Application of the Matrix Pencil Method for Estimating the SEM (Singularity Expansion Method) Poles of Source-Free Transient Responses from Multiple Look Directions

Tapan Kumar Sarkar, *Fellow, IEEE*, Sheeyun Park, *Member, IEEE*, Jinhwan Koh, and Sadasiva M. Rao

***Abstract*—In this paper, the matrix pencil method has been utilized for estimating the natural resonances from different transient responses recorded along multiple look directions as a function of time after the incident field has passed the structure. The novelty of this article is that a single estimate for all the poles are done utilizing multiple transient waveforms emanating from the structure along multiple look directions. The SEM poles are independent of the angle at which the transient response is recorded. The only difference between the various waveforms are that the residues at the various poles are of different magnitudes. Some of the residues may even be zero for some of the poles indicating that the contribution from certain SEM poles may not be significant along that look direction. Here all the waveforms are utilized providing a single estimate for the poles without performing an arithmetic mean of the various waveforms.**

***Index Terms*—Electromagnetic scattering, natural resonances.**

I. Introduction

It is well known in the electromagnetics literature that after the incident field had crossed the structure of interest, the time-domain responses can be modeled by a sum of complex exponentials [1], [2]. In the Laplace domain this is equivalent to modeling the transfer function of the system by the poles along with its residues or in terms of a ratio of two polynomials whose roots provide the poles and zeros of the system.

Many methods exist in the published literature to carry out such a parameterization of the source-free transient responses of the system. A partial survey of such techniques is available in [3], [9], and [10]. Out of most of the techniques, the matrix pencil method had proved to be quite useful [4], [5] because of its low sensitivity to background noise and its computational ease and efficiency.

Now when the transient responses from the object of interest whose SEM poles need to be found out is looked at from different angles both in azimuth and in elevation, the residues of the poles are angle dependent whereas the SEM poles modeling the time-domain waveforms are not angle dependent. In addition, for each look direction there are two possible polarizations. One could also use both polarizations to increase the number of waveforms available. Conventionally, to estimate the SEM poles from multiple look angle data one takes the average of all the various look directions waveforms and then obtains a single waveform. Then a sum of complex exponentials is used to fit the single waveform and an estimate of the SEM poles is obtained along with the averaged values of the residues. However, this is not a good approach if the signal-to-noise ratios of the different waveforms are different—namely in some of them the transient response dies down quite fast whereas in some of the responses this may continue to ring for a long time. Hence, taking an average of those two classes of waveshapes actually deteriorates the signal-to-noise ratio of the data. This is because by taking an average of the signal along with waveforms where the signal has died down may lead to an unnecessary contamination of the signal by noise. In this paper, the matrix pencil approach is applied to obtain a single estimate for the SEM poles utilizing simultaneously all the transient waveforms from multiple look directions and without averaging them.

In Section II, the matrix pencil method is presented for the simultaneous estimation of all the SEM poles from multiple look directions without averaging. In Section III, the computational procedure utilizing the total least squares singular-value decomposition based approach is presented for the estimation of the SEM poles from multiple look directions. This approach has been found to be most robust in obtaining estimates for the poles in the presence of random noise [6]–[8]. Section IV provides some numerical examples utilizing sample simulated data followed by conclusion and a selected set of references where additional materials are available.

Manuscript received August 11, 1999; revised January 10, 2000.

T. K. Sarkar, S. Park, and J. Koh are with the Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY 13244-1240 USA.

S. M. Rao is with the Department of Electrical Engineering, University of Alabama, Auburn, AL 36849 USA.

Publisher Item Identifier S 0018-926X(00)03255-5.

II. Application of the Matrix Pencil Method for Simultaneous Estimation of the SEM-Poles Utilizing Waveforms from Multiple Look Directions

