The Singularity Expansion Method and Its Application to Target Identification

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The singularity expansion method (SEM) for quantifying the transient electromagnetic scattering from targets illuminated by pulsed EM radiation is reviewed. SEM representations for both induced currents and scattered fields are presented. Natural-resonance-based target identification schemes, based upon the SEM, are described. Various techniques for the extraction of natural-resonance modes from measured transient response waveforms are reviewed. Discriminant waveforms for target identification, synthesized-based upon the complex natural-resonance frequencies of the relevant targets, are exposed. Particular attention is given to the aspect-independent (extinction) E-pulse and (single-mode) S-pulse discriminant waveforms which, when convolved with the late-time pulse response of a matched target, produce null or mono-mode responses, respectively, through natural-mode annihilation. Extensive experimental results for practical target models are included to validate the E-pulse target discrimination technique. Finally, anticipated future extensions and areas requiring additional research are identified.

I. INTRODUCTION

The singularity expansion method (SEM) was introduced in 1971 as a way to represent the solution of electromagnetic interaction or scattering problems in terms of the singularities in the complex-frequency (s or two-sided-Laplace-transform) plane [3]. Particularly for the pole terms associated with a scatterer (natural frequencies), their factored form separates the dependencies on various parameters of the incident field, observer location, and scatterer characteristics, with an equally simple form in both frequency (poles) and time (damped sinusoids) domains. Besides the application to EMP (nuclear electromagnetic pulse) interaction problems, it was recognized from the beginning that SEM was useful for scatterer identification due to the aspect-independent nature of the pole locations in the complex frequency plane. There has been quite a lot of work done on SEM since the basic structure was outlined in 1971 [45]. The complete bibliography is far too lengthy to be included here, but is included in [29]. There are several book chapters and review papers which summarize the major parts of SEM theory [17], [19], [23], [24], [45]; one of these summarizes numerical examples of surface currents [19]. Here we also mention the early papers which began SEM [3]–[5].

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II. SINGULARITY EXPANSION OF CURRENTS ON SCATTERERS

As in Fig. 1, let there be some finite-size object in free space. While this is typically taken as a perfectly conducting object with only a surface current density on the surface S (with coordinate $\vec{r}_s$) the results are readily generalized to volume current density. The general coordinate $\vec{r}$ is chosen referenced to the center of the minimum circumscribing sphere (radius a) of the scatterer for optimum aspect-independent convergence of the pole series [30], [57]. The incident field is taken as a plane wave with electric field

$$\tilde{\vec{E}}^{(\text{inc})}(\vec{r},s) = E_o\tilde{f}(s)\vec{1}_p e^{-\gamma\vec{1}_1\cdot\vec{r}},$$
$$\vec{E}^{(\text{inc})}(\vec{r},t) = E_o f(t-\frac{\vec{1}_1\cdot\vec{r}}{c})\vec{1}_p \qquad (2.1)$$

where

γ	=		s/c,
s	≡		complex frequency (Laplace-transform variable),
c	=	$(\mu_0\epsilon_0)^{-1/2}$	speed of light,
$\vec{1}_1$	≡		direction of incidence,
$\vec{1}_p$	≡		direction of polarization ($\vec{1}_p\cdot\vec{1}_1 = 0$),
$f(t)$	≡		waveform,
$\sim$	≡		Laplace transform (two-sided).

The surface current density is related to the incident field via an integral equation:

$$\langle\tilde{\overleftrightarrow{Z}}_t(\vec{r}_s,\vec{r}_s^{\,\prime};s);\tilde{\vec{J}}_s(\vec{r}_s^{\,\prime},s)\rangle$$

