impulse response from time limited input and output: Theory and experiment; IEEE Trans. Inst. & Meas. Dec. 1985, pp.541-546.
H. Chen, T. K. Sarkar, S. A. Dianat and J. Brule, "Adaptive Spectral Estimation utilizing the conjugate gradient method," IEEE Trans. on Acoustics speech and signal proc., April 1986, pp.272-284.

Tapan Sarkar to be Co-Associate Editor for Feature Articles

Beginning with the current issue, Prof. Tapan K. Sarkar will share the position of Associate Editor for Feature Articles on the AP-S Newsletter staff with Prof. Arlon T. Adams. Tapan will take primary responsibility for soliciting and editing feature articles. This addition to the Newsletter staff has been made at the request and suggestion of Arlon, who is beginning to feel the demands of chairing the 1988 AP-S International Symposium and National Radio Science Meeting to be held at Syracuse University.

To be accurate, Arlon originally tried to resign. However, your Editor refused to let the Newsletter so easily lose Arlon's experience and expertise. It is appropriate to pause and thank Arlon for his outstanding work. In the approximately one and one-half years since Arlon began as Associate Editor for Features, he has set a new standard for this critical position. The articles he has obtained have been of uniformly high quality. As this issue goes to press, the Newsletter has committments for two feature articles per issue throughout the next year of publication. Tapan and Arlon have set a goal to maintain this "lead time", and use the resultant flexibility to expand the scope of Newsletter features in response to suggestions from the AP-S membership. In particular, it is hoped that over the next year a practice of having at least one article per issue by an author or authors from outside the US can be established. The possibility of having an additional feature article (i.e., three) in some - and eventually most - issues is also being persued.

It is perhaps fitting that Tapan takes on this position with the issue in which he is also the author of one of the feature articles. His photo and biographical sketch appear adjacent to the article. If any reader has suggestions or comments regarding the Newsletter's feature articles - or, best of all, an article or a suggestion for a topic and an author - please contact Tapan at the address listed inside the front cover.

Your Editor welcomes Tapan to the Newsletter staff. His willingness to serve our Society and his enthusiasm are greatly appreciated.

Introducing Carl E. Baum Feature Article Author

Carl Baum was born in Binghamton, New York, on February 6, 1940. He received the BS (with honor), MS, and PhD degrees in electrical engineering from the California Institute of Technology, in 1962, 1963, and 1969, respectively.

Dr. Baum was commissioned in the United States Air Force in 1962 and was stationed at the Air force Weapons Laboratory from 1963 to 1967 and from 1968 to 1972. Since 1971 he has served as a civil servant with the position of Senior Scientist at the Air Force Weapons Laboratory. He is an advisor to numerous US Army, Navy, Air Force, and tri-service agencies on EMP-related matters, and is a US representative in exchanging EMP information with various countries.

Dr. Baum is a Fellow of the IEEE. He is a member of Tau Beta Pi and Sigma Xi. He is a member of Commissions B and E of USNC/URSI, and of the New Mexico Academy of Science. He is past president of the Electromagentics Society, and founder and president of SUMMA Foundation. He received the IEEE EMC Society Richard R. Stoddart award, and has received numerous other awards as author of technical papers and as editor of EMP technical publications.

Used and Out-of-Print Books and AP-S Transactions

Joseph R. Jahoda is seeking out-of-print antenna books. He also wants back issues of the AP-S Transactions, AP-S International Symposia Digests, and Microwave Theory and Techniques Transactions, all prior to 1985. He will pay a reasonable price and shipping. Contact him at Astron Corporation, 929 W. Broad St., Suite 249, Falls Church, VA 22046 (703-241-1490).

Feature Article

The Singularity Expansion Method: Background and Developments

Carl E. Baum

Air Force Weapons Laboratory (NTAAB)
Kirtland AFB, NM 87117

PREFACE

When A.T. Adams asked me to contribute a feature article on the singularity expansion method (SEM) we discussed what it should be. Considering that it should be an overview we decided that a recent paper by myself was most suitable. Thus the paper that follows was originally published as the first paper in the Special Issue on the Singularity Expansion Method edited by L. Wilson Pearson and Lennart Marin. This was Vol. 1, No. 4, October-December 1981 of the journal Electromagnetics, published by Hemisphere Publishing Corporation. Hemisphere has graciously granted permission to reprint that article here. Following the article itself I have added an appendix concerning some developments since this article was originally published. For interested readers a very extensive bibliography is included in the above-mentioned special issue.

ABSTRACT

The singularity expansion method (SEM) arose from the observation that the transient response of complex electromagnetic scatterers appeared to be dominated by a small number of damped sinusoids. In the complex frequency plane, these damped sinusoids are poles of the Laplace-transformed response. The question is then one of characterizing the object response (time and frequency domains) in terms of all the singularities (poles, branch cuts, entire functions) in the complex frequency plane (hence singularity expansion method). Building on the older concept of natural frequencies, formulae were developed for the pole terms from an integral-equation formulation of the scattering process. The resulting factoring of the pole terms has important application consequences. Later developments include the eigenmode expansion method (EEM) which diagonalizes the integral-equation kernels and which can be used as an intermediate step in ordering the SEM terms. Additional concepts which have appeared include eigenimpedance synthesis and equivalent electrical networks. Of current interest is the use of the theoretical formulae to efficiently analyze and order experimental data. Related to this is the application of SEM results to target identification. This paper does not delve into the mathematical details; it presents an overview of the history and major concepts and results in SEM and EEM and related matters.

1. BACKGROUND

1.1. Natural Frequencies

An important antecedent physical concept is that of natural frequencies. These are thought of as frequencies for which there is a response with no forcing function. Also called natural oscillations or resonances, these in general exhibit a damping phenomenon (in the case of passive objects) which can be interpreted as one part of a complex frequency. In electromagnetic responses of various scatterers/antennas, there are various examples of early work on natural frequencies. The perfectly conducting thin wire and circular loop were treated by numerous investigators including Pocklington in 1897 [12], Abraham [1,2], Oseen [4-7], Hallén [3], and Rayleigh [13,14]. This was extended to perfectly conducting prolate spheroids by Page and Adams [8-11] and perfectly conducting spheres by Stratton [15]. An important contribution was made by Schwinger [41] who treated the special case of electromagnetic fields internal to perfectly conducting cavities. In this case the natural frequencies are all on the $j\omega$ axis (pure imaginary) in the complex-frequency (s) or Laplace-transform plane, and the natural modes have a convenient orthogonality property.

1.2 Laplace Transform

Various mathematical tools had been in use in electrical engineering and provided some starting point for constructing basic SEM formulae when the time was ripe. One such tool was certainly the Laplace (or Fourier) transform which we take in the two-sided sense as

$$\tilde{F}(s) \equiv \int_{-\infty}^{\infty} F(t)\, e^{-st} dt \ , \quad F(t) = \frac{1}{2\pi j} \int_{\Omega_0 - j\infty}^{\Omega_0 + j\infty} \tilde{F}(s)\, e^{st} ds$$

$$\begin{aligned} t &\equiv \text{time} \ , \ \sim \text{(above)} \equiv \text{Laplace transform} \\ s &\equiv \Omega + j\omega = \text{Laplace transform variable} = \text{complex frequency} \\ F(t) &\equiv \text{any Laplace transformable time function or operator (scalar, vector, tensor, etc.)} \end{aligned} \qquad (1.1)$$

where the Bromwich contour, $Re[s] = \Omega_0$, for inversion is chosen in the strip of convergence, say $\Omega_a < Re[s] < \Omega_b$.

