(\frac{\pi}{2} - \frac{11}{3} + 2\sqrt{2}\right)
\end{aligned}
$$

8. $\text{RADIUS} = \sqrt{5}\, y^2$

$$\text{AREA} = \pi \cdot \left(\sqrt{5}\, y^2\right)^2 = 5\pi y^4$$

$$
\begin{aligned}
\text{VOLUME} &= \int_{-1}^{1} 5\pi y^4 \, dy \\
&= \pi y^5 \Big|_{-1}^{1} \\
&= \pi - (-\pi) \\
&= 2\pi
\end{aligned}
$$

9. $\text{RADIUS} = \frac{\sqrt{2y}}{y^2+1}$

$\text{AREA} = \pi \cdot \left(\frac{\sqrt{2y}}{y^2+1}\right)^2 = \pi \cdot \frac{2y}{(y^2+1)^2}$

$$
\begin{aligned}
\text{VOLUME} &= \int_0^1 \pi \cdot \frac{2y}{(y^2+1)^2}\,dy \\
&= \int_1^2 \pi \cdot \frac{1}{u^2}\,du \\
&= \pi \cdot -\frac{1}{u}\bigg|_1^2 \\
&= -\frac{\pi}{2} + \pi \\
&= \frac{\pi}{2}
\end{aligned}
$$

10. To find the volume of this solid, we are going to use the method of washers. We have outer radius $R(x) = 1$ and inner radius $r(x) = x$.

$$
\begin{aligned}
\text{VOLUME} &= \int_0^1 \pi \cdot \left[R(x)^2 - r(x)^2\right]\,dx \\
&= \int_0^1 \pi \cdot \left(1 - x^2\right)\,dx \\
&= \pi \cdot \left(x - \frac{1}{3}x^3\right)\bigg|_0^1 \\
&= \pi \cdot \left(1 - \frac{1}{3}\right) - \pi \cdot \left(0 - \frac{1}{3} \cdot 0\right) \\
&= \frac{2\pi}{3}
\end{aligned}
$$

11. We have the outer radius $R(x) = \sec x$ and inner radius $r(x) = \tan x$.

$$
\begin{aligned}
\text{VOLUME} &= \int_0^1 \pi \cdot \left(\sec^2 x - \tan^2 x\right)\,dx \\
&= \int_0^1 \pi\,dx \qquad\qquad (\sec^2 x = 1 + \tan^2 x) \\
&= (\pi x)\big|_0^1 \\
&= \pi
\end{aligned}
$$

12. a. Rotating about the x-axis

To find the volume of the solid, we will use the method of washers. The outer radius $R = 2$ and the inner radius $r = \sqrt{x}$.

$$
\begin{aligned}
\text{VOLUME} &= \int_0^4 \pi \cdot \left(2^2 - \left(\sqrt{x}\right)^2\right)\, dx \\
&= \pi \int_0^4 4 - x\, dx \\
&= \pi \cdot \left(4x - \frac{1}{2}x^2\right)\Big|_0^4 \\
&= \pi \cdot \left(16 - \frac{1}{2} \cdot 16\right) \\
&= 8\pi
\end{aligned}
$$

b. Rotating about the y-axis

In this case we will want to integrate with respect to y. Therefore we need to rewrite the the functions with y as the variable. This gives $0 \le y \le 2$ and the radius $r = y^2$.

$$
\begin{aligned}
\text{VOLUME} &= \int_0^2 \pi \cdot \left(y^2\right)^2\, dy \\
&= \pi \int_0^2 y^4\, dy \\
&= \frac{\pi}{5} y^5\Big|_0^2 \\
&= \frac{32\pi}{5}
\end{aligned}
$$

c. Rotating about the line $y = 2$

In this case, we are integrating with respect to x. We have $0 \le x \le 4$. The radius is the distance between the line $y = 2$ and the graph $y = \sqrt{x}$. Therefore, the radius is given by $r = 2 - \sqrt{x}$.

$$
\begin{aligned}
\text{VOLUME} &= \int_0^4 \pi \cdot \left(2-\sqrt{x}\right)^2 \, dx \\
&= \pi \int_0^4 4 - 4\sqrt{x} + x \, dx \\
&= \pi \cdot \left(4x - \frac{8}{3}x^{3/2} + \frac{1}{2}x^2\right)\Bigg|_0^4 \\
&= \pi \cdot \left(16 - \frac{8}{3}\cdot 8 + \frac{1}{2}\cdot 16\right) \\
&= \frac{8\pi}{3}
\end{aligned}
$$

d. Rotating about the line $x = 4$

In this case we are going to use the washer method, and we will integrate with respect to y. We have $0 \leq y \leq 2$. The outer radius is $R = 4$ and the inner radius is $r = 4 - y^2$.