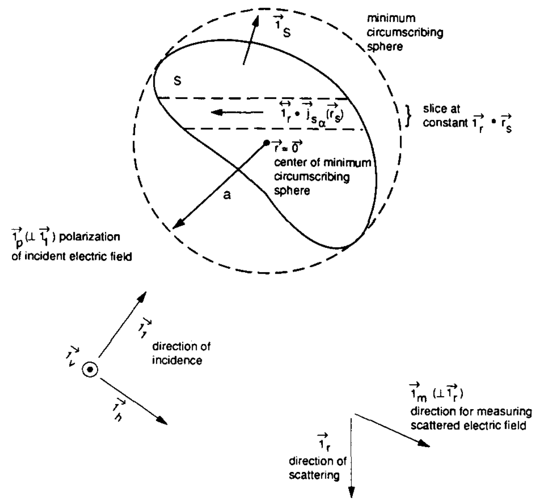


Fig. 1. Finite-size object in free space illuminated by plane wave.

$$
\begin{aligned}
&= \tilde{\vec{E}}_t^{(\mathrm{inc})}(\vec{r}_s, s) \\
&= \overleftrightarrow{1}_S(\vec{r}_s) \cdot \tilde{\vec{E}}^{(\mathrm{inc})}(\vec{r}_s, s) \qquad (2.2)
\end{aligned}
$$

$$
\underset{S}{\overleftrightarrow{1}}(\vec{r}_s) = \overleftrightarrow{1} - \vec{1}_S(\vec{r}_s)\vec{1}_S(\vec{r}_s)
$$

$$
\overleftrightarrow{1} \equiv \text{ identity } = \vec{1}_x\vec{1}_x + \vec{1}_y\vec{1}_y + \vec{1}_z\vec{1}_z
$$

$$
\vec{1}_S(\vec{r}_s) \equiv \text{ outward pointing normal to } S.
$$

Here we have taken the impedance or E-field integral equation. Any other such equation will also do since here we are concerned only with the general form of the solution, rather than numerical computations. The kernel $\tilde{\overleftrightarrow{Z}}_t(\vec{r}_s, \vec{r}_s\,'; s)$ here is symmetric and involves the free-space dyadic Green's function [58]. Denote the symmetric product (no implied conjugation) as $\langle , \rangle$ involving integration over the common coordinates. The SEM form of the solution is

$$
\begin{aligned}
&\tilde{\vec{J}}_s(\vec{r}_s, s) \\
&= E_o \sum_{\alpha} \tilde{f}(s_\alpha)\eta_\alpha(\vec{1}_1, \vec{1}_p)\vec{j}_{s_\alpha}(\vec{r}_s)[s - s_\alpha]^{-1} e^{-(s-s_\alpha)t_o} \\
&+ \text{ singularities of } \tilde{f}(s) + \text{ possible entire function} \qquad (2.3)
\end{aligned}
$$

where only first-order poles have been included, but poles of higher order are possible in special circumstances [17], [24]. We have the terms:

$$
\langle \tilde{\overleftrightarrow{Z}}_t(\vec{r}_s, \vec{r}_s\,'; s_\alpha); \vec{j}_{s_\alpha}(\vec{r}_s\,') \rangle = \vec{0}
$$

$s_\alpha \equiv$ natural frequency, $\gamma_\alpha \equiv \frac{s_\alpha}{c}, \vec{j}_{s_\alpha}(\vec{r}_s) \equiv$ natural mode

$$
\eta_\alpha\left(\vec{1}_1, \vec{1}_p\right) = \frac{\vec{1}_p \cdot \left\langle e^{-\gamma_\alpha \vec{1}_1 \cdot \vec{r}_s}, \vec{j}_{s_\alpha}(\vec{r}_s\,')\right\rangle}{\left\langle \vec{j}_{s_\alpha}(\vec{r}_s); \frac{\partial}{\partial s}\tilde{\overleftrightarrow{Z}}_t(\vec{r}_s, \vec{r}_s\,'; s)|_{s=s_\alpha}; \vec{j}_{s_\alpha}(\vec{r}_s\,')\right\rangle}
$$

$$
\equiv \text{coupling coefficient.} \qquad (2.4)
$$

In time domain the poles $[s - s_\alpha]^{-1} e^{-(s-s_\alpha)t_o}$ are replaced by $e^{s_\alpha t}u(t - t_o)$. Note the inclusion of a turn-on time t_o since the definition of $t = 0$ is arbitrary (say, a/c, or first arrival at the scatterer) [30]. While we are not concerned here with numerical computations *per se*, it is instructive to think of the general integral equation (2.2) as a matrix equation (via the moment method) with N expansion and N testing functions [19] (here chosen symmetrically) as

$$
(\tilde{Z}_{t_{n,m}}(s)) \cdot (\tilde{J}_{s_n}(s)) = (\tilde{E}_{t_n}^{(\mathrm{inc})}(s)) \qquad (2.5)
$$