1.3 Complex Variable Theory

Considering the response of some antenna or scatterer as a function of s in the complex s plane one can describe the s-plane behavior in terms of the singularities (or boundaries of analyticity) in the complex plane, including the behavior at infinity (entire function). Appropriate contour integrals can be used to describe the response; the contours can be deformed to give separate terms for each singularity in both complex-frequency and time domains [18].

1.4 Circuit and System Theory

In electrical engineering there has been a considerable body of knowledge developed concerning electrical networks. This is summarized in circuit analysis and circuit synthesis theory which (especially in the linear case) is documented in numerous texts. This is further extended to linear system theory and control theory which are now major subject areas with an extensive literature. The use of the Laplace transform is quite extensive in these areas, and expansions in terms of poles are often used. Our problem of electromagnetic interaction (scattering) is related in that a scatterer can be thought of as a distributed network or system of a special kind (with response described by the Maxwell equations). Furthermore, it is possible to describe the scattering process by an equivalent circuit by using circuit synthesis concepts to synthesize (perhaps approximately) the appropriate complex transfer functions and impedances of the scatterer.

1.5 Integral Equations for Electromagnetic Scatterers and Antennas

For perfectly conducting objects (as well as for certain types of impedance loading) an integral equation reduces the problem from three space dimensions (the Maxwell differential equations) to two dimensions (the scatterer surface). Perhaps more important, the radiation condition at infinity for the scattered fields is explicitly incorporated into the integral equation so that one need not be concerned with the analytic continuation of the radiation condition into the left half of the s plane. Well-known integral equations include the electric-field integral equation and the magnetic-field integral equation (in various forms). In one-dimensional approximate forms (for wires) there are the Hallén and Pocklington equations. The details of these equations do not concern us here. The important point is that they all have the form

$$\langle \tilde{\overleftrightarrow{\Gamma}}(\vec{r},\vec{r}\,';s) \overset{\cdot}{,} \tilde{\vec{J}}(\vec{r}\,',s)\rangle = \tilde{\vec{I}}(\vec{r},s) \tag{1.2}$$

$\tilde{\vec{I}}(\vec{r},s) \equiv$ incident or source field of some kind (specified)

$\tilde{\overleftrightarrow{\Gamma}}(\vec{r},\vec{r}\,';s) \equiv$ kernel (related to Green's function) which may be a distribution

$\tilde{\vec{J}}(\vec{r}\,',s) \equiv$ typically current density or surface current density

Here

$$\langle\, , \rangle \equiv \text{symmetric product} \tag{1.3}$$

is our convenient way to indicate multiplication (of the two terms separated by the comma) followed by integration with respect to the common spatial coordinates over the domain of the scatterer; the type of multiplication (e.g., dot (•) or cross (×) product) is indicated by appropriate symbols above the comma. With additional commas this symmetric product is extended to as many terms and integrations as desired.

One can in principal solve the integral equation by inverting the integral operator. One formally determines an inverse kernel (which may be a distribution) which gives a solution

$$\tilde{\vec{J}}(\vec{r},s) = \langle \tilde{\overleftrightarrow{\Gamma}}^{-1}(\vec{r},\vec{r}\,';s) \overset{\cdot}{,} \tilde{\vec{I}}(\vec{r}\,',s)\rangle$$

$$\langle \tilde{\overleftrightarrow{\Gamma}}(\vec{r},\vec{r}\,'') \overset{\cdot}{,} \tilde{\overleftrightarrow{\Gamma}}^{-1}(\vec{r}\,'',\vec{r}\,';s)\rangle = \overleftrightarrow{1}(\vec{r}-\vec{r}\,') \tag{1.4}$$

$\equiv$ identity on scatterer

where the identity is taken in the sense of the relevant vector components and domain of integration (e.g., two or three dimensions for surfaces or volumes, respectively.)

For SEM these integral equations have proven to be very useful in constructing formulae for the various terms. Singularity expansions can be constructed for both the response $\tilde{J}$ and the inverse kernel $\tilde{\overleftrightarrow{\Gamma}}^{-1}$ (related to the class 1 and class 2 forms of the coupling coefficient, respectively). Furthermore, the integral-equation kernels can be used to construct eigenmode expansions which give additional insight into the SEM terms.

1.6 Matrix and Operator Theory

Integral equations have been cast in approximate numerical form by the moment method (MoM). In this numerical solution procedure (typically for use with large digital computers) the current density (response) is expanded in a set of functions (of finite number in practice) called expansion functions; the incident or source field is similarly expanded in a set of testing functions [46]. The vectors of coefficients of these two sets (taken with equal numbers of components) are related by a matrix (square) which replaces the integral-equation operator in the form

$$(\tilde{\Gamma}_{n,m}(s)) \cdot (\tilde{J}_n(s)) = (\tilde{I}_n(s)) \tag{1.5}$$

Inverting the matrix, one has an approximate solution to the original equation (1.2) in a form analogous to (1.4). In matrix form our equation is more familiar to electrical engineers because such types of equations appear in circuit problems. The arsenal of matrix theory is now at our disposal. Eigenvectors and eigenvalues can be constructed for representing the solution and understanding its properties. Combining matrix (or operator) theory with complex variable theory is essential to SEM. This paper will not delve into the mathematical theory of such operators, this subject being left to others.

2. EARLY DEVELOPMENT OF SEM

2.1 The Beginning

In early 1971 the question was posed (by this author). Experimental observations of damped sinusoids in EMP experiments [47] suggested poles in the corresponding Laplace transforms. Then in Laplace-transform or complex-frequency domain this led to the idea of expanding the response in terms of all the singularities in the complex frequency plane. Besides poles, such singularities might include branch points and associated integrals, essential singularities, and (for completeness) entire function(s) corresponding to any singularities at infinity.

Concentrating on the poles it was observed that, except for poles in the exciting waveform (transformed), these were the natural frequencies of the scatterer or antenna because integral equations describing the object response would admit non-trivial responses at such frequencies with no excitation. Said another way, the response at an object pole is infinite if the excitation is non-zero at such a complex frequency. Interpreting (1.2) in this sense gives

$$\langle \tilde{\vec{\Gamma}}(\vec{r},\vec{r}\,';s_\alpha) \,;\, \vec{j}_\alpha(\vec{r}\,')\rangle = \vec{0}\ ,\quad s_\alpha \equiv \text{natural frequency} \tag{2.1}$$

$\vec{j}_\alpha(\vec{r}) \equiv$ natural mode corresponding to s_α

or from (1.5) in MoM form

$$(\tilde{\Gamma}_n(s_\alpha)) \cdot (j_n)_\alpha = (0_n)\ ,\quad \det((\tilde{\Gamma}_n(s_\alpha)) = 0 \tag{2.2}$$

which gives a way of computing natural frequencies. Noting that the matrix is singular (and hence so is its transpose) we can write

$$(\mu_n)_\alpha \cdot (\tilde{\Gamma}_n(s_\alpha)) = (0_n)\ ,\ \langle \vec{\mu}_\alpha(\vec{r}) \,;\, \tilde{\vec{\Gamma}}(\vec{r},\vec{r}\,';s_\alpha)\rangle = \vec{0} \tag{2.3}$$

$\vec{\mu}_\alpha(\vec{r}) \equiv$ coupling model corresponding to s_α

where the use of the coupling mode will become clear later. For the common case of a symmetric kernel (as in the E-field or impedance integral equation) the coupling mode can be set equal to the natural mode. The choice of a normalization for these modes is somewhat arbitrary.