Let us denote the transient response of length $N+1$ along a particular look direction $-k$ by the set $[Y_k]$. So that the column vector $[Y_k]$ is represented by

$$[Y_k]_{(N+1)\times 1} = [y_k(0);\, y_k(1);\, \cdots;\, y_k(N)]^T_{1\times(N+1)} \tag{1}$$

where T denotes the transpose of a matrix. The elements $y_k(j)$ represents the values of the transient response at the jth time sample, so that

$$y_k(j) = \sum_{i=1}^{M} A_k(i)\, \exp[s_i j \Delta T] \cdots \quad \text{for } j = 0,\, 1,\, 2,\, \cdots,\, N \tag{2}$$

where ΔT is the sampling time. Each transient response consists of the same M SEM poles s_i, which are to be solved for along their amplitudes $A_k(i)$ for a particular look direction k. Please note that M is also an unknown along with the SEM poles and their residues. The SEM poles s_i are look direction independent but not their residues $A_k(i)$. In the sampled domain, (2) can be rewritten as

$$y_k(j) = \sum_{i=1}^{M} A_k(i) z_i^j \qquad \text{for } j = 0,\, 1,\, 2,\, \cdots,\, N \tag{3}$$

where

$$z_i = \exp[s_i \Delta T]. \tag{4}$$

It has been further assumed that all the waveforms for different look angles $k = 1,\, 2,\, \cdots,\, K$, have been sampled uniformly at the same sampling rate ΔT and that each waveform contains the same number of samples $N + 1$.

Next we consider two matrices $[B_1]$ and $[B_2]$ defined as

$$[B_1]_{N\times K} = \begin{bmatrix} y_1(0) & y_2(0) & \cdots & y_K(0) \\ y_1(1) & y_2(1) & \cdots & y_K(1) \\ \vdots & \vdots & & \vdots \\ y_1(N-1) & y_2(N-1) & \cdots & y_K(N-1) \end{bmatrix}_{N\times K} \tag{5}$$

$$[B_2]_{N\times K} = \begin{bmatrix} y_1(1) & y_2(1) & \cdots & y_K(1) \\ y_1(2) & y_2(2) & \cdots & y_K(2) \\ \vdots & \vdots & & \vdots \\ y_1(N) & y_2(N) & \cdots & y_K(N) \end{bmatrix}_{N\times K}. \tag{6}$$

Now, it can be shown that the two matrices $[B_1]$ and $[B_2]$ can be decomposed as follows:

$$[B_1]_{N\times K} = [Z_1]_{N\times M}[I]_{M\times M}[A]_{M\times K} \tag{7}$$

$$[B_2]_{N\times K} = [Z_1]_{N\times M}[Z_o]_{M\times M}[A]_{M\times K} \tag{8}$$

where

$$[Z_1]_{N\times M} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ z_1^2 & z_2^2 & \cdots & z_M^2 \\ \vdots & & & \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_M^{N-1} \end{bmatrix}_{N\times M} \tag{9}$$

$$[I]_{M\times M} = \text{is a diagonal matrix} \equiv \text{identify matrix} \tag{10}$$

and a diagonal matrix $[Z_o]$ containing the SEM pole set

$$[Z_o]_{M\times M} = \begin{bmatrix} z_1 & & O & \\ & z_2 & & \\ & & \ddots & \\ & O & & z_M \end{bmatrix}_{M\times M} \tag{11}$$

with

$$[A] = \begin{bmatrix} A_1(1) & A_2(1) & \cdots & A_k(1) \\ A_1(2) & A_2(2) & \cdots & A_k(2) \\ \vdots & \vdots & & \vdots \\ A_1(M) & A_2(M) & \cdots & A_k(M) \end{bmatrix}_{M\times K}. \tag{12}$$

Now if we consider the matrix pencil

$$[B_2] - \lambda[B_1] \tag{13}$$

then we observe

$$[B_2] - \lambda[B_1] = [Z_1]\{[Z_o] - \lambda[I]\}[A]. \tag{14}$$

This matrix pencil becomes linearly dependent when λ is one of the system poles as then the rank of $\{[Z_o] - \lambda[I]\}_{M\times M}$ is reduced by one as $\lambda = z_i$. Equation (14) can be transformed into a computationally palatable form by considering the ordinary eigenvalue problem in either of the following forms:

$$[B_2][B_1]^+ - \lambda[I] \tag{15}$$

$$[I] - \lambda[B_1][B_2]^+ \tag{16}$$

where the superscript $+$ is the pseudo inverse of the respective matrices. The pseudo-inverse is defined in terms of the singular value decompositions of the respective matrix. Let

$$\begin{aligned}[B_1]_{N\times K} &= [U_1]_{N\times N} \begin{bmatrix} \sigma_1^2 & & O & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_k^2 \\ & O & & \vdots \end{bmatrix}_{N\times K} [V_1]^H_{K\times K} \\ &= [U_1][\Sigma][V_1]^H \end{aligned} \tag{17}$$