Then (2.4) becomes

$$
(\tilde{Z}_{t_{n,m}}(s_\alpha)) \cdot (\tilde{j}_{s_n})_\alpha = (0_n), \det((\tilde{Z}_{t_{n,m}}(s_\alpha))) = 0
$$

$$
\eta_\alpha\left(\vec{1}_1, \vec{1}_p\right) = \frac{1}{E_0\tilde{f}(s_\alpha)} \frac{\left(\tilde{E}_{t_n}^{(\mathrm{inc})}(s_\alpha)\right) \cdot (j_{s_n})_\alpha}{(j_{s_n})_\alpha \cdot \frac{\partial}{\partial s}\left(\tilde{Z}_{t_{n,m}}(s)\right)|_{s=s_\alpha} \cdot (j_{s_n})_\alpha}. \qquad (2.6)
$$

Since these terms are experimentally observable and can be obtained from scattering data [53], then all correct formulations (integral equations or other) must give the same results. As can be seen in these formulas the s_α are aspect-independent (i.e., independent of the incident-field parameters), this being a powerful result which will be discussed later in the context of scatterer identification. In the context of computations, the determinant equation in (2.6) gives a means of calculating the s_α. While one can find the zeros of $\det((\tilde{Z}_{t_{n,m}}(s)))$ by various iterative procedures, there are two powerful contour-integral techniques involving the argument number (generalized) and the residue theorem which rely on the property of the determinant as an analytic function in the complex s plane [24].

One of the important early SEM results was that the response (2.3) included no branch integrals provided we are dealing with a finite-size, perfectly conducting (or suitable-simple-media) object and the exciting waveform had no branch cuts [3], [4]. Two-dimensional objects (infinite in one direction), on the other hand, do have a branch contribution [19]. If the object is embedded in an infinite lossy medium, there is also a branch cut introduced [31]. Branch cuts are readily included in the SEM formalism when needed and can be thought of as a continuous distribution of poles [24].

There is the case of the elusive entire function (or singularity at ∞). While this is an area of continuing research, let us briefly summarize what is currently known. Assume that the turn-on time t_o in (2.3) is judiciously chosen so that it is when or before the incident wave reaches the observation position on S [19], and no sooner than the earliest time that the pole series converges [30]. The numerical results for step-function incident waves ($\tilde{f}(s) = 1/s$) show that no entire function is required in this case for the various example problems [19]. Furthermore, there is no pole at $s = 0$ due to the lack of scatterer response there [3]. The impulse (δ function) response is another matter. As discussed in [40] this leads to an "essential entire function" related to the physical optics terms. Similarly, if one considers an antenna (such as a gap in a wire), one can have such an entire function as a simple additive constant in the input admittance [24], [27]. Asymptotic behavior as $s \to 0$ and $s \to \infty$ can help in establishing the best form to use [24]. Note that this entire-function determination is separate from what is the most efficient early-time representation. Instead of summing up a large number of poles, one can use a small number of high-frequency terms (GTD) involving physical optics and creeping waves [40].

Mentioning a few related topics, there is the eigenmode expansion method (EEM) in which the integral operator in (2.2) is diagonalized to give s-dependent eigenmodes which can be used to order the natural modes [17], [24]. This is not unique in the sense that there are various integral equations that one can use, giving different sets of eigenmodes. The impedance integral equation is of interest, in that one can consider the synthesis of eigenimpedances (shifting natural frequencies by impedance loading of the scatterer). There is also the whole subject of synthesis of equivalent circuits from the SEM representation of antennas and scatterers [24].

