$$\int_N^P g^2(t)\,dt = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\int_N^P \frac{\sin^2 \pi t}{(t+n)(t+m)}\frac{1}{n^{\frac{1}{2}+\epsilon}}\frac{1}{m^{\frac{1}{2}+\epsilon}}\,dt$$

$$> \frac{1}{2}\sum_1^{\infty}\sum_1^{\infty}\int_{N+\frac{1}{2}}^{P-\frac{1}{2}} \frac{dt}{(t+n)(t+m)}\frac{1}{n^{\frac{1}{2}+\epsilon}}\frac{1}{m^{\frac{1}{2}+\epsilon}},$$

(cf. Ref. 9, p. 309)

$$> \frac{1}{2}\int_{N+\frac{1}{2}}^{P-\frac{1}{2}} dt \int_1^{\infty} dt \int_1^{\infty} dy\, \frac{1}{(t+x)(t+y)x^{\frac{1}{2}+\epsilon}y^{\frac{1}{2}+\epsilon}},$$

since $1/[x^{1+\epsilon}(t+x)]$ is monotone decreasing in x. Now

$$\int_1^{\infty}\frac{dx}{x^{\frac{1}{2}+\epsilon}(t+x)} = \frac{1}{t^{\frac{1}{2}+\epsilon}}\int_{1/t}^{\infty}\frac{du}{u^{\frac{1}{2}+\epsilon}(u+1)}$$

$$= \frac{1}{t^{\frac{1}{2}+\epsilon}}\left[\int_0^{\infty}\frac{du}{u^{\frac{1}{2}+\epsilon}(u+1)} - \int_0^{1/t}\frac{du}{u^{\frac{1}{2}+\epsilon}(u+1)}\right]$$

$$> \frac{1}{t^{\frac{1}{2}+\epsilon}}\left[\frac{\pi}{\sin\frac{\pi}{2}(1+2\epsilon)} - \int_0^{1/(N+\frac{1}{2})}\frac{du}{u^{\frac{1}{2}+\epsilon}(u+1)}\right]$$

for $t > N + \frac{1}{2}$

$$= \frac{1}{t^{\frac{1}{2}+\epsilon}}[\pi + 0(\epsilon) + 0(N^{-\frac{1}{2}})].$$

Therefore

$$\int_N^P g^2(t)\,dt > \frac{\pi^2}{2}\int_{N+\frac{1}{2}}^{P-\frac{1}{2}}\frac{dt}{t^{1+2\epsilon}}(1 + 0(\epsilon) + 0(N^{-\frac{1}{2}}) + 0(P^{-\frac{1}{2}}))$$

$$= \frac{\pi^2}{2}\left[\frac{1}{2\epsilon}\left(\frac{1}{N^{2\epsilon}} - \frac{1}{P^{2\epsilon}}\right)\right](1 + 0(\epsilon) + 0(N^{-\frac{1}{2}}) + 0(P^{-\frac{1}{2}})),$$

whence

$$\frac{\int_N^P g^2(t)\,dt}{\int_{-\infty}^{\infty} g^2(t)\,dt} = \frac{1}{2}\left(\frac{1}{N^{2\epsilon}} - \frac{1}{P^{2\epsilon}}\right)$$

plus smaller terms. If $P = N^{\alpha}$ for $\alpha > 1$, and $\epsilon = \log\alpha/[(\alpha-1)\log N]$, then

$$\frac{1}{N^{2\epsilon}} - \frac{1}{P^{2\epsilon}} = \alpha^{-2/(\alpha-1)} - \alpha^{-2\alpha/(\alpha-1)} > 0.$$

Hence