LOCAL RADON TRANSFORMS VIA GENERALIZED DECONVOLUTION

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ABSTRACT

The Radon transform is a powerful method that has been used to filter coherent noise from seismic records and to reconstruct seismic data. In addition, it has a long history in image processing as a tool for feature extraction. An important shortcoming in exploration seismology, however, is the requirement of simple integration paths that often do not match well enough the spatio-temporal structure of seismic waveforms. The latter can be avoided by adopting local Radon transforms.

We show that local Radon transforms (with arbitrary integration paths) can be implemented with a generalized deconvolution approach. In this case, the data are represented via the convolution of Local Wavefield Operators (LWO) and an ensemble of filters that represent the local Radon panels. Deconvolving the LWOs from the data yields the unknown suite of filters that can be used to reconstruct seismic data, reject coherent noise, and analyze seismic waveforms.

This paper also discusses two regularization strategies for the computation of the local Radon transform. The first one is based on conventional regularized least squares; the second one is based on a sparse reconstruction approach where we attempt to estimate sparse local Radon panels.

1. INTRODUCTION

A chief problem in seismic data processing is the filtering of unwanted events like ground roll and multiples. Methods to deal with this problem often exploit moveout or curvature differences between offending events and the events one would like to preserve [1]. In particular, removal of multiple reflections based on moveout discrimination can be attained via parabolic and hyperbolic Radon transforms. In the parabolic transform, seismic data (after normal-moveout correction) are assumed to be composed of a superposition of parabolas; in the second case, the data are assumed to be a superposition of hyperbolas. Methods exist to enhance the resolution of both hyperbolic [2] and parabolic Radon transforms [3]. In both cases, the operator capable of inverting the Radon transform is designed in such a way that the Radon panel exhibits minimum entropy or maximum sparseness. The latter is equivalent to finding a Radon panel where waveforms are focused to impulsive signals. The sparseness assumption might not be optimal when there is a mismatch between the integration path of the Radon operator and the spatial-temporal signature of the seismic event. The latter can be overcome by constructing local operators [4]. In other words, we propose to use operators that match the structure of the wavefield on small spatio-temporal apertures. Alternatively, one could attempt a much more ambitious path where operators are directly extracted from the data (data driven process). This paper describes a method to construct local Radon operators. This new class of Radon operators is implemented through a strategy that is based on Generalized Convolution (GC) and Generalized Deconvolution (GD) [4]. We describe this idea in the following section.

2. THEORY

In classical applications of the Radon transform to seismic data processing [1], we attempt to represent the data, D, with a finite number of waveforms defined over the complete seismic signal aperture by means of the following expansion:

$$D = \sum_k \alpha_k \Phi_k, \tag{1}$$

where $\Phi_k, k = 1..N$ are the basis functions (waveforms with linear, hyperbolic or parabolic paths) and $\alpha_k, k = 1,N$ are the coefficients of the expansion. In general, the coefficients $\alpha_k, k = 1..N$ represent the non-zero coefficients of the Radon panel. In the new approach we propose to adopt basis functions that are local (waveforms that can operate on a sub-aperture of the seismic data). In this case we propose to represent the data using the following model (GC):

$$D = \sum_{k=1}^{N} F_k \otimes B_k \tag{2}$$

In equation (2), the data are represented via the convolution of compact Local Wavefield Operators (LWO) B_k and an unknown suite of filters $F_k, k = 1,N$. The symbol $\otimes$ represents multi-dimensional convolution. The problem reduces to finding the filters F_k given the data D and the operators B_k. It is clear that noise in the data must be considered and, therefore, the filters are computed by minimizing the following cost function:

$$J = \left\| D - \sum_{k=1}^{N} F_k \otimes B_k \right\|^2 + \mu \sum_{k=1}^{N} \left\| F_k \right\|^2 \qquad (3)$$

In the previous expression we have incorporated a regularization term. The trade-off parameter μ controls the degree of fitting of the modelled data to the observations. Equation (3) is minimized using the method of conjugate gradients It is important to stress that the inner products required by the CG algorithm are implemented via optimized convolution and cross-correlation algorithms computed with multi-dimensional FFTs. We designate the solution computed by minimizing equation (3) the damped least-squares solution (DLS).

The Local Wavefield Operators (LWO) proposed in [4] consists of waveforms of constant ray parameter defined on a small aperture (5-7 traces). A suite of N=25 LWOs were numerically designed for the purpose of computing local linear Radon transforms (local slant stacks). The operators are shown in Figure 1. Each operator is parameterized with a ray parameter, a band-limiting seismic source function and an operator aperture. The ensemble of operators was constructed with *N=25* ray parameters or dips.

