$$\frac{\int_N^{N^\alpha} g^2(t)\,dt}{\int_{-\infty}^{\infty} g^2(t)\,dt}$$

remains bounded from 0 as N becomes large, by some k (depending on α), so that

$$\int_{-\infty}^{\infty} g^2(t)\,dt - \int_{t \notin (N,N^\alpha)} g^2(t)\,dt > k \int_{-\infty}^{\infty} g^2(t)\,dt,$$

or

$$\int_{-\infty}^{\infty} g^2(t)\,dt > \frac{1}{1-k} \int_{t \notin (N,N^\alpha)} g^2(t)\,dt. \tag{11.1}$$

But $g(t) = g(t) - \sum_1^\infty g(n)[\sin \pi(t-n)]/[\pi(t-n)]$ since $g(n) = 0$, $n = 1,2,3, \cdots$. Therefore, letting

$$f(t) = g\left(t + \left[\frac{N^\alpha + N}{2}\right]\right),$$

we find, using (11.1),

$$\int_{-\infty}^{\infty} f^2(t)\,dt = \int_{-\infty}^{\infty} \left| f(t) - \sum_{|n| \leqq [(N^\alpha+N)/2]} f(n) \frac{\sin \pi(t-n)}{\pi(t-n)} \right|^2 dt$$

$$> \frac{1}{1-k} \int_{|t| > [(N^\alpha - N)/2]} f^2(t)\,dt.$$

If we now think of $[(N^\alpha - N)/2]$ as T, then $[(N^\alpha + N)/2]$ behaves like $T^\beta + T$, for $\beta = 1/\alpha$, and with $\delta = (1-k)^{-1} - 1$, Theorem 11 is proved.

12. We turn next to the case in which $f(t)$ is neither precisely band-limited nor precisely time-limited.

Theorem 12: If $f(t) \in \mathfrak{L}^2$ with $\| f \| = 1$, and if

$$\| Df \|^2 = 1 - \epsilon_T^2$$

and

$$\| Bf \|^2 = 1 - \eta_W^2,$$

then for some constants $a_n = a_n(f)$,

$$\| f - \sum_0^{[2WT]} a_n \psi_n \|^2 \leqq 12(\epsilon_T + \eta_W)^2 + \eta_W^2.$$