Having equations for the natural frequencies and modes then construct a solution in the form

$$\tilde{\vec{U}}(\vec{r},s) = \sum_\alpha \tilde{\eta}_\alpha \vec{j}_\alpha(\vec{r})(s - s_\alpha)^{-1} + \text{other singularity terms}$$

$\equiv$ normalized (delta-function) response to incident or source field $E_0^{-1}\tilde{f}^{-1}(s)\,\tilde{\vec{I}}(\vec{r},s)$ (2.4)

$\tilde{\vec{J}}(\vec{r},s) = E_0 \tilde{f}(s)\,\tilde{\vec{U}}(\vec{r},s)$, $\tilde{\eta}_\alpha \equiv$ coupling coefficient

$\tilde{f}(s) \equiv$ incident or source waveform (Laplace transformed)

$E_0 \equiv$ scaling amplitude for incident waveform

where the postulated coupling coefficient contains the spatial characteristics of the incident field. Here first order poles have been assumed, although higher order poles can be included. One can also include the incident waveform in the pole residues as

$$\tilde{\vec{J}}(\vec{r},s) = E_0 \sum_\alpha \tilde{f}(s_\alpha)\,\tilde{\eta}_\alpha \vec{j}_\alpha(\vec{r})(s - s_\alpha)^{-1} + \text{other singularity terms} \tag{2.5}$$

This was the general state of knowledge on this subject when in September 1971 a special meeting was held at Northrop Corporate Laboratories office in Pasadena, California. Many prominent electromagnetic specialists participated in this discussion of SEM. The basic concepts were presented as outlined above to stimulate basic ideas and potential application to areas such as EMP data analysis, target identification, equivalent circuits, etc.

2.2 Evaluation of the Coupling Coefficient

In late 1971 a key discovery was made in that formulae for the coupling coefficient were developed in terms of the integral-equation terms in (1.2). This was done independently with different approaches by Baum [20] and by Marin and Latham [24]. The details of these derivations need not concern us here as they were rather involved. Subsequent papers have simplified this somewhat.

Noting that the kernel $\tilde{\vec{\Gamma}}$ and normalized incident

$$\tilde{\vec{I}}^{(n)}(\vec{r},s) = E_0^{-1}\tilde{f}^{-1}(s)\,\tilde{\vec{I}}(\vec{r},s) \tag{2.6}$$

or source field are analytic functions of s near s_α, expand them in a power series in $s - s_\alpha$. Collecting terms and applying the coupling vector leads to the class 1 coupling coefficient

$$\tilde{\eta}_\alpha^{(1)} = e^{-(s-s_\alpha)t_0} \frac{\langle \vec{\mu}_\alpha(\vec{r}) \,;\, \tilde{\vec{I}}^{(n)}(\vec{r},s_\alpha)\rangle}{\langle \vec{\mu}_\alpha(\vec{r}) \,;\, \frac{d}{ds}\tilde{\vec{\Gamma}}(\vec{r},\vec{r}\,';s)\Big|_{s=s_\alpha} \,;\, \vec{j}_\alpha(\vec{r}\,')\rangle} \tag{2.7}$$

where the turn-on time t_0 can be a function of the observer position $\vec{r}$. An alternate form is the class 2 coupling coefficient which results from first finding the SEM representation (strict) of $\tilde{\vec{\Gamma}}^{-1}$ and then as in (1.4) operating on $\tilde{\vec{I}}^{(n)}(\vec{r}\,',s)$ with the poles of $\tilde{\vec{\Gamma}}^{-1}$ giving

$$\tilde{\eta}^{(2)} = \frac{\langle e^{-(s-s_\alpha)t_0}\vec{\mu}_\alpha(\vec{r}) \,;\, \tilde{\vec{I}}^{(n)}(\vec{r},s)\rangle}{\langle \vec{\mu}_\alpha(\vec{r}) \,;\, \frac{d}{ds}\tilde{\vec{\Gamma}}(\vec{r},\vec{r}\,';s)\Big|_{s=s_\alpha} \,;\, \vec{j}_\alpha(\vec{r}\,')\rangle} \tag{2.8}$$

where the turn-on time t_0 can here be a function of both $\vec{r}$ and $\vec{r}\,'$. See [16] for a more complete derivation.

The two classes of coupling coefficients have some significant differences. Except for a delay factor the class 1 form is particularly simple, being independent of s, so that in time domain the normalized response in (2.4) takes the form

$$\vec{U}(\vec{r},t) = \sum_\alpha \tilde{\eta}_\alpha^{(0)} \vec{j}_\alpha(\vec{r})\, e^{s_\alpha t}\, u(t - t_0) + \text{other singularity terms} \tag{2.9}$$

Here the coupling coefficient at $s = s_\alpha$ is

$$\tilde{\eta}_\alpha^{(0)} = \tilde{\eta}_\alpha^{(1)}\Big|_{s=s_\alpha} = \tilde{\eta}_\alpha^{(2)}\Big|_{s=s_\alpha} = \frac{\langle \vec{\mu}_\alpha(\vec{r}) \,;\, \tilde{\vec{I}}^{(n)}(\vec{r},s_\alpha)\rangle}{\langle \vec{\mu}_\alpha(\vec{r}) \,;\, \frac{d}{ds}\tilde{\vec{\Gamma}}(\vec{r},\vec{r}\,';s)\Big|_{s=s_\alpha} \,;\, \vec{j}_\alpha(\vec{r}\,')\rangle} \tag{2.10}$$

so that both classes reduce to the same thing at the pole ($s = s_\alpha$). While the class 1 form gives simple damped sinusoids the class 2 form gives a convolution as

$$\vec{U}(\vec{r},t) = \sum_\alpha \vec{j}_\alpha(\vec{r})\, \eta_\alpha^{(2)} \circ [e^{s_\alpha t}\, u(t)] + \text{other singularity terms}$$

$\circ \equiv$ convolution with respect to time

$$\eta_\alpha^{(2)} = \frac{\left\langle \vec{\mu}_\alpha(\vec{r})[e^{s_\alpha t_0} u(t-t_0)] \; ; \; \vec{I}^{(n)}(\vec{r},t) \right\rangle}{\left\langle \vec{\mu}_\alpha(\vec{r}) \; ; \; \frac{\partial}{\partial s} \tilde{\overleftrightarrow{\Gamma}}(\vec{r},\vec{r}\,';s)\Big|_{s=s_\alpha} \; ; \; \vec{j}_\alpha(\vec{r}\,') \right\rangle} \tag{2.11}$$

At late time the time-domain pole terms in (2.9) and (2.11) give the same simple damped sinusoids. For $t_0 = 0$ in class 2, and t_0 (typically used) in class 1 chosen on or before the wave reaches the scatterer, class 1 and class 2 give identical pole terms after the wave passes the body. There are numerous details concerning the properties of the two classes omitted here. A recent paper goes into this topic in greater depth [23].