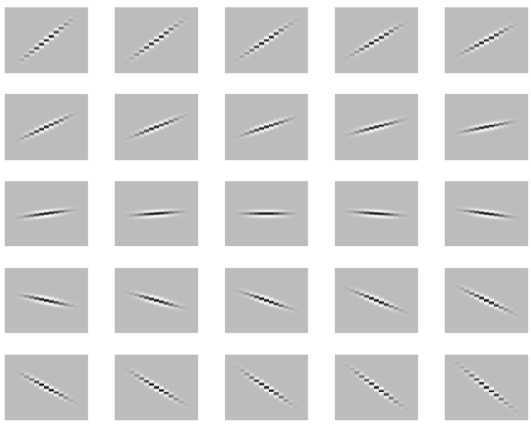

Figure 1. Linear Local Wavefield Operators (N=25).

In Figure 2 we examine the decomposition of a seismic record containing hyperbolic and linear events. Multi-dimensional generalized filters F_k are first estimated by inverting equation (2). Then, a subset of operators $B_k, k = kl,...kh$ is used to reconstruct the data. The reconstructed data are computed with the following expression:

$$\widehat{D} = \sum_{k=kl}^{kh} D_k, \quad D_k = F_k \otimes B_k \qquad (4)$$

In our example, *kl=11, kh=15*. Each member of the sum in equation (3) (D_k) is called a mode. The *k*-mode is a panel of size equal to the size of the data; it captures waveforms primarily and locally modelled by the operator B_k. We have reconstructed the data using dips that locally model the hyperbolas. It can be seen that residual energy from linear events leaks in the reconstructed model of hyperbolas. The signals are non-orthogonal to each other and therefore, some degree of leakage is expected. Sparse regularization strategies for inverse problems can be adopted to alleviate the aforementioned problem. This is discussed in the following section.

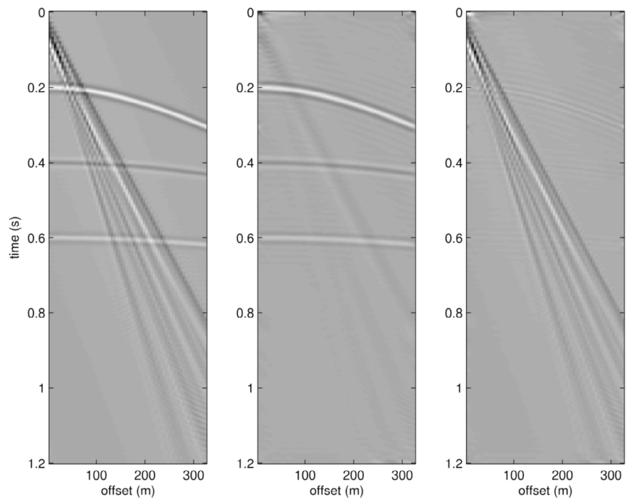

Figure 2. Synthetic shot gather (left). Reconstruction using modes *k=11…14* associated to the Local Wavefield Operators in Figure 1 (center). Residual panel (right).

The procedure outlined above was also used to eliminate ground roll (surface waves) from a multichannel seismic record from the Western Canadian Sedimentary Basin (Figure 3). In this example, *N=41* LWOs were adopted for the generalized deconvolution. A subset of *11* modes was retained to reconstruct the data. Low velocity linear coherent noise has been eliminated and the resulting filtered seismic record reveals quite well the seismic reflections.

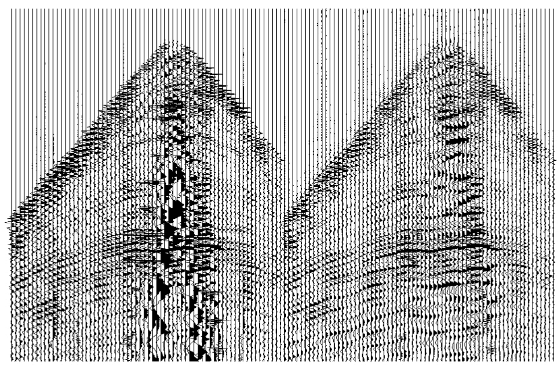

Figure 3. Linear noise removal using Generalized Deconvolution. Linear Local Wavefield Operators were deconvolved from the data. The modes capturing dips associated to the ground roll were eliminated from the data (right).