where $[U_1]$ and $[V_1]$ are two orthogonal matrixes, i.e.,

$$[U_1]^{-1} = [U_1]^H \tag{18}$$

$$[V_1]^{-1} = [V_1]^H \tag{19}$$

where the superscript H denotes the conjugate transpose of a matrix. Here $[\Sigma]$ is a rectangular matrix whose diagonal elements are related to the singular values of $[B_1]$. In summary, we have the following relationship:

$$\begin{aligned}&[B_1]_{N\times K}[\{V_c\}]_{K\times 1} \\ &\quad = \sigma_c[\{U_c\}]_{N\times 1} \qquad \text{for } c = 1,\, 2,\, \cdots,\, K\end{aligned} \tag{20}$$

and

$$[U_1]_{N\times N} = \Big[\{u_1\}_{N\times 1} \vdots \{u_2\}_{N\times 1} \vdots \cdots \vdots \{u_N\}\Big]_{N\times N} \cdots \tag{21}$$

$$[V_1]_{K\times K} = \Big[\{v_1\}_{K\times 1} \vdots \{v_2\}_{K\times 1} \vdots \cdots \vdots \{v_K\}_{K\times 1}\Big]_{K\times K} \cdots . \tag{22}$$

Now the pseudo-inverse of $[B_1]$ can be computed from

$$[B_1]^+ = [V_1][\Sigma]^{-1}[U_1]^H \tag{23}$$

where

$$[\Sigma]^{-1} = \begin{bmatrix} \frac{1}{\sigma_1^2} & & & O \\ & \frac{1}{\sigma_2^2} & & \\ & & \ddots & \\ & & & \frac{1}{\sigma_K^2} \\ & O & & \end{bmatrix}_{N\times K} . \tag{24}$$

It is interesting to observe from (7), (8), (15), and (16) is that the matrix pencil has a solution provided

$$K \geq M \tag{25}$$

i.e., the multiple look directions must be greater than or equal to the number of poles of the system to be estimated. This can be a serious limitation in many cases as described in [11] as the number of SEM poles can be quite large for many practical systems and it may not be possible to provide as many sensors for each look directions. Hence, this method is extended to the case where one may have $K < M$. If $K < M$, we assume that $N \gg K$ or M.

To deal with the more general situation we consider the two matrices $[D_1]$ and $[D_2]$. They are defined by (27) and (28), shown at the bottom of the next page. Next, it can be shown that the two matrices $[D_1]$ and $[D_2]$ can be factored into

$$\begin{aligned} &[D_1]_{(L+1)\times K\cdot(N-L)} \\ &\quad = [P]_{(L+1)\times M}[I]_{M\times M}[R]_{M\times(L\cdot M)}[Q]_{(K\cdot M)\times[K\cdot(N-L)]} \end{aligned} \tag{29}$$

and

$$\begin{aligned} &[D_2]_{(L+1)\times[K\cdot(N-L)]} \\ &\quad = [P]_{(L+1)\times M}[Z_o]_{M\times M}[R]_{M\times(K\cdot M)}[Q]_{(K\cdot M)\times[K\cdot(N-L)]} \end{aligned} \tag{30}$$

where as shown in (31)–(33), shown at the top of page 616, and

$$[Q_1]_{M\times(N-L)} = \begin{bmatrix} 1 & z_1 & z_1^2 & \cdots & z_1^{N-L-1} \\ 1 & z_2 & z_2^2 & \cdots & z_2^{N-L-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & z_M & z_M^2 & \vdots & z_M^{N-L-1} \end{bmatrix}_{M\times(N-L)} . \tag{34}$$

In addition $[I]$ and $[Z_o]$ are two diagonal matrices, which have been defined by (10) and (11), respectively. Now if we consider the matrix pencil

$$[D_2] - \lambda[D_1] \tag{35}$$

and their equivalent ordinary eigenvalue form of the type

$$[D_2][D_1]^+ - \lambda[I] \tag{36}$$

or

$$[I] - \lambda[D_1][D^2]^+ \tag{37}$$

then when λ becomes an eigenvalue of either (36) or (37), and its value is equivalent to a system pole.