2.3 Example Problems

Now that the floodgates were open numerous investigators considered specific finite-size scatterers in free space. The early examples were the sphere (analytically) [20], the thin wire (approximate) [28], and the thin wire by numerical (MoM) computation [33]. The reader can consult the bibliography in this special issue for many more examples. A review book chapter by this author [16] summarizes most of the early examples of this type.

3. LATER DEVELOPMENTS

3.1 Natural Modes for Radiated or Scattered Fields

An early extension of the SEM concepts was to go from the currents and charges on an object to the radiated or scattered fields in the space surrounding the object. In 1973 there were papers by Tesche [34] concerning the numerical calculation of the far fields from linear antennas in terms of natural modes, and by Baum [21] concerning the formalism of such natural modes for near and far fields. These results established a concept of transient antenna (or scatterer) patterns in terms of natural frequencies, modes, and coupling coefficients.

3.2 Analysis of Experimental Data

Since the original impetus toward SEM came from observations of the general properties of the transient electromagnetic response of systems, it is understandable that the general SEM theory should be applied to such experimental data. Certain SEM parameters are in principle experimentally observable. In 1974 a paper (USNC/URSI meeting, Boulder, Colorado, October 1974, later in [40] by Van Blaricum and Mittra applied the Prony technique to transient EM scattering waveforms to find the natural frequencies and residues by fitting the waveform with a sum of damped sinusoids. Since then many investigators have tried various other techniques in attempts to increase speed of computation, minimize the effect of noise in the waveform, and maximize the accuracy in determining the true poles in the scattering data.

3.3 Eigenmode Expansion Method (EEM)

In 1975 this author introduced the eigenmode expansion method to find more properties of the SEM [22]. One defines eigenvalues and eigenmodes for the integral operator (kernel) in (1.2) via

$$\left\langle \tilde{\overleftrightarrow{\Gamma}}(\vec{r},\vec{r}\,';s) \; ; \; \tilde{\vec{j}}_\beta(\vec{r}\,',s) \right\rangle = \tilde{\lambda}_\beta(s) \; \tilde{\vec{j}}_\beta(\vec{r},s)$$

$$\left\langle \tilde{\vec{\mu}}_\beta(\vec{r},s) \; ; \; \tilde{\overleftrightarrow{\Gamma}}(\vec{r},\vec{r}\,';s) \right\rangle = \tilde{\lambda}_\beta(s) \; \tilde{\vec{\mu}}_\beta(\vec{r}\,',s) \tag{3.1}$$

$\tilde{\lambda}_\beta(s)$ = eigenvalue

$\tilde{\vec{j}}_\beta(\vec{r},s)$ = right eigenmode , $\tilde{\vec{\mu}}_\beta(\vec{r},s)$ = left eigenmode

Unlike the natural modes the eigenmodes can be generally biorthonormalized as

$$\left\langle \tilde{\vec{\mu}}_{\beta_1}(\vec{r},s) \; ; \; \tilde{\vec{j}}_{\beta_2}(\vec{r},s) \right\rangle = 1_{\beta_1,\beta_2} = \begin{cases} 1 \text{ for } \beta_1 = \beta_2 \\ 0 \text{ otherwise} \end{cases} \tag{3.2}$$

giving the representations for the kernel (and its inverse)

$$\tilde{\overleftrightarrow{\Gamma}}^{n}(\vec{r},\vec{r}\,';s) = \sum_\beta \tilde{\lambda}_\beta^{n}(s) \; \tilde{\vec{j}}_\beta(\vec{r},s) \; \tilde{\vec{\mu}}_\beta(\vec{r}\,',s) \tag{3.3}$$

and the response

$$\tilde{\vec{U}}(\vec{r},s) = \sum_\beta \tilde{\lambda}_\beta^{-1}(s) \left\langle \tilde{\vec{\mu}}_\beta(\vec{r}\,',s) \; ; \; \tilde{\vec{I}}^{(n)}(\vec{r}\,',s) \right\rangle \tilde{\vec{j}}_\beta(\vec{r},s) \tag{3.4}$$

While there are various mathematical problems to be considered concerning completeness, root vectors, sense of convergence, etc., there are some approximate ways to view this matter. Casting the integral equation (1.2) into matrix (MoM) numerical form as in (1.5), the EEM is considered as a problem of finding the eigenvalues and left and right eigenvectors of $(\tilde{\Gamma}_{n,m}(s))$.

Summarizing some of the SEM related results we have

$$\tilde{\lambda}_\beta(s_{\beta,\beta'}) = 0 \quad , \quad s_{\beta,\beta'} \equiv s_\alpha \tag{3.5}$$

so that the natural frequencies are zeros of particular eigenvalues (hence $\alpha \to (\beta,\beta')$), so that the eigenvalues order or partition the set of natural frequencies. Similarly for the modes (with appropriate normalizations)

$$\tilde{\vec{j}}_\beta(\vec{r},s_{\beta,\beta'}) = \vec{j}_{\beta,\beta'}(\vec{r}) \quad , \quad \tilde{\vec{\mu}}_\beta(\vec{r},s_{\beta,\beta'}) = \vec{\mu}_{\beta,\beta'}(\vec{r}) \tag{3.6}$$

For the denominator in the coupling coefficients we have

$$\left\langle \vec{\mu}_\alpha(\vec{r}) \; ; \; \frac{d}{ds} \tilde{\overleftrightarrow{\Gamma}}(\vec{r},\vec{r}\,';s)\Big|_{s=s_\alpha} \; ; \; \vec{j}_\alpha(\vec{r}\,') \right\rangle = \frac{d}{ds} \tilde{\lambda}_\beta(s)\Big|_{s=s_{\beta,\beta'}} \tag{3.7}$$

which allows us to represent class 1 (in (2.7)) and class 2 (in (2.8)) in terms of EEM quantities.

Another application of EEM is to the synthesis of transient responses via changing the eigenvalues. Eigenimpedance synthesis considers the eigenvalues $\tilde{Z}_\beta(s)$ of the impedance (or E-field) integral equation and notes that, if the scatterer or antenna is impedance loaded in certain ways ($\tilde{Z}_\ell(s)$), the eigenimpedances are modified as

$$\tilde{Z}_\beta(s) \to \tilde{Z}_\beta(s) + \tilde{Z}_\ell(s) \tag{3.8}$$

which allows one to synthesize a $\tilde{Z}_\ell(s)$ to move the natural frequencies $s_{\beta,\beta'}$ to other more desirable positions in the complex s plane. These EEM matters are necessarily quite abbreviated here. More complete reviews are included in [17,18]. Of special note is the recent extension of Sancer et al. [25] in which the eigenmodes of the "pseudosymmetric" H-field integral equation are paired with corresponding eigenvalues (normalized) adding to 1.0.

3.4 Target Identification

In the original development of the SEM concept (section 2.1) it was noted that the natural frequencies of a scatterer were independent of the exciting fields. This was considered a potentially useful property for target identification purposes. In 1975 two groups published papers proposing techniques for this general kind of target identification [42,43], based on work dating from about 1974. Another group [44] gave a spoken paper on this subject in 1975 also. This was also about the time (1975) of the introduction of the concept of eigenimpedance synthesis for modifying the pole pattern in the s plane to make the identification more difficult [22].

