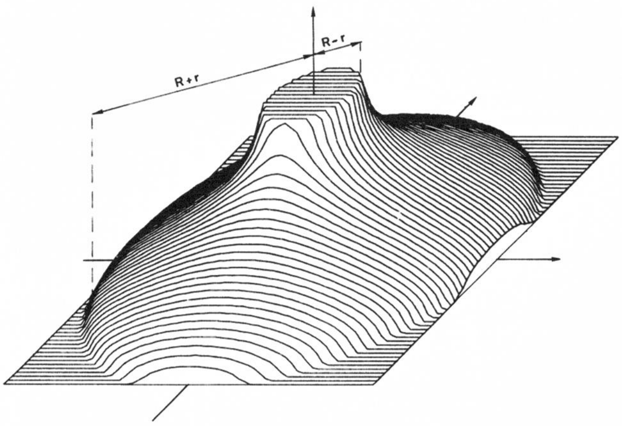

Fig. 15. Hough plane in the 1-to-m case of detection of circles in a circular retina of radius R and in polar coordinates. The representation here shown is for a given value of a circle (r) and can be considered as a cut in the complete Hough domain: the 3-D volume with coordinates $\{r, z, \theta\}$.

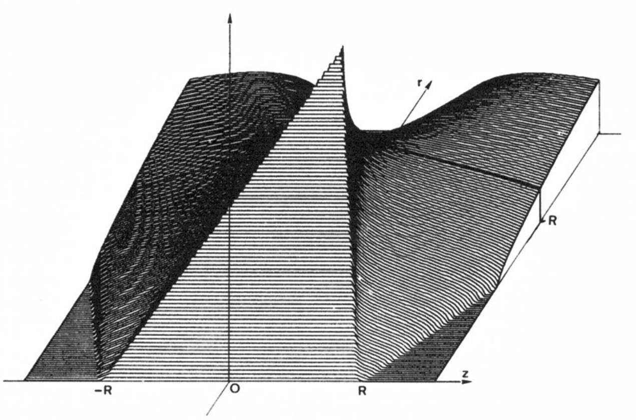

Fig. 16. The same probability for a constant value of θ, versus r and z.

$dadb$, we obtain the limits of the acceptance set C_3 of M_3 just considering E and F. The internal limit is the locus of the symmetric of M_1 with respect to any line going through E; thus, it is the circle of center E and radius EM_1. The external limit is the circle of center F and radius FM_1 (Fig. 14).

Equation (1) is now written

$$P(A)\ dadb = \int_{C_1} P(M_1)\ dM_1 \int_{C_2} P(M_2)\ dM_2 \int_{C_3} P(M_3)\ dM_3.$$

Unfortunately, even with simple probabilities this equation remains heavy, and only approximate solutions have been obtained (see [14]).

The use of the third Hough parameter (the radius r) makes this integral a little simpler (five integration instead of six), but the integration domains are more complex (they are $3 \times D$ volumes).

In the 1-to-m case and three-parameter Hough plane, the derivation of $P(A)$ is simple because only one feature point is taken at a time. In the circular retina case (with R as a retina radius and $\{z, \theta\}$ as the polar coordinates of the circle center), the conditional probability of z for a given value of r is independent of θ:

$$P(z/r) = 2r/R^2 \qquad \text{if } z < R - r$$

$$P(z/r) = \frac{2r}{\pi R^2} \cdot \cos^{-1} \frac{z^2 - R^2 + r^2}{2zr} \qquad \text{if } R - r \leq z < R + r.$$

This equation is simpler than the equation in [8, p. 97], but it is the same. This function is presented in Fig. 15 and in Fig. 16. Thus, in the controversy between [8] and [9], we propose a very simple explanation: in [8], implicitly, only the 1-to-m case is considered; on the other hand, in [9] it is the m-to-1. This closes the controversy, but opens a new one because this explanation is in opposition to the one proposed in [9]!

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Comments on "Scale-Based Description and Recognition of Planar Curves and Two-Dimensional Shapes"

ARDESHIR GOSHTASBY

In a recent paper,[1] Mokhtarian and Mackworth have described a technique for registration of a Landsat image with a map. In this technique, the map is assumed to be the reference image and the Landsat image is assumed to be the sensed image. The registration

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[1] F. Mokhtarian and A. Mackworth, *IEEE Trans. Pattern Anal. Machine Intell.*, vol. PAMI-8, pp. 34–43, Jan. 1986.

process has involved the following steps: 1) Isolation of closed contours in the map and preparation of scale-space images of the contours. 2) Segmentation of the Landsat image and isolation of closed-boundary regions in the image. 3) Determination of scale-space images of the isolated region boundaries in the Landsat image. 4) Matching of scale-space images of contours in the map and scale-space images of region boundaries in the Landsat image.