Once all the poles z_1, $i = 1, \cdots, M$ have been computed the residues at the poles can be computed from the following equation:

$$\begin{bmatrix} y_1(0) & y_2(0) & \cdots & y_K(0) \\ y_1(1) & y_2(1) & \cdots & y_K(1) \\ \vdots & \vdots & & \vdots \\ y_1(N) & y_2(N) & & y_K(N) \end{bmatrix}_{(N+1)\times K} \tag{38}$$

$$= \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ z_1^2 & z_2^2 & \cdots & z_M^2 \\ \vdots & \vdots & & \vdots \\ z_1^N & z_2^N & \cdots & z_M^N \end{bmatrix}_{(N+1)\times M} \cdot \begin{bmatrix} A_1(1) & A_2(1) & \cdots & A_k(1) \\ A_1(2) & A_2(2) & \cdots & A_k(2) \\ \vdots & \vdots & & \vdots \\ A_1(M) & A_2(M) & \cdots & A_k(M) \end{bmatrix}_{M\times K} \tag{39}$$

or equivalently

$$[Y] = [Z]\cdot[A]. \qquad 40$$

The various residues can now be computed from the least squares solution of (31) from

$$[A] = [Z]^+[Y] = \left\{[Z]^H[Z]\right\}^{-1}[Z]^H[Y]. \tag{41}$$

III. Computation of the SEM Poles Utilizing the Total Least Squares

In order to deal with noisy data, the formulation of the Section II is made more robust to noise. We now consider the composite matrix $[D]$ as shown in (42), shown on page 616. Please note that $[D_1]$ is obtained from $[D]$ by eliminating the last row and $[D_2]$ is obtained from $[D]$ by eliminating the first row. We now perform a singular value decomposition of $[D]$ according to (17) as follows:

$$\begin{aligned} &[D]_{(L+2)\times[K\cdot(N-L)]} \\ &\quad = [U]_{(L+2)\times(L+2)}[\Sigma]_{(L+2)\times[K\cdot(N-L)]} \\ &\qquad \cdot [V]^H_{K\cdot(N-1)\times[K\cdot(N-L)]}. \end{aligned} \tag{43}$$

To combat the effects of noise and to determine the order M, we perform a singular value filtering of $[\Sigma]$ by retaining its M dominant singular values. The details are available in [4]–[8] and are omitted in this paper. Also, it can be seen that the ordinary eigenvalue problem of

$$[D_2] - \lambda[D_1] \tag{44}$$

can be transformed into the following:

$$\begin{aligned}&[U_2]_{(L+1)\times(L+2)}[\Sigma]_{(L+2)\times[K\cdot(N-L)]}[V]^H_{[K\cdot(N-1)]\times[K\cdot(N-L)]}\\ &\quad = \lambda[U_1]_{(L+1)\times(L+2)}[\Sigma]_{(L+2)\times[K\cdot(N-L)]}\\ &\qquad \cdot [V]^H_{[K\cdot(N-1)]\times[K\cdot(N-L)]}\end{aligned} \tag{45}$$

or

$$[U_2] - \lambda[U_1] \tag{46}$$

where $[U_2]$ and $[U_1]$ of (45) are obtained from $[U]$ by eliminating the first and last row, respectively.

Then the poles are obtained from the solution of either one of the following four ordinary eigenvalue problem:

$$[U_2]^H[U_2] - \lambda[U_2]^H[U_1]$$

$$[U_2][U_2]^H - \lambda[U_1][U_2]^H \tag{47}$$

$$[U_2][U_1]^H - \lambda[U_1][U_1]^H$$

$$[U_1]^H[U_2] - \lambda[U_1]^H[U_1]. \tag{48}$$

Once the poles are obtained the residues at the poles due to different signals measured at various look directions are obtained from the solution of (40) through the use of (41).

IV. Numerical Results

As an example consider a square plate of dimensions 1 m $\times$ 1 m (lying in the x–y plane) irradiated by an electromagnetic pulse which is of 6 ℓm (light-meters) in duration and is oriented along the E_θ direction with magnitude -377 V/m. We are observing the current at the center of the plate. The waveforms are sampled every $\Delta T = 0.118\,75$ ℓm and the incident pulse dies down after 12 ℓm. The waveshape for the y-directed current is observed after 130 time samples so as to ensure that the incident field has passed the metal plate. The next 50 samples are taken to estimate the SEM poles for the plate by observing the transient field arriving from different angles of incidence. It has been observed that only five poles are required as the singular values drop off beyond 10^{-8} in evaluating $[D]$ of (42). In the first table we present the SEM poles along with their residues for seven separate incident angles of Θ and ϕ.