III. Extension to Scattered Far Fields

Consideration of the currents on the scatterer has already led to the location of the s_α in the s plane (aspect independent) as a useful property for identification. Extending to the far fields one can ask if there are other potentially useful properties. Early considerations of this were in terms of far natural modes [6], [15], [17], [19].

Recently a more complete theory has emerged [57]. The far scattered field is written in SEM form as

$$\begin{aligned}\tilde{\vec{E}}_f(\vec{r},s) &= \frac{E_0}{4\pi r}e^{-\gamma r}\sum_\alpha \tilde{f}(s_\alpha)W_\alpha \overleftrightarrow{C}_{f_\alpha}\left(\vec{1}_r,\vec{1}_1\right) \\ &\quad \cdot \vec{1}_p[s-s_\alpha]^{-1}e^{-(s-s_\alpha)t_0} \\ W_\alpha &\equiv w_\alpha^2 \equiv -s_\alpha\mu_0 \\ &\quad \left\langle \vec{j}_{s_\alpha}(\vec{r}_s); \frac{\partial}{\partial s}\overleftrightarrow{Z}_t(\vec{r}_s,\vec{r}_s\,';s)\Big|_{s=s_\alpha}; \vec{j}_{s_\alpha}(\vec{r}_s\,')\right\rangle^{-1} \\ \overleftrightarrow{C}_{f_\alpha}\left(\vec{1}_r,\vec{1}_1\right) &= \vec{C}_{r_\alpha}\left(\vec{1}_r\right)\vec{C}_\alpha\left(\vec{1}_1\right) \\ \vec{C}_\alpha\left(\vec{1}_1\right) &\equiv \left\langle \overleftrightarrow{1}_1 e^{-\gamma_\alpha \vec{1}_1\cdot\vec{r}_s\,'}; \vec{j}_{s_\alpha}(\vec{r}_s\,')\right\rangle, \\ \vec{C}_{r_\alpha}\left(\vec{1}_r\right) &= \left\langle \overleftrightarrow{1}_r e^{\gamma_\alpha \vec{1}_r\cdot\vec{r}_s\,'}; \vec{j}_{s_\alpha}(\vec{r}_s\,')\right\rangle \\ \overleftrightarrow{1}_1 &\equiv \overleftrightarrow{1} - \vec{1}_1\vec{1}_1, \\ \overleftrightarrow{1}_r &\equiv \overleftrightarrow{1} - \vec{1}_r\vec{1}_r \text{ (transverse identities).}\end{aligned} \tag{3.1}$$

Note the reciprocity relationship:

$$\vec{C}_{r_\alpha}(\vec{1}_r) = \vec{C}_\alpha(-\vec{1}_r), \overleftrightarrow{C}_{f_\alpha}(\vec{1}_r,\vec{1}_1) = \overleftrightarrow{C}_{f_\alpha}^{T}(\vec{1}_1,\vec{1}_r). \tag{3.2}$$

The scattering residue for the s_α pole takes the form of a single dyad (for a single mode $\vec{j}_{s_\alpha}$ (nondegenerate)) which, taken as a 2×2 matrix, has zero determinant, this property being observable in experimental scattering data. Note that for each pole the polarization of the far field is determined by the vector $\vec{C}_{r_\alpha}(\vec{1}_r)$, which in combination with the complex exponential $e^{s_\alpha t}u(t-t_0)$ gives what can be termed elliptical spiral polarization. This is a characteristic of the scatterer, not the incident field, and so can be termed a scatterer polarization vector (as seen at the observer). Referring to Fig. 1, one can interpret this scatterer polarization as the average direction of the natural mode currents $({}_\perp\vec{1}_r)$ as weighted by the integral with γ_α along the $\vec{1}_r$ direction. For long slender objects this gives a simple geometric interpretation.

Figure 1 gives the unit vectors for incident and far scattered fields. Note that in the far-field expansion one cannot in general let $|s| \to \infty$ since the transition from near to far field is in general a function of s. In time domain this appears in the form of errors in the expression for very small time changes which do not concern us here.