It is argued in the paper[1] that matching was carried out on the scale-space images of the contours rather than the contours themselves because 1) the scales of details in the map and the image were different, and 2) a scale-space image contains information about a contour at a continuum of scales. The purpose of this note is to show that if the scale of details in the map and the Landsat image are different, scale-space images of contours in the map and scale-space images of region boundaries in the image will not match reliably.

If we assume that a closed contour in the map provides the most detailed representation of an area on the ground then the corresponding region in the segmented Landsat image will have a boundary that is a smoothed version of the contour in the map. Smoothing is considered to be convolution with a 2-D Gaussian filter. If we fill the closed contours in the map so that we obtain solid regions, then if we convolve a region in the map with a 2-D Gaussian filter and isolate the region boundary that is obtained by the zero-crossings, and keep doing this by continuously increasing the scale of the filter, we will arrive at a point where the obtained region boundary matches perfectly with the corresponding region boundary in the image. If we take the contour in the map and smooth it by convolving it with a 1-D Gaussian filter, as has been done by Mokhtarian and Mackworth, then by increasing the filter size, there is no guarantee that we will reach a point at which the obtained contour matches the region boundary in the Landsat image. As a matter of fact there is evidence to the contrary.

Yuille and Poggio [1] have shown that zero-crossings obtained from convolution of an image with 2-D Gaussian filter are such that when the scale of the filter is changed, the zero-crossing contours could split into two or two neighboring zero-crossing contours could merge into one. An excellent example exhibiting this important fact has been shown in Fig. 3 of [2]. In this figure, it is shown that the zero-crossing contour of a dumbbell changes drastically as the scale changes. A single contour divides into two contours as the scale increases and joins into one again as the scale increases further. This never happens when convolving a contour with a 1-D Gaussian filter. In 1-D, no matter what the scale, a contour never breaks into two nor do two neighboring contours merge into one.

The important point that is intended to be made here is that convolving a region boundary with a 1-D Gaussian would not be the same as convolving the region as a solid with a 2-D Gaussian and then extracting the obtained region boundary. Images obtained at different resolutions are examples of convolution with 2-D Gaussian filters. Region boundaries obtained in real images cannot be compared to contours obtained from convolution of a contour in a map by 1-D Gaussian filters. If done so, the matching will not be accurate as is evident by the poor registration results reported by Figs. 9 and 10 of the subject paper.[1]

A region boundary in an image is not an independent entity when considering multiple-scales but rather it depends on the region as a whole and also on the nearby regions.

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Authors' Reply[2]

FARZIN MOKHTARIAN AND ALAN MACKWORTH

Given a binary image there is a choice to be made. One can use our approach and smooth the boundary representations or one can, indeed, smooth the image itself with a 2-D filter, extracting zero-crossing contours from the smoothed image data. We considered both approaches and adopted the former. The following brief points are relevant.

First, 1-D smoothing based on path length in boundary representations is very well behaved. Because of the properties referred to in the paper[1] the top down best-first matching process can exploit the topological tree structure of the contours. These properties do not obtain for 2-D smoothing, as pointed out. In addition, experiments show difficulties in the interference between adjacent regions.

Second, the smoothing is more efficient in 1-D than 2-D even though the 2-D filter can be made separable.

Third, boundary curvature based methods are well-established in the machine and human vision literature (whether they are based on extrema or zero-crossings of curvature) and our method is a generalization of those methods with well defined criteria.

Fourth, the discrepancies between the map and the Landsat image are almost all due to boundary-based effects rather than image blurring. In the Landsat image, the jaggies are due to spatial quantization noise and in the vector based map data coarse piecewise linear approximations to the boundary are to blame for the lack of fidelity as shown in Figs. 9 and 11 of the subject paper.[1] Both of these effects are best handled by boundary smoothing.

Fifth, nowhere in our paper[1] do "we assume that a closed contour in a map provides the most detailed representation." In fact, the data given explicitly show that that assumption does not hold. As shown in Figs. 9 and 11 the map and image island data are provided to about the same level of detail, as can be checked by looking at the boundaries or by comparing the heights of the corresponding contours in the scale-space images. Thus, the analysis provided in Goshtasby's comments does not apply and the boundary effects described above dominate.

Sixth, we agree that the registration of the map and the image could be better. (We did state "exact registration . . . is not a goal of this paper.") The residual error is, we believe, mostly due to deformations of the image not modeled by our transforms, not to the putative cause given by Goshtasby.

Seventh, the relationship between 1-D and 2-D scale-space filtering still yields many intriguing unsolved research problems with theoretical and applied facets.

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Comments on "Low Level Segmentation: An Expert System"

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In a recent paper[1] Nazif and Levine showed comparisons of various segmentation methods, including the split-and-merge algo-

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