$$[D_1]_{(L+1)\times K\cdot(N-L)} = \begin{bmatrix} y_1(0) & y_1(1) & \cdots & y_1(N-L-1) & y_2(0) & y_2(1) & \cdots & y_2(N-L-1) & \cdots & y_K(0) & y_K(1) & \cdots & y_K(N-L-1) \\ y_1(1) & y_1(2) & \cdots & y_1(N-L) & y_2(1) & y_2(2) & \cdots & y_2(N-L) & \cdots & y_K(1) & y_K(2) & \cdots & y_K(N-L) \\ \vdots & \vdots & & \vdots & \vdots & & & & & \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots & \vdots & & & & & \vdots & \vdots & & \vdots \\ y_1(L) & y_1(L+1) & \cdots & y_1(N-1) & y_2(L) & y_2(L+1) & \cdots & y_2(N-1) & \cdots & y_K(L) & y_K(L+1) & \cdots & y_K(N-1) \end{bmatrix}_{(L+1)\times[K\cdot(N-L)]} \tag{27}$$

$$[D_2]_{(L+1)\times K\cdot(N-L)} = \begin{bmatrix} y_1(1) & y_1(2) & \cdots & y_1(N-L) & y_2(1) & y_2(2) & \cdots & y_2(N-L) & \cdots & y_K(1) & y_K(2) & \cdots & y_K(N-L) \\ y_1(2) & y_1(3) & \cdots & y_1(N-L+1) & y_2(2) & y_2(3) & \cdots & y_2(N-L+1) & \cdots & y_K(2) & y_K(3) & \cdots & y_K(N-L+1) \\ \vdots & \vdots & & \vdots & \vdots & & & & & \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots & \vdots & & & & & \vdots & \vdots & & \vdots \\ y_1(L+1) & y_1(L+2) & \cdots & y_1(N) & y_2(L+1) & y_2(L+2) & \cdots & y_2(N) & \cdots & y_K(L+1) & y_K(L+2) & \cdots & y_K(N) \end{bmatrix}_{(L+1)\times[K\cdot(N-L)]} \tag{28}$$

$$[P]_{(L+1)\times M}=\begin{bmatrix}1 & 1 & \cdots & 1\\ z_1 & z_2 & \cdots & z_M\\ z_1^2 & z_2^2 & \cdots & z_M^2\\ \vdots & \vdots & & \cdots\\ z_1^L & z_2^L & \cdots & z_M^L\end{bmatrix}_{(L+1)\times M} \tag{31}$$

$$[R]_{M\times(K-M)} = \begin{bmatrix} A_1(1) & & O & \vdots & A_2(1) & & O & \vdots & \cdots & \vdots & A_K(1) & & O\\ & A_1(2) & & \vdots & & A_2(s) & & \vdots & \cdots & \vdots & & A_K(2) & \\ O & & A_1(M) & \vdots & 0 & & A_2(M) & \vdots & & \vdots & O & & A_K(M)\end{bmatrix}_{M\times M} \tag{32}$$

$$[Q]_{(K\cdot M)\times[K\cdot(N-L)]}=\begin{bmatrix}[Q_1]_{M\times(N=L)} & & \\ & [Q_1]_{M\times(N-L)} & \\ & \ddots & \\ & & [Q_1]\end{bmatrix}_{(K\cdot M)\times[K\cdot(N-L)]} \tag{33}$$

$$[D]=\begin{bmatrix} y_1(0) & y_1(1) & \cdots & y_1(N-L-1) & y_2(0) & y_2(1) & \cdots & y_2(N-L-1) & \cdots & y_K(0) & y_K(1) & \cdots & y_K(N-L-1)\\ y_1(1) & y_1(2) & \cdots & y_1(N-L) & y_2(1) & y_2(2) & \cdots & y_2(N-L) & \cdots & y_K(1) & y_K(2) & \cdots & y_K(N-L)\\ \vdots & \vdots & & \vdots & \vdots & & & & & \vdots & \vdots & & \vdots\\ \vdots & \vdots & & \vdots & \vdots & & & & & \vdots & \vdots & & \vdots\\ y_1(L) & y_1(L+1) & \cdots & y_1(N-1) & y_2(L) & y_2(L+1) & \cdots & y_2(N-1) & \cdots & y_K(L) & y_K(L+1) & \cdots & y_K(N-1)\\ y_1(L+1) & y_1(L+2) & \cdots & y_1(N) & y_2(L+1) & y_2(L+2) & \cdots & y_2(N) & \cdots & y_K(L+1) & y_K(L+2) & \cdots & y_K(N)\end{bmatrix}_{(L+2)\times[K\cdot(N-L)]} \tag{42}$$