3.5 Equivalent Circuits for Antennas and Scatterers

In 1976 this author showed how to construct formal equivalent circuits at an antenna/scatterer port from the SEM representation [27]. A review of this development is included in [18]. The key to this development is to note that the admittance and short-circuit current (or the impedance and open-circuit voltage) have the same pole locations in the s plane because they have the same integral-equation operator; only the source fields are different. For the short-circuit boundary value problem this leads to a parallel combination of series "resonant" circuits with series voltage sources. For the open-circuit boundary value problem one has the dual situation of a series combination of parallel "resonant" circuits with parallel current sources. More recent investigations have centered on canonical problems for exploring the realizability of such networks. Results have been obtained by Pearson et al. [29-31], Singaraju and Baum [26], and Sharpe and Roussi [32].

3.6 Calculation of Natural Frequencies

Initial computations of the natural frequencies from the MoM matrix determinant in (2.2) were by classical Newton and Muller zero-searching techniques [36]. Following an early paper in 1974 [35], Baum, Giri, and Singaraju developed contour integral techniques including computer programs to efficiently and accurately compute all the natural frequencies in a given portion of the s plane [37,39]. This is also reviewed in [18]. Also of interest is the variational technique based on EEM concepts proposed by Mittra and Pearson [38].

3.7 Fora and Reviews

An important milestone in SEM development was the first special session at a USNC/URSI meeting in Boulder, Colorado, August 1973. Since that time there have been many SEM sessions at the various USNC/URSI meetings and IEEE Antennas and Propagation symposia. Reviews on the subject have been given at the triennial URSI General Assemblies beginning with the one in Lima, Peru, in 1975. This author has written three major review papers and book chapters on this subject [16-18]; these can be consulted for more complete developments and numerous references. A review [19] by Dolph and Scott treats some of the applicable mathematical theory. Now SEM has reached another milestone with the recent symposium: "Mathematical Foundations of the Singularity Expansion Method," University of Kentucky, November 1980. This special SEM issue is the proceedings of that symposium.

4. CONTINUED DEVELOPMENT

Quo vadimus? Quo vadit SEM? These are difficult questions. SEM is currently being pursued on two levels. First there is the engineering theory and applications oriented to meeting the practical needs of transient and broadband EM applications such as EMP, lightning, and target identification. This is even finding application in acoustic target identification (see references to [45]). It is these applications oriented developments that I have concentrated on in this paper. On another level the mathematicians are pursuing a rigorous exploration of the SEM theory with a view to defining the precise limits of applicability. Other papers in this issue address such points.

From an applications point of view I see some important areas, both theoretical and experimental, for future development. For experimental description of complex electronic equipment we need to apply all our powerful insights concerning the SEM description to obtaining <u>all</u> the SEM pole (and other) parameters from the experimental scattering (or interaction) data. Using (2.5) (in frequency and/or time domains) one can use the factoring of the pole terms to exhibit the dependence of the response on the various separate parameters of the scattering problem. This gives a much more compact representation of the data (in the resonant region) allowing one to much more readily see the important features, including worst cases, etc., of the response. This factorization can also likely be used to more accurately evaluate the SEM parameters by having (2.5) simultaneously fit many data records corresponding to different locations and excitation conditons.

The construction of equivalent circuits needs much more development. Alternate canonical forms (such as ladder networks, etc.) need to be developed. Perhaps other expansions such as a low-frequency expansion [17] could be useful in conjunction with SEM and EEM. Both a deeper understanding of SEM/EEM decomposition of scatterer response, and more accurate and efficient obtaining of these parameters from experimental data, are needed for the target identification problem. This area has a very great practical potential.

References

1. Abraham, M., "Die electrischen Schwingungen um einen stabförmigen Leiter, behandelt nach der Maxwell'schen Theorie," <u>Ann. d. Physik</u>, Vol. 66, pp. 435-472, October, 1898.
2. Abraham, M., "Elektrische Schwingungen in einem freiendigenden Draht," <u>Ann. d. Physik</u>, Vol. 2, pp. 32-61, 1900.
3. Hallén, E., Über die elektrischen Schwingungen in drahtformigen Leitern," <u>Uppsala Universitets Arsskrift</u>, No. 1, pp. 1-102, 1930.
4. Oseen, C.W., "Über die elektromagnetischen Schwingungen an dünnen Ringen," <u>Arkiv för Matematik, Astronomi O. Fysik</u>, Vol. 9, No. 12, pp. 1-30, 1913.
5. Oseen, C.W., "Über die elektromagnetischen Schwingungen an einem dünnen Ankerring," <u>Phys. Zeitschr.</u>, Vol. 14, pp. 1222-1226, 1913.
6. Oseen, C.W., "Über das elektromagnetische Spektrum eines dünnen Ringes," <u>Arkiv för Matematik, Astronomi O. Fysik</u>, Vol. 9, No. 28, pp. 1-34, 1914.
7. Oseen, C.W., "Über die elektromagnetischen Schwingungen an dünnen Stäben," <u>Arkiv för Matematik, Astronomi O. Fysik</u>, Vol. 9, No. 30, pp. 1-27, 1914.