$$
\begin{aligned}
\text{VOLUME} &= \int_0^2 \pi \cdot \left[4^2 - \left(4-y^2\right)\right] \, dy \\
&= \pi \int_0^2 16 - \left(16 - 8y^2 + y^4\right) \, dy \\
&= \pi \int_0^2 8y^2 - y^4 \, dy \\
&= \pi \cdot \left(\frac{8}{3}y^3 - \frac{1}{5}y^5\right)\Bigg|_0^2 \\
&= \pi \cdot \left(\frac{64}{3} - \frac{32}{5}\right) \\
&= \frac{224\pi}{15}
\end{aligned}
$$

13. We start with the disc given by $x^2 + y^2 \leq a^2$, and we want to revolve it around the line $x = b$ for $b > a$. We will use the method of washers, integrating with respect to y where $-a \leq y \leq a$. The outer radius is $R = b - \left(-\sqrt{a^2 - y^2}\right) = b + \sqrt{a^2 - y^2}$ and the inner radius is given by $r = b - \sqrt{a^2 - y^2}$.

$$
\begin{aligned}
\text{VOLUME} &= \int_{-a}^{a} \pi \cdot \left[\left(b + \sqrt{a^2 - y^2}\right)^2 - \left(b - \sqrt{a^2 - y^2}\right)^2 \right] dy \\
&= \pi \int_{-a}^{a} \left[\left(b^2 + 2b\sqrt{a^2 - y^2} + (a^2 - y^2)\right) - \left(b^2 - 2b\sqrt{a^2 - y^2} + (a^2 - y^2)\right) \right] dy \\
&= \pi \int_{-a}^{a} 4b\sqrt{a^2 - y^2}\, dy \\
&= 4b\pi \int_{-a}^{a} \sqrt{a^2 - y^2}\, dy \\
&= 4b\pi \cdot \frac{\pi a^2}{2} \\
&= 2a^2 b\pi^2
\end{aligned}
$$

Section 6.2

1. We are going to integrate with respect to x where $0 \le x \le 2$. We have the shell radius $r = x$ and the shell height $h = 1 + \frac{x^2}{4}$.

$$
\begin{aligned}
\text{VOLUME} &= \int_0^2 2\pi r h\, dx \\
&= 2\pi \int_0^2 x \left(1 + \frac{x^2}{4}\right) dx \\
&= 2\pi \int_0^2 x + \frac{x^3}{4}\, dx \\
&= 2\pi \cdot \left(\frac{1}{2}x^2 + \frac{x^4}{16}\right)\Big|_0^2 \\
&= 2\pi \cdot \left(\frac{1}{2} \cdot 4 + \frac{1}{16} \cdot 16\right) \\
&= 6\pi
\end{aligned}
$$

2. We are going to integrate with respect to y where $0 \le y \le \sqrt{3}$. We have the shell radius $r = y$ and the shell height $h = 3 - (3 - y^2) = y^2$.

$$
\begin{aligned}
\textsc{Volume} &= \int_0^{\sqrt{3}} 2\pi \cdot y \cdot y^2 \, dy \\
&= 2\pi \int_0^{\sqrt{3}} y^3 \, dy \\
&= 2\pi \cdot \frac{1}{4} y^4 \Big|_0^{\sqrt{3}} \\
&= 2\pi \cdot \frac{9}{4} \\
&= \frac{9\pi}{2}
\end{aligned}
$$

3. We are going to integrate with respect to x where $0 \leq x \leq 1$. We have the shell radius $r = x$ and the shell height $h = 2x - \frac{x}{2} = \frac{3x}{2}$.

$$
\begin{aligned}
\textsc{Volume} &= \int_0^{1} 2\pi x \left(\frac{3x}{2}\right) dx \\
&= 2\pi \int_0^{1} \frac{3x^2}{2} \, dx \\
&= 2\pi \cdot \frac{x^3}{2} \Big|_0^{1} \\
&= 2\pi \cdot \frac{1}{2} \\
&= \pi
\end{aligned}
$$

4. Let $f(x) = \begin{cases} (\sin x)/x & 0 < x \leq \pi \\ 1 & x = 0 \end{cases}$

   a. Show that $xf(x) = \sin(x)$ for $0 \leq x \leq \pi$.

      Begin by looking at $xf(x) = \begin{cases} \sin x & 0 < x \leq \pi \\ x & x = 0 \end{cases}$.

      Notice that $\sin x = x$ when $x = 0$.