Case I—$\theta = 0$ and $\phi = 90°$:

$$\begin{aligned} s_{1,2} &= -1.2 \pm j1.73 & A_{1,2} &= 0.45\underline{/\pm 130°}\\ s_{3,4} &= -0.49 \pm j0.721 & A_{3,4} &= 0.37\underline{/\pm 18.5°}\\ s_5 &= 0.703 & A_5 &= 0.0083. \end{aligned}$$

In this case the incident electric field E_θ of amplitude -377 V/m is impinging on the plate from $\theta = 0°$ and $\phi = 90°$. There are two sets of complex conjugate poles and one growing exponential, which is of small amplitude. However, the growing exponential is nonphysical it is the error due to curve fitting of the data.

Case II—$\theta = 10°$ and $\phi = 80°$:

$$\begin{aligned} s_{1,2} &= -1.35 \pm j1.71 & A_{1,2} &= 0.27\underline{/\mp 38.2°}\\ s_{3,4} &= -0.276 \pm j1.241 & A_{3,4} &= 0.19\underline{/\mp 150°}\\ s_5 &= 0.204 & A_5 &= 0.113. \end{aligned}$$

Case III—$\theta = 30°$ and $\phi = 50°$:

$$\begin{aligned} s_{1,2} &= -1.06 \pm j1.75 & A_{1,2} &= 0.37\underline{/\mp 32°}\\ s_{3,4} &= -0.281 \pm j1.18 & A_{3,4} &= 0.29\underline{/\mp 143°}\\ s_5 &= 0.041 & A_5 &= 0.219. \end{aligned}$$

Case IV—$\theta = 50°$ and $\phi = 75°$:

$$\begin{aligned} s_{1,2} &= -0.941 \pm j1.82 & A_{1,2} &= 0.44\underline{/\mp 30°}\\ s_{3,4} &= -0.283 \pm j1.17 & A_{3,4} &= 0.36\underline{/\mp 138°}\\ s_5 &= 0.07 & A_5 &= 0.325. \end{aligned}$$

Case V—$\theta = 20°$ and $\phi = 70°$:

$$\begin{aligned} s_{1,2} &= -1.57 \pm j1.49 & A_{1,2} &= 0.9\underline{/\mp 55.3°}\\ s_{3,4} &= -0.278 \pm j1.22 & A_{3,4} &= 0.35\underline{/\mp 158°}\\ s_5 &= 0.14 & A_5 &= 0.454. \end{aligned}$$