8. Page, L., and N. Adams, "The Electrical Oscillations of a Prolate Spheroid, Paper I," Phys. Rev., Vol. 53, pp. 819-831, 1938.
9. Page, L., "The Electrical Oscillations of a Prolate Spheroid, Paper II," Phys. Rev., Vol. 65, pp. 98-110, 1944.
10. Page, L., "The Electrical Oscillations of a Prolate Spheroid, Paper III," Phys. Rev., Vol. 65, pp. 111-116, 1944.
11. Page, L., and N.T. Adams, Electrodynamics, Dover Publications, Inc., New York, 1965, pp. 341-369.
12. Pocklington, H.C., "Electrical Oscillations in Wires," Proc. Cambridge Phil. Soc., Vol. 9, pp. 324-332, 1897.
13. Lord Rayleigh, "On the Electrical Vibrations Associated with Thin Terminated Conducting Rods," Phil. Mag., Vol. 8, pp. 105-107, July, 1904.
14. Lord Rayleigh, "Elektrische Schwingungen an einem dunnen Ankerring," Phys. Zeitschr., Vol. 13, pp. 842-848, 1912.
15. Stratton, J.A., Electromagnetic Theory, McGraw-Hill Book Company, New York and London, 1941, pp. 554-563.
16. Baum, C.E., "The Singularity Expansion Method," in L.B. Felsen, ed., Transient Electromagnetic Fields, New York: Springer, 1976.
17. Baum, C.E., "Emerging Technology for Transient and Broad-Band Analysis and Synthesis of Antennas and Scatterers," Proc. IEEE, Vol. 64, pp. 1598-1616, November, 1976; also Interaction Note 300, Air Force Weapons Laboratory, Albuquerque, New Mexico, November, 1976.
18. Baum, C.E., "Toward an Engineering Theory of Electromagnetic Scattering: The Singularity and Eigenmode Expansion Methods," in P.L.E. Ushlenghi, ed., Electromagnetic Scattering, New York: Academic Press, 1978.
19. Dolph, C.L., and R.A. Scott, "Recent Developments in the Use of Complex Singularities in Electromagnetic Theory and Elastic Wave Propagation," in P.L.E. Ushlenghi, ed., Electromagnetic Scattering, New York: Academic Press, 1978.
20. Baum, C.E., "On the Singularity Expansion Method for the Solution of Electromagnetic Interaction Problems," Interaction Note 88, Air Force Weapons Laboratory, Albuquerque, New Mexico, December 1971.
21. Baum, C.E., "Singularity Expansion of Electromagnetic Fields and Potentials Radiated from Antennas or Scattered from Objects in Free Space," Sensor and Simulation Note 179, Air Force Weapons Laboratory, Albuquerque, New Mexico, May, 1973.
22. Baum, C.E., "On the Eigenmode Expansion Method for Electromagnetic Scattering and Antenna Problems, Part 1: Some Basic Relations for Eigenmode Expansions and Their Relation to the Singularity Expansion," Interaction Note 229, Air Force Weapons Laboratory, Albuquerque, New Mexico, January, 1975.
23. Baum, C.E., and L.W. Pearson. "On the Convergence and Numerical Sensitivity of the SEM Pole-Series in Early Time Scattering Response," Electromagnetics, Vol. 1, pp. 209-228, April-June, 1981.
24. Marin, L., and R.W. Latham, "Analytical Properties of the Field Scattered by a Perfectly Conducting, Finite Body," Interaction Note 92, Northrop Corporate Laboratories, Pasadena, California, January, 1972.
25. Sancer, M.I., A.D. Varvatsis, and S. Siegel, "Pseudosymmetric Eigenmode Expansion for the Magnetic Field Integral Equation and SEM Consequences," Interaction Note 355, TDR, Inc., Los Angeles, California, October, 1978.
26. Baum, C.E., and B.K. Singaraju, "The Singularity and Eigenmode Expansion Methods with Application to Equivalent Circuits and Related Topics," in V.K. Varadan and V.V. Varadan, eds., Acoustic, Electromagnetic, and Elastic Wave Scattering--Focus on the T-Matrix Approach, Oxford: Pergamom Press, 1980; also Interaction Note 377, Air Force Weapons Laboratory, Albuquerque, New Mexico, September, 1978.
27. Baum, C.E., "Single Port Equivalent Circuits for Antennas and Scatterers," Interaction Note 295, Air Force Weapons Laboratory Albuquerque, New Mexico, March, 1976.
28. Lee, S.W., and B. Leung, "The Natural Resonance Frequency of a Thin Cylinder and Its Application to EMP Studies," Interaction Note 96, Antenna Laboratory, University of Illinois, Urbana, Illinois, February, 1972.
29. Melson, G.B., and L.W. Pearson, "SEM Characterization of the Electromagnetic Scattering from Thin and Thick Open-Ended Cylinders," Interaction Note 402, University of Kentucky, Lexington, Kentucky, November, 1980.
30. Michalski, K.A., and L.W. Pearson, "Synthesis Methods Applicable to SEM Pole-Pair Admittances," Interaction Note 407, University of Kentucky, Lexington, Kentucky, December, 1979.
31. Pearson, L.W., and D.R. Wilton, "Theoretical Aspects of the Physical Realizability of Broad-Band Equivalent Circuits for Energy Collecting Structures," IEEE Trans. Antennas Propagat., Vol. AP-29, pp. 697-707, September, 1981; also Interaction Note 386, University of Kentucky, Lexington, Kentucky, and the University of Mississippi, University, Mississippi, July, 1979.
32. Sharpe, C.B., and C.J. Roussi, "Equivalent Circuit Representation of Radiation Systems," Interaction Note 361, The University of Michigan Radiation Laboratory, Department of Electrical and Computer Engineering, Ann Arbor, Michigan, April, 1979.
33. Tesche, F.M., "On the Analysis of Scattering and Antenna Problems Using the Singularity Expansion Technique," IEEE Trans. Antennas Propagat., Vol. AP-21, pp. 53-62, January, 1973; also Interaction Note 102, The Dikewood Corporation, Albuquerque, New Mexico, April, 1972.
34. Tesche, F.M., "The Far-Field Response of a Step-Excited Linear Antenna Using SEM," IEEE Trans. Antennas Propagat., Vol. AP-23, pp. 834-838, November, 1975; also Sensor and Simulation Note 177, The Dikewood Corporation, Albuquerque, New Mexico, May, 1973.
35. Baum, C.E., "On the Use of Contour Integration for Finding Poles, Zeros, Saddles, and Other Function Values in the Singularity Expansion Method," Mathematics Note 35, Air Force Weapons Laboratory, Albuquerque, New Mexico, February, 1974.
36. Crow, T.T., B.D. Graves, and C.D. Taylor, "Numerical Techniques Useful in the Singularity Expansion Method as Applied to Electromagnetic Interaction Problems," Mathematics Note 27, Mississippi State University, Mississippi State, Mississippi, December, 1972.
37. Giri, D.V., and C.E. Baum, "Application of Cauchy's Residue Theorem in Evaluating the Poles and Zeros of Complex Meromorphic Functions and Apposite Computer Programs," Mathamatics Note 55, Air Force Weapons Laboratory, Albuquerque, New Mexico, 1978.
38. Mittra, R., and L.W. Pearson, "A Variational Method for Efficient Determination of SEM Poles," IEEE Trans. Antennas Propagat., Vol. AP-26, pp. 354-358, March, 1978; also Mathematics Note 62, University of Illinois at Urbana-Champaign, Urbana, Illinois, and University of Kentucky, Lexington, Kentucky, December, 1976.
39. Singaraju, B.K., D.V. Giri, and C.E. Baum, "Further Developments in the Application of Contour Integration to the Evaluation of the

Zeros of Analytic Functions and Relevant Computer Programs," Mathematics Note 42, Air Force Weapons Laboratory, Albuquerque, New Mexico, March, 1976.
40. Van Blaricum, M.L., and R. Mittra, "A Technique for Extracting the Poles and Residues of a System Directly from Its Transient Response," IEEE Trans. Antennas Propagat., Vol. AP-23, pp. 777-781, November, 1975; also Interaction Note 245, University of Illinois, Urbana, Illinois, February, 1975.
41. Schwinger, J., "The Theory of Obstacles in Resonant Cavities and Waveguides," MIT Radiation Laboratory Report, May, 1943.
42. Pearson, L.W., M.L. Van Blaricum, and R. Mittra, A New Method for Radar Target Recognition Based on the Singularity Expansion Method, Record of IEEE International Radar Conference, Arlington, Virginia, April, 1975, pp. 452-457.
43. Moffatt, D.L., and R.K. Mains, "Detection and Discrimination of Radar Targets," IEEE Trans. Antennas Propagat., May 1975, pp. 358-367.
44. Deadrick, F.J., H.G. Hudson, E.K. Miller, J.A. Landt, and A.J. Poggio, Object Identification via Pole Extraction from Transient Fields, USNC/URSI Meeting, University of Illinois, 3-5 June, 1975, p. 67.
45. Überall, H., and C.G. Gaunard, "Resonances and Surface Waves: The Inverse Scattering Problem," Electromagnetics, Vol. 1, pp. 473-479, October-December, 1981.
46. Harrington, R.F., Field Computation by Moment Methods, Macmillan, 1968.
47. Joint Special Issue on the Nuclear Electromagnetic Pulse, IEEE Trans. Antennas Propagat., AP-26, January, 1978, and IEEE Trans. EMC, EMC-20, February, 1978.