      Thus $xf(x) = \begin{cases} \sin x & 0 < x \leq \pi \\ \sin x & x = 0 \end{cases} = \sin x$

   b. We are going to integrate with respect to x where $0 \leq x \leq \pi$. We have the shell radius $r = x$ and the shell height is given by $h = f(x)$.

$$\begin{aligned}
\textsc{Volume} &= \int_0^{\pi} 2\pi x f(x)\, dx \\
&= 2\pi \int_0^{\pi} \sin x\, dx \qquad\qquad (\textit{by part a}) \\
&= 2\pi \cdot (-\cos x)\big|_0^{\pi} \\
&= 2\pi[-1-(-1)] \\
&= 4\pi
\end{aligned}$$

5. We are going to integrate with respect to y where $0 \le y \le 2$. We have the shell radius $r = y$ and the shell height is given by $h = y^2 - (-y) = y^2 + y$.

$$\begin{aligned}
\textsc{Volume} &= \int_0^{2} 2\pi \cdot y\left(y^2+y\right)\, dy \\
&= 2\pi \int_0^{2} y^3 + y^2\, dy \\
&= 2\pi \cdot \left(\frac{1}{4}y^4 + \frac{1}{3}y^3\right)\Bigg|_0^2 \\
&= 2\pi \cdot \left(4 + \frac{8}{3}\right) \\
&= \frac{40\pi}{3}
\end{aligned}$$

6. We are going to integrate with respect to y where $0 \le y \le 1$. We have the shell radius $r = y$ and the shell height is given by $h = y - (-y) = 2y$.

$$\begin{aligned}
\textsc{Volume} &= \int_0^{1} 2\pi \cdot (y)(2y)\, dy \\
&= 2\pi \int_0^{1} 2y^2\, dy \\
&= 2\pi \cdot \frac{2}{3}y^3\Bigg|_0^1 \\
&= 2\pi \cdot \frac{2}{3} \\
&= \frac{4\pi}{3}
\end{aligned}$$

7. In each part, we are going to integrate with respect to y where $0 \leq y \leq 1$. The shell height will always be $h = 12y^2 - 12y^3$, and the shell radius will change depending on which horizontal line we use.

   a. In this case we have the shell radius $r = y$ (the distance between y and the x-axis).

$$\begin{aligned}
\text{VOLUME} &= \int_0^1 2\pi \cdot y \left(12y^2 - 12y^3\right)\, dy \\
&= 2\pi \int_0^1 12y^3 - 12y^4\, dy \\
&= 2\pi \cdot \left(3y^4 - \frac{12}{5}y^5\right)\Big|_0^1 \\
&= 2\pi \left(3 - \frac{12}{5}\right) \\
&= \frac{6\pi}{5}
\end{aligned}$$

   b. In this case we have the shell radius $r = 1 - y$ (the distance between the line $y = 1$ and y).

$$\begin{aligned}
\text{VOLUME} &= \int_0^1 2\pi(1 - y)\left(12y^2 - 12y^3\right)\, dy \\
&= 2\pi \int_0^1 12y^2 - 24y^3 + 12y^4\, dy \\
&= 2\pi \cdot \left(4y^3 - 6y^4 + \frac{12}{5}y^5\right)\Big|_0^1 \\
&= 2\pi \cdot \left(4 - 6 + \frac{12}{5}\right) \\
&= \frac{4\pi}{5}
\end{aligned}$$

   c. In this case we have the shell radius $r = \frac{8}{5} - y$ (the distance between the line $y = \frac{8}{5}$ and y).

$$\begin{aligned}
\text{VOLUME} &= \int_0^1 2\pi \left(\frac{8}{5} - y\right)\left(12y^2 - 12y^3\right)\, dy \\
&= 2\pi \int_0^1 \frac{96}{5}y^2 - \frac{156}{5}y^3 + 12y^4\, dy \\
&= 2\pi \left(\frac{32}{5}y^3 - \frac{39}{5}y^4 + \frac{12}{5}y^5\right)\Bigg|_0^1 \\
&= 2\pi \left(\frac{32}{5} - \frac{39}{5} + \frac{12}{5}\right) \\
&= 2\pi
\end{aligned}$$

d. In this case we have the shell radius $r = y - \left(-\frac{2}{5}\right) = y + \frac{2}{5}$ (the distance between y and the line $y = -\frac{2}{5}$)

$$\begin{aligned}
\text{VOLUME} &= \int_0^1 2\pi \left(y + \frac{2}{5}\right)\left(12y^2 - 12y^3\right)\, dy \\
&= 2\pi \int_0^1 \frac{24}{5}y^2 + \frac{36}{5}y^3 - 12y^4\, dy \\
&= 2\pi \left(\frac{8}{5}y^3 + \frac{9}{5}y^4 - \frac{12}{5}y^5\right)\Bigg|_0^1 \\
&= 2\pi \left(\frac{8}{5} + \frac{9}{5} - \frac{12}{5}\right) \\
&= 2\pi
\end{aligned}$$