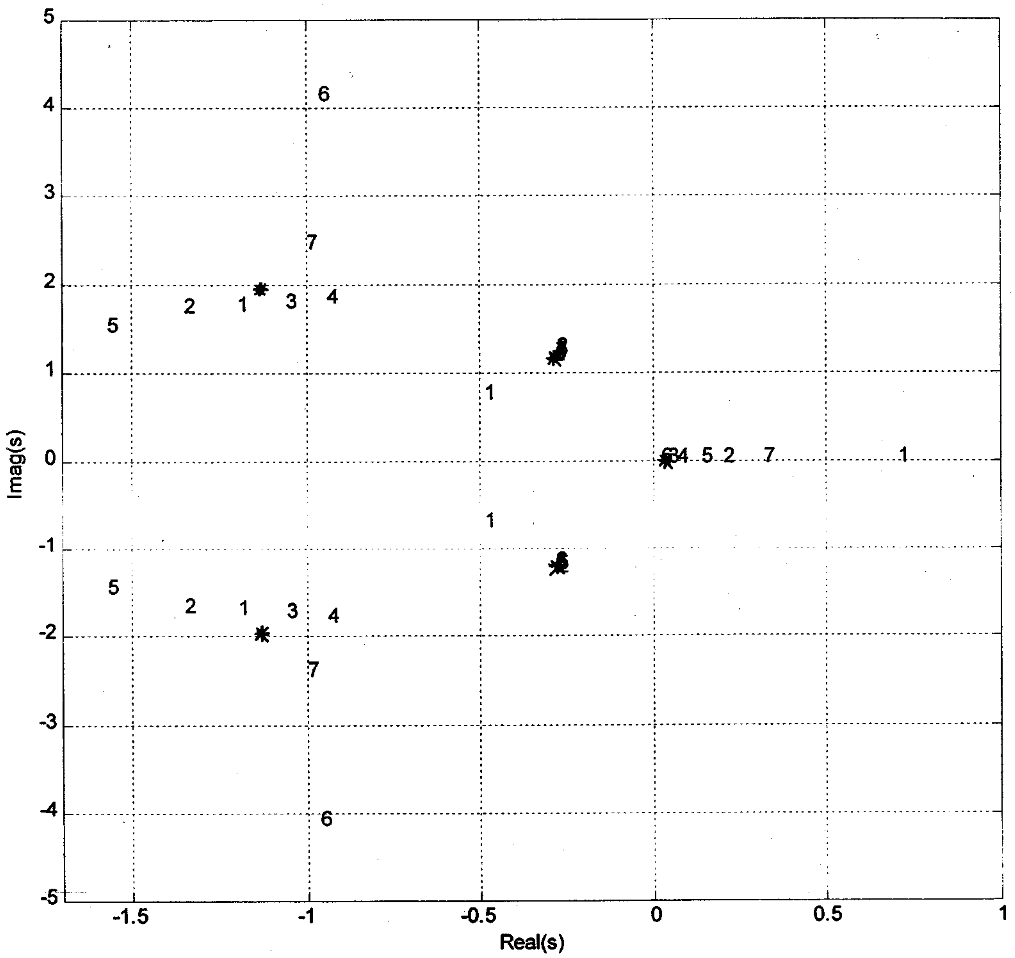

Fig. 1. Clustering of the poles.

Case VI—$\theta = 10°$ and $\phi = 170°$:

$$s_{1,2} = -0.963 \pm j4.11 \qquad A_{1,2} = 0.145\underline{/\mp 55°}$$
$$s_{3,4} = -0.282 \pm j1.18 \qquad A_{3,4} = 0.212\underline{/\mp 112°}$$
$$s_5 = 0.023 \qquad A_5 = 0.208.$$

Case VII—$\theta = 30°$ and $\phi = 140°$:

$$s_{1,2} = -0.999 \pm j2.42 \qquad A_{1,2} = 0.116\underline{/\mp 31°}$$
$$s_{3,4} = -0.282 \pm j1.19 \qquad A_{3,4} = 0.139\underline{/\mp 126°}$$
$$s_5 = 0.316 \qquad A_5 = 0.068.$$

As can be seen from the various results, only one set of poles around $-0.28 \pm j1.2$ is stable and the others move around. The various poles are marked in Fig. 1 through the various numericals representing the seven cases described above by roman numerals.

Next, we utilize all the seven data sets to obtain a single estimate for the poles. Again as before five poles are obtained. This single estimate of the poles is markes by "$*$" in Fig. 1

$$s_{1,2} = -1.13 \pm j1.95$$
$$s_{3,4} = -0.28 \pm j1.17$$
$$s_5 = 0.039.$$

However, the estimates for the residues are different for different look angles.

Case I—$\theta = 0°$ and $\phi = 90°$:

$$A_{1,2} = 0.35\underline{/\mp 37.6°}$$
$$A_{3,4} = 0.273\underline{/\mp 139°}$$
$$A_5 = 0.218.$$

Case II—$\theta = 10°$ and $\phi = 80°$:

$$A_{1,2} = 0.29\underline{/\mp 30.1°}$$
$$A_{3,4} = 0.252\underline{/\mp 131°}$$
$$A_5 = 0.211.$$

Case III—$\theta = 30°$ and $\phi = 50°$:

$$A_{1,2} = 0.125\underline{/\mp 45.2°}$$
$$A_{3,4} = 0.152\underline{/\mp 91.1°}$$
$$A_5 = 0.13.$$

Case IV—$\theta = 50°$ and $\phi = 70°$:

$$A_{1,2} = 0.126\underline{/\mp 55°}$$
$$A_{3,4} = 0.143\underline{/\mp 86°}$$

$$A_5 = 0.117.$$

Case V—$\theta = 20°$ and $\phi = 70°$:

$$\begin{aligned} A_{1,2} &= 0.195\underline{/\mp 11.7°} \\ A_{3,4} &= 0.213\underline{/\mp 118°} \\ A_5 &= 0.186. \end{aligned}$$