APPENDIX: RECENT DEVELOPMENTS

Since this paper was originally published many additional papers concerned with SEM topics have been published. With no attempt to be exhaustive I would like to point out a few areas where progress has been made. For brevity I will pass over various numerical applications in favor of more general results.

A. PROCESSING OF DATA TO OBTAIN SEM PARAMETERS

Much work in the past has dealt with the analysis of experimental transient waveforms and frequency spectra to obtain estimates of the poles (and perhaps residues) contained therein. The interest might be EMP interaction or radar target identification. While this continues to refine the algorithms there is potentially more (and perhaps more accurate) information to be obtained by processing a number of transient waveforms or frequency spectra corresponding to the same scatterer, but at different locations and/or incident wave conditions. A recent paper by this author presents several concepts concerning the use of the SEM and EEM representations to enforce conditions concerning the information to be extracted from the data [A1].

The group at University of Michigan has pursued some concepts using multiple spatial locations for frequency spectra (using scale models) [A2,A3]. Dudley [A4] has pursued such matters in time domain. Beginning with some natural mode extraction in [A5] by Lin and Cordaro, general algorithms involving both natural modes and coupling coefficients in time domain have been developed by Park and Cordaro [A6]. This latter has led so far to the determination of the coupling coefficients for several symmetric natural modes of an actual flying NASA F-106 aircraft [A7].

References

A1. Baum, C.E., "A Priori Application of Results of Electromagnetic Theory to the Analysis of Electromagnetic Interaction Data," Interaction Note 444, Air Force Weapons Laboratory, Albuquerque, New Mexico, February, 1985.
A2. Ksienski, D.A., "Pole and Residue Extraction from Measured Data in the Frequency Domain Using Multiple Data Sets," Mathematics Note 80, University of Michigan, Ann Arbor, Michigan, October, 1984, and Radio Science, January-February, 1985, pp. 13-19.
A3. Ksienski, D.A., and T.M. Willis, "Numerical Methods of Noise Reduction for Frequency Domain SEM," University of Michigan, Ann Arbor, Michigan, October, 1984, and Electromagnetics, 1984, pp. 393-405.
A4. Dudley, D.G., "Complex Resonance Identification from Data at Multiple Spatial Locations," Mathematics Note 84, University of Arizona, Tucson, Arizona, January, 1985.
A5. Lin, C.A., and J.T. Cordaro, "Singularity Expansion Method Parameter Measurement," Interaction Note 409, University of New Mexico, Albuquerque, New Mexico, May, 1981, and (same authors) "Determination of the SEM Parameters for an Aircraft Model from the Transient Surface Current," Electromagnetics, 1983, pp. 65-75.
A6. Park, S.-W., and J.T. Cordaro, "Preprocessing Techniques for the Estimation of SEM Parameters from Multiple Measurements," Mathematics Note 85, University of New Mexico, Albuquerque, N. Mexico, August, 1985.
A7. Park, S.-W., and J.T. Cordaro, "Application of the AIPA to F-106 Data," Interaction Application Memo 42, University of New Mexico, Albuquerque, New Mexico, August, 1985.

B. ENTIRE-FUNCTION CONTRIBUTION

From the beginning the question of an entire-function contribution to the SEM representation has been a difficult one. Since the entire function can be thought of as a singularity at infinity, it is basically a high-frequency (or early-time) contribution. A series of papers [B1-B3] considers cases under which an entire function can be avoided, based on asymptotic estimates of terms appearing in the integral equations (which includes [23] as well). Nevertheless at early times convergence troubles occur [B5]. It is pointed out in [B6] that scattered fields (instead of say surface current density) do require an entire function in their representation. A series of short papers [B7-B9] go further into this early-time problem, including relation to other early-time representations (also see part D).

It is interesting to note that at least for surface current density and surface charge density, numerous numerical results for step-function excitation show that at least in such cases no entire-function is required to get the correct answer [16,B4]. Perhaps, though, a delta-function excitation does require an entire function (say a delta function) in the response. One example, the input admittance of a finite-length cylindrical antenna has been treated in [34] and shown to have such an entire function.

I expect that this entire-function question will remain somewhat controversial, for a while at least.

Fortunately for some practical applications this is not critical.

References

B1. Pearson, L.W., "Evidence that Bears on the Left Half Plane Asymptotic Behavior of the SEM Expansion of Surface Currents," Electromagnetics, 1981, pp. 395-402.

B2. Wilton, D.R., "Large Frequency Asymptotic Properties of Resolvent Kernels," Electromagnetics, 1981, pp. 403-411.

B3. Pearson, L.W., D.R. Wilton, and R. Mittra, "Some Implications of the Laplace Transform Inversion on SEM Coupling Coefficients in the Time Domain," Electromagnetics, 1982, pp. 181-200.

B4. Michalski, K.A., "On the Class 1 Coupling Coefficient Performance in the SEM Expansion for the Current Density on a Scattering Object," Electromagnetics, 1982, pp. 201-209.

B5. Morgan, M.A., "Singularity Expansion Representations of Fields and Currents in Transient Scattering," IEEE Trans. Antennas Propagat., 1984, pp. 466-473.

B6. Pearson, L.W., "A Note on the Representation of Scattered Fields as a Singularity Expansion," IEEE Trans. Antennas Propagat., 1984, pp. 520-524.

B7. Felsen, L.B., "Comments on Early Time SEM," IEEE Trans. Antennas Propagat., 1985, pp. 118-119.

B8. Dudley, D.G., "Comments on SEM and the Parametric Inverse Problem," IEEE Trans. Antennas Propagat., 1985, pp. 119-120.

B9. Morgan, M.A., "Response to Comments Regarding SEM Representations," IEEE Trans. Antennas Propagat., 1985, p. 120.

C. BRANCH CONTRIBUTION

In a general SEM formalism there can be contributions from branch cuts. General formulae for this are found in [18]. One recent paper [C1] has actually evaluated a branch integral which appears in the case of an antenna in a lossy medium. Another paper [C2] evaluates a branch integral associated with an infinitely large body (a circular cylinder). As discussed in another paper [C3] the presence of a large number of scatterers, when treated statistically for the multiple scattering, also leads to what is a branch integral.

Another question arises concerning the eigenvalues of integral-equation operators. The zeros of these eigenvalues are natural frequencies of the scatterer (or antenna). It is well established now that perfectly conducting scatterers in free space have no branch contributions in the SEM representation. However, this does not necessarily mean that the eigenvalues have no branch integrals. A recent paper [C4] makes a case that some scatterers admit eigenvalues with branch cuts.

References

C1. Giri, D.V., and F.M. Tesche, "On the Use of Singularity Expansion Method for Analysis of Antennas in Conducting Media," Electromagnetics, 1981, pp. 455-471.

C2. Chang, C.-I., D.P. Nyquist, K.-M. Chen, and B.C. Drachman, "Singularity Expansion Method Formulation for Impulse Response of a Perfectly Conducting Thick Cylinder," Radio Science, 1985, pp. 1025-1030.

C3. Riley, D.J., W.A. Davis, and I.M. Besieris, "The Singularity Expansion Method and Multiple Scattering," Radio Science, 1985, pp. 20-234.