3. PARABOLIC LOCAL RADON TRANSFORM WITH SPARSE REGULARIZATION

We adopt the same mathematical structure to model seismic data but now the LWOs represent waveforms with parabolic moveout (a good approximation, for instance, to model diffracted multiple reflections). Each waveform is parameterized by a curvature parameter. Figure 4 displays the synthetic seismic record used to test our algorithm. The goal is to separate the two events using generalized deconvolution. A suite of *11* LWOs with parabolic moveout is depicted in Figure 5. In Figure 6 we portrayed the filters F_k; the associated modes are portrayed in Figure 7. In this case the filters were computed using the damped least-squares method. The modal decomposition cannot capture the individual waveforms in the original data. The least squares method yields a solution where the energy is distributed over all the modes. The problem can be circumvented by introducing sparse regularization into the solution of equation (2) [2]-[5]. In this case, we minimize

$$J = \left\| D - \sum_{k=1}^{N} F_k \otimes B_k \right\|^2 + \mu \sum_{k=1}^{N} R(F_k), \qquad (5)$$

where the function $R(x)$ used to regularize the problem is a Cauchy norm [2]-[5]:

$$R(x) = \sum_j \ln\left(1 + \frac{x_j^2}{\sigma^2}\right). \qquad (6)$$

The Cauchy norm has been proposed in [2] to estimate sparse solutions of inverse problems arising in seismic signal processing scenarios. In addition, an application to data reconstruction in the Fourier domain was proposed in [5]. Other norms capable of retrieving sparse models could have also been used, for instance, the l_1 norm [6]. The cost function in equation (5) is non-quadratic. Therefore, equation (5) is minimized using iterative re-weighted least squares [7]-[8]

We observe that the ensemble of filters computed with the sparse regularization can capture the two signals quite well (Figure 8). The modal decomposition in Figure 9 has correctly identified the two waveforms. The full reconstruction of the data (sum of all the modes) has provided the right reconstruction for both the DLS solution and the Sparse Least-Squares solution. The advantage of using a solver with a sparseness regularization term is quite evident: we have achieved simplicity in the filters and event separation in the modes.

4. SUMMARY

We have presented a generalized convolution/deconvolution approach to solve the problem of waveform separation and filtering. The methodology is designed to represent seismic data in terms of Local Wavefield Operators. The ideas presented in this paper have numerous applications: random and coherent (aliased) noise attenuation, interpolation beyond aliasing, wavefield separation, filtering of diffracted multiples, etc. Similarly, these ideas can lead to interesting algorithms for migration velocity analysis where the focusing power of the filter ensemble may well be used for velocity estimation.

5. REFERENCES

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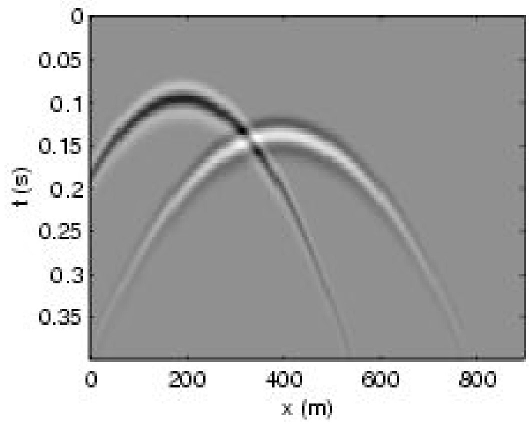

Figure 4. Synthetic example used to test the decomposition with parabolic Local Wavefield Operators.

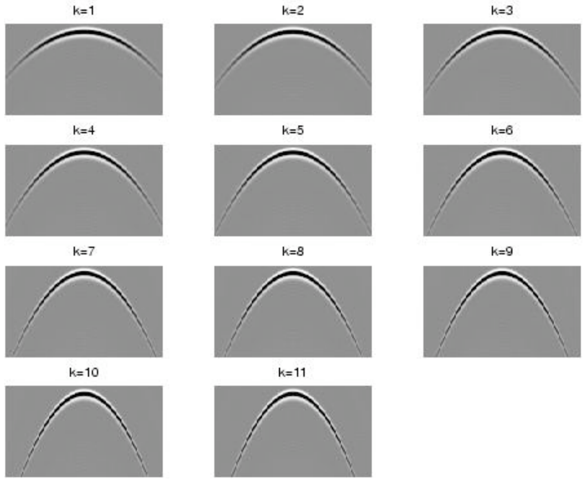

Figure 5. Local Wavefield Operators with parabolic moveout.