Case VI—$\theta = 10°$ and $\phi = 170°$:

$$\begin{aligned} A_{1,2} &= 0.077\underline{/\mp 47.7°} \\ A_{3,4} &= 0.05\underline{/\mp 152°} \\ A_5 &= 0.036. \end{aligned}$$

Case VII—$\theta = 30°$ and $\phi = 140°$:

$$\begin{aligned} A_{1,2} &= 0.236\underline{/\mp 46°} \\ A_{3,4} &= 0.159\underline{/\mp 149°} \\ A_5 &= 0.117. \end{aligned}$$

From the theory presented in this paper where only one set of poles has been estimated using all the waveforms, it appears that there is a possible harmonic relationship between the two decaying exponentials, at least in the imaginary part, and, as expected, the higher frequency is damped more. However, such a relationship is missing when the poles are estimated from each waveshape separately.

V. Conclusion

The matrix pencil method is presented for estimating the SEM poles due to different transient responses. The novelty of this approach is that only one set of the SEM poles are estimated from the multiple waveforms. However the residues at the pole sets are different for different waveshapes. It is hoped that such a single estimate for the SEM poles will be more accurate and robust to different orientation of polarizations of the incident fields and due to various effects introduced by noise.

Acknowledgment

The authors would like to thank the reviewers for enhancing the readability of the manuscript.

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Tapan Kumar Sarkar (S'69–M'76–SM'81–F'92) received the B.Tech. degree from the Indian Institute of Technology, Kharagpur, India, in 1969, the M.Sc.E. degree from the University of New Brunswick, Fredericton, Canada, in 1971, and the M.S. and Ph.D. degrees from Syracuse University, Syracuse, NY, in 1975.

From 1975 to 1976, he was with the TACO Division of the General Instruments Corporation. He was with the Rochester Institute of Technology, Rochester, NY, from 1976 to 1985. He was a Research Fellow at the Gordan McKay Laboratory, Harvard University, Cambridge, MA, from 1977 to 1978. He is now a Professor in the Department of Electrical and Computer Engineering, Syracuse University, NY. He was an Associate Editor for feature articles of the *IEEE Antennas and Propagation Society Newsletter* and has authored or coauthored more than 210 journal articles and numerous conference papers and has written chapters, 28 books, and ten books, including *Iterative and Self-Adaptive Finite-Elements in Electromagnetic Modeling* (Norwood, MA: Artech House, 1998). His current research interests deal with numerical solutions of operator equations arising in electromagnetics and signal processing with application to system design.

Dr. Sarkar received one of the "Best Solution" Awards in May 1977 at the Rome Air Development Center (RADC) Spectral Estimation Workshop. He is a Registered Professional Engineer in the State of New York. He received the Best Paper Award of the IEEE Transactions on Electromagnetic Compatibility in 1979 and at the 1997 National Radar Conference. He received the College of Engineering Research Award in 1996 and the Chancellor's Citation for Excellence in Research in 1998 at Syracuse University. He was the Technical Program Chairman for the 1998 IEEE Antennas and Propagation Society International Symposium and URSI on Time Domain Metrology (1990–1996). He is a member of Sigma Xi and International Union of Radio Science Commissions A and B. He received the title *Docteur Honoris Causa* from Universite Blaise Pascal, Clermont Ferrand, France, in 1998.

Sheeyun Park (S'96–M'98) received the B.S. degree in electrical engineering from Massachusetts Institute of Technology (MIT), Cambridge, MA, in 1994, and the M.S. degree in electrical engineering from Syracuse University, Syracuse, NY, in 1996. He is currently working toward the Ph.D. degree in electrical engineering from Syracuse University.

From 1995 to 1998, he was a Research Assistant in the Microwave Laboratory, Syracuse University. He is currently a Staff Scientist at Mission Research Corporation, Santa Barbara, CA, where he has been working since 1998. His current interests lie in time-frequency analysis, multiresolution harmonic analysis, and subband coding.

Jinhwan Koh was born in Taegu, Korea. He received the B.S. degree in electronics from Inha University, Korea. He is currently working toward the Ph.D. degree in the Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY.

His main research areas include various problems associated with digital signal processing, electromagnetic, and adaptive array system.

Sadasiva M. Rao, biography and photograph not available at the time of publication.