8. Suppose we try to use the method of shells. We would want to integrate with respect to y where $1 \leq y \leq \sqrt{3}$. The shell radius $r = y$ and the shell height $h = \ln 3 - \ln y^2 = \ln\left(3 - y^2\right)$.

$$\begin{aligned}
\text{VOLUME} &= \int_1^{\sqrt{3}} 2\pi y \ln\left(3 - y^2\right)\, dy \\
&= \int_2^0 -\pi \ln u\, du \\
&= \int_0^2 \pi \ln u\, du
\end{aligned}$$

Notice, that we do not want to take the integral of $\ln x$, so let's rethink our choice of method.

Suppose we try the method of washers. In this case we will integrate with respect to x where $0 \leq x \leq \ln 3$. The outer radius $R = e^{x/2}$ and the inner radius $r = 1$.

$$
\begin{aligned}
\text{VOLUME} &= \int_0^{\ln 3} \pi \left(\left(e^{x/2}\right)^2 - 1^2 \right) dx \\
&= \pi \int_0^{\ln 3} e^x - 1\, dx \\
&= \pi \left(e^x - x\right)\Big|_0^{\ln 3} \\
&= \pi \left[(3 - \ln 3) - (1 - 0)\right] \\
&= \pi(2 - \ln 3)
\end{aligned}
$$

Section 6.3

1. $y = \frac{1}{3}(x^2+2)^{3/2}$ from $x = 0$ to $x = 3$.

$$
\frac{dy}{dt} = \frac{1}{3} \cdot \frac{3}{2}(x^2+2)^{1/2} \cdot 2x = x\sqrt{x^2+2}
$$

$$
\begin{aligned}
L &= \int_0^3 \sqrt{1 + \left(\frac{dy}{dt}\right)^2}\, dt \\
&= \int_0^3 \sqrt{1 + \left(x\sqrt{x^2+2}\right)^2}\, dt \\
&= \int_0^3 \sqrt{1 + x^2\left(x^2+2\right)}\, dt \\
&= \int_0^3 \sqrt{x^4 + 2x^2 + 1}\, dt \\
&= \int_0^3 \sqrt{\left(x^2+1\right)^2}\, dt \\
&= \int_0^3 x^2 + 1\, dt \\
&= \left(\frac{1}{3}x^3 + x\right)\Big|_0^3 \\
&= 9 + 3 \\
&= 12
\end{aligned}
$$

2. $x = \frac{y^3}{6} + \frac{1}{2y}$ from $y = 2$ to $y = 3$.

$$\frac{dx}{dt} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4 - 1}{2y^2}$$

$$\begin{aligned}
L &= \int_2^3 \sqrt{1 + \left(\frac{dx}{dt}\right)^2}\, dt \\
&= \int_2^3 \sqrt{1 + \left(\frac{y^4 - 1}{2y^2}\right)^2}\, dt \\
&= \int_2^3 \sqrt{1 + \frac{y^8 - 2y^4 + 1}{4y^4}}\, dt \\
&= \int_2^3 \sqrt{\frac{y^8 + 2y^4 + 1}{4y^4}}\, dt \\
&= \int_2^3 \sqrt{\left(\frac{y^4 + 1}{2y^2}\right)^2}\, dt \\
&= \int_2^3 \frac{y^4 + 1}{2y^2}\, dt \\
&= \int_2^3 \frac{y^2}{2} + \frac{1}{2y^2}\, dt \\
&= \left(\frac{y^3}{6} - \frac{1}{2y}\right)\Bigg|_2^3 \\
&= \left(\frac{9}{2} - \frac{1}{6}\right) - \left(\frac{4}{3} - \frac{1}{4}\right) \\
&= \frac{13}{4}
\end{aligned}$$

18. $x = \displaystyle\int_0^y \sqrt{\sec^4 t - 1}\, dt \quad \text{for} \quad -\tfrac{\pi}{4} \leq y \leq \tfrac{\pi}{4}.$

$$\frac{dx}{dy} = \sqrt{\sec^4 y - 1}$$

$$
\begin{aligned}
L &= \int_{-\pi/4}^{\pi/4} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \\
&= \int_{-\pi/4}^{\pi/4} \sqrt{1 + \sec^4 y - 1}\, dy \\
&= \int_{-\pi/4}^{\pi/4} \sqrt{\sec^4 y}\, dy \\
&= \int_{-\pi/4}^{\pi/4} \sec^2 y\, dy \\
&= \tan y|_{-\pi/4}^{\pi/4} \\
&= 1 - (-1) \\
&= 2
\end{aligned}
$$