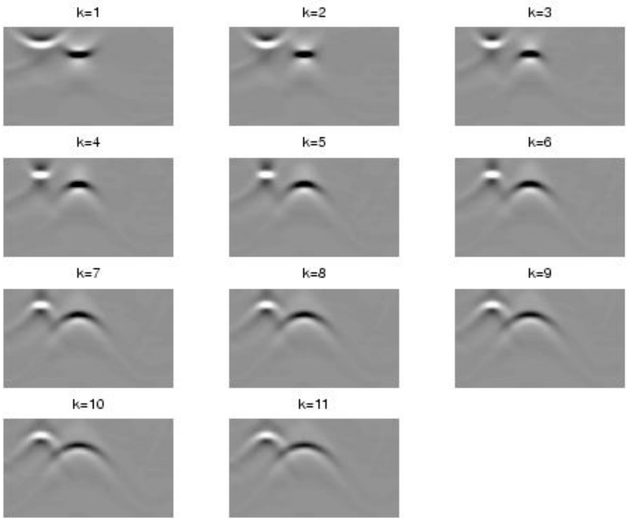

Figure 6. Filters computed using the damped least-squares method.

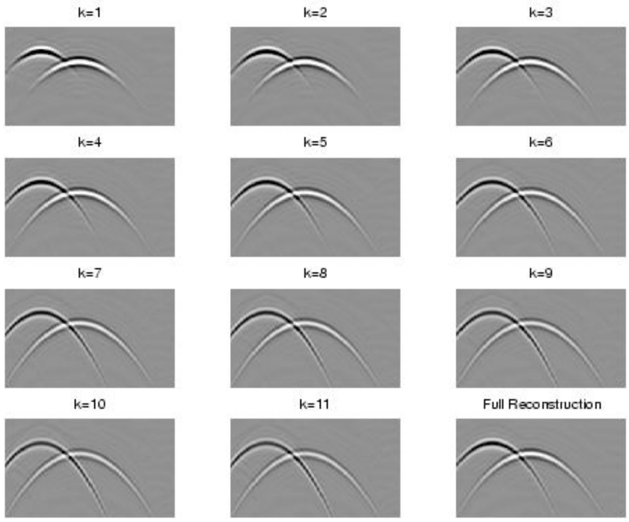

Figure 7. Modes obtained by convolving the filters from Figure 6 with parabolic Local Wavefield Operators. The last sub panel is the full data reconstruction (sum of all modes).

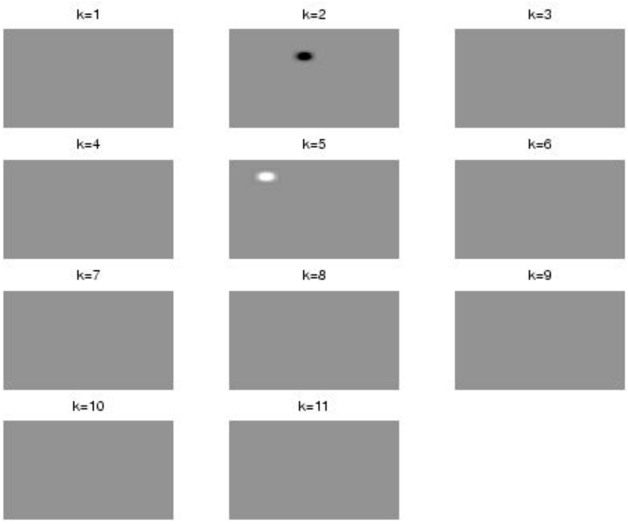

Figure 8. Filters computed using the least-squares method with sparseness constraint regularization.

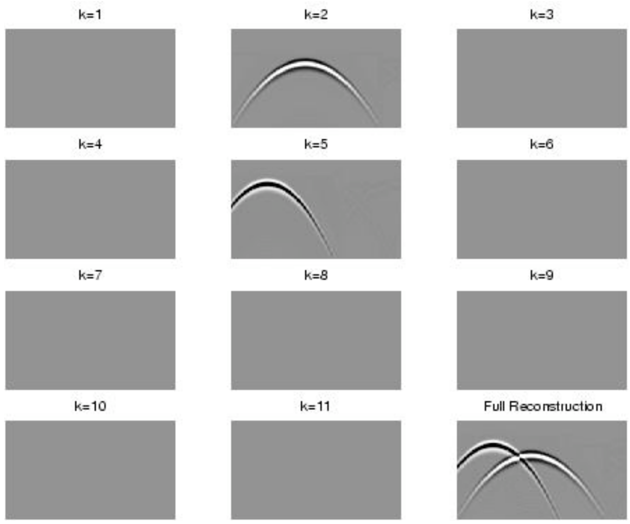

Figure 9. Modes obtained by convolving the filters from Figure 7 with parabolic Local Wavefield Operators. The last sub panel is the full data reconstruction (sum of all modes).