C4. Wilton, D.R., K.A. Michalski, and L.W. Pearson, "On the Existence of Branch Points in the Eigenvalues of the Electric Field Integral Equation Operator in the Complex Frequency Plane," IEEE Trans. Antennas Propagat., 1983, pp. 86-91.

D. CONNECTION WITH HIGH-FREQUENCY REPRESENTATIONS

Knowing that SEM works best at intermediate and late times, there has been some interest in combining an SEM representation with a more appropriate early-time (high-frequency) representation (including specular, edge-diffraction, creeping-wave, etc., contributions). Some success has been reported [D1-D5]. It appears that there are various ways the SEM poles can be ordered, giving in this view what have been referred to as layers (roughly parallel to the $j\omega$ axis of the s plane) [D1-D4]. One paper [D5] gives a very general theory of somewhat arbitrary partitioning the response between poles and these early-time contributions.

References

D1. Howard, A.Q., "A Geometric Theory of Natural Oscillation Frequencies in Exterior Scattering Problems," Interaction Note 378, University of Arizona, Tucson, Arizona, October, 1979.

D2. Gaunard, G.C., and H. Überall, "Complex-Frequency Poles and Creeping-Wave Transients in Electromagnetic-Wave Scattering," IEEE Proc., 1983, pp. 172-174.

D3. Heyman, E., and L.B. Felsen, "Creeping Waves and Resonances in Transient Scattering by Smooth Convex Objects," IEEE Trans. Antennas Propagat., 1983, pp. 426-437.

D4. Überall, H., and C.G. Gaunard, "Relation Between the Ringing of Resonances and Surface Waves in Radar Scattering," IEEE Trans. Antennas Propagat., 1984, pp. 1071-1079.

D5. Heyman, E., and L.B. Felsen, "A Wavefront Interpretation of the Singularity Expansion Method," IEEE Trans. Antennas Propagat., 1985, pp. 706-718.

E. EQUIVALENT CIRCUITS

One of the early-recognized applications of SEM was for the synthesis of equivalent circuits for antennas and scatterers. Recall [C1], in which a branch integral was approximated in an equivalent-circuit sense. In addition two papers have recently appeared from the University of Mississippi group on this subject [E1,E2].

References

E1. Pearson, L.W., and D.R. Wilton, "On the Physical Realizability of Broad-Band Lumped-Parameter Equivalent Circuits for Energy-Collecting Structures," Interaction Note 386, University of Mississippi, Oxford Mississippi, July, 1979, and IEEE Trans. Antennas Propagat., 1981, pp. 697-707.

E2. Michaelski, K.A., and L.W. Pearson, "Synthesis of SEM-Derived Equivalent Circuits for Electromagnetic Energy Collecting Structures," Interaction Note 423, University of Mississippi, October, 1981, and "Equivalent Circuit Synthesis for a Loop Antenna Based on the Singularity Expansion Method," IEEE Trans. Antennas Propagat., 1984, pp. 433-441.

F. TARGET IDENTIFICATION

The use of SEM for target identification has proceeded considerably. The K-pulse concept was introduced by Kennaugh [F1] with the idea of suppressing the poles of the target by providing zeros in the Laplace transform of the excitation waveform to cancel them and thereby measure them. A related concept, the E-pulse has been pursued with considerable success by the Michigan State group [F2-F4]. This problem has also been pursued from a different viewpoint by Howell and Überall [F5].

Two papers from the Ohio State group [F6,F7] apply SEM concepts to removing undesirable poles in waveforms. In particular transmitting and receiving (radar) antennas introduce poles in the waveforms; these can be removed via data processing.

References

F1. Kennaugh, E.M., "The K-Pulse Concept," IEEE Trans. Antennas Propagat., 1981, pp. 327-331.
F2. Chen, K.M., "Radar Waveform Synthesis Method--A New Radar Detection Scheme," IEEE Trans. Antennas Propagat., 1981, pp. 553-566.
F3. Chen, K.-M., D.P. Nyquist, D. Westmoreland, C.-I. Chuang, and B. Drachman, "Radar Waveform Synthesis for Single-Mode Scattering by a Thin Cylinder and Application for Target Discrimination," IEEE Trans. Antennas Propagat., 1982, pp. 867-880.
F4. Rothwell, E.J., D.P. Nyquist, K.-M. Chen, and B. Drachman, "Radar Target Discrimination Using the Extinction-Pulse Technique," IEEE Trans. Antennas Propagat., 1985, pp. 929-937.
F5. Howell, W.E., and H. Überall, "Complex Frequency Poles of Radar Scattering from Coated Conducting Spheres," IEEE Trans. Antennas Propagat., 1984, pp. 624-627.
F6. Chan, L.-C., L. Peters, and D. Moffatt, "Improved Performance of a Subsurface Radar Target Identification System Through Antenna Design," IEEE Trans. Antennas Propagat., 1981, pp. 307-311.
F7. Volakis, J.L., and L. Peters, "Improved Identification of Underground Targets Using Video Pulse Radars by Elimination of Undesired Natural Resonances," IEEE Trans. Antennas Propagat., 1983, pp. 334-340.

Chapter News

John K. Schindler
AP-S Chapter Activity Chairman
RADC/EEA
Hanscom AFB
Bedford, MA 01731
(617) 377-3932

COLUMBUS CHAPTER WINS AP-S BEST CHAPTER AWARD

The AP-S Columbus Ohio Chapter was presented the Society Best Chapter Award for 1985-1986 at the Philadelphia International AP-S Symposium on June 11. Receiving the award was Dr. Roberto Rojas, newly elected Secretary of the Chapter, on behalf of the Chapter Chairman, Dr. Inder Gupta. Dr. Edward Newman was the Columbus Chapter Vice Chairman and Dr. Allen Dominek was Secretary. All of the Columbus Chapter officers are from the Ohio State University ElectroScience Laboratory. This was the second consecutive year that the Columbus Chapter has won this important Society award.

The Columbus Chapter conducted an outstanding program of technical meetings for its members. The program consisted of ten technical meetings covering a variety of topics in antennas, propagation, scattering and fundamental electromagnetic theory. Of special note was a "back to school" meeting during which 185 students heard Dr. Jonathan Young describe the Ohio State contributions to the understanding of electromagnetic scattering. This special program was held to inform students of the interesting work of the Society and encourage students to join the IEEE and AP-S.

The AP-S Best Chapter Award is presented each year to the Chapter which provides the best service to Society members at the Section level. The Award results from a vote of current year Chapter Chairmen. Their vote is based on a detailed report from each Chapter describing meeting topics, duration and attendance during the year.

SIGNIFICANT GROWTH IN SOCIETY CHAPTERS

The number of AP-S Chapters has grown by more than a third during this past year. Moreover, it appears that over 90% of these Chapters are active in providing service to their Society members. For the first time, the Society maintains an organizational presence on five continents -- the Americas, Europe, Africa and Asia.

This record of growth and interest in Society technical activities is impressive and due to the outstanding efforts of our Chapter officers, speakers and Distinguished Lecturers. As the following pages of Chapter news indicate, our Chapters have an enviable record of accomplishment in organizing and conducting technical programs throughout the year.

The Society recognizes this Chapter vitality and supports Chapter activities as an important means for providing member services. The Distinguished Lecturer program, described in the June issue of the Newsletter, permits each Chapter to hear