In terms of coupling coefficients (scalars) we have

$$\begin{aligned}\eta_{f_\alpha}(\vec{1}_r,\vec{1}_m;\vec{1}_1,\vec{1}_p) &= W_\alpha\vec{1}_m\cdot\overleftrightarrow{C}_{f_\alpha}(\vec{1}_r,\vec{1}_1)\cdot\vec{1}_p \\ &\equiv \text{far coupling coefficient.} \\ &= \eta_{r_\alpha}(\vec{1}_r,\vec{1}_m)\eta_\alpha(\vec{1}_1,\vec{1}_p)\end{aligned} \tag{3.3}$$

$$\eta_\alpha(\vec{1}_1,\vec{1}p) = -\frac{W_\alpha}{s_\alpha\mu_0}\vec{1}_p\cdot\vec{C}_\alpha(\vec{1}_1) \equiv \text{ coupling coefficient}$$

$$\eta_{r_\alpha}(\vec{1}_r,\vec{1}_m) = -s_\alpha\mu_0\vec{1}_m\cdot\vec{C}_{r_\alpha}(\vec{1}_r) \equiv \text{ recoupling coefficient.}$$

If we normalize η_α so that it has value 1 (and peak magnitude) at $\vec{1}_1 = \vec{1}_{1_0}, \vec{1}_p = \vec{1}_{p_0}$, and similarly for η_{r_α}, then we can have

$$\vec{1}_{m_0} = \vec{1}_{p_0}, \vec{1}_{r_0} = -\vec{1}_{1_0} \tag{3.4}$$

which gives the reciprocity-related result

$$\eta_\alpha^{(n)}(\vec{1}_1,\vec{1}_p) = \eta_{r_\alpha}^{(n)}(-\vec{1}_1,\vec{1}_p) \tag{3.5}$$

with superscript n denoting the normalized coefficients. Then the normalized far coupling coefficient is

$$\eta_{f_\alpha}^{(n)}(\vec{1}_r,\vec{1}_m;\vec{1}_1,\vec{1}_p) = \eta_{r_\alpha}^{(n)}(\vec{1}_r,\vec{1}_m)\eta_\alpha^{(n)}(\vec{1}_1,\vec{1}_p). \tag{3.6}$$

For the case of backscattering with measurement parallel to the incident field we have

$$\begin{aligned}\eta_{b_\alpha}(\vec{1}_1,\vec{1}_p) &= \eta_{r_\alpha}(-\vec{1}_1\;\vec{1}_p)\eta_\alpha(\vec{1}_1,\vec{1}_p) \\ &= W_\alpha\vec{1}_p\cdot\overleftrightarrow{C}_{b_\alpha}(-\vec{1}_1,\vec{1}_1)\cdot\vec{1}_p \\ \overleftrightarrow{C}_{b_\alpha}(-\vec{1}_1,\vec{1}_1 &= \vec{C}_\alpha(\vec{1}_1)\vec{C}_\alpha(\vec{1}_1) \text{ (symmetric dyad).}\end{aligned} \tag{3.7}$$

In normalized form this is

$$\eta_{b_\alpha}^{(n)}(\vec{1}_1,\vec{1}_p) = \eta_{r_\alpha}^{(n)}(-\vec{1}_1,\vec{1}_p)\eta_\alpha^{(n)}(\vec{1}_1,\vec{1}_p) = [\eta_\alpha^{(n)}(\vec{1}_1,\vec{1}_p)]^2. \tag{3.8}$$

This is a powerful result in that it implies that one can measure either $\eta_{b_\alpha}^{(n)}$ (far field) or $\eta_\alpha^{(n)}$ (on S) and infer the other. Note that this applies for a case (typical) of nondegenerate modes.

If one has the various polarizations in transmission and reception available, then one can deal with the scattering residue dyadic. For backscattering, we have the usual (for radar) h,v coordinate orientations as in Fig. 1. Then it is convenient to introduce

$$\overleftrightarrow{c}_{b_\alpha}(\vec{1}_1) = W_\alpha \overleftrightarrow{C}_{b_\alpha}(\vec{1}_1) = \vec{c}_\alpha(\vec{1}_1)\vec{c}_\alpha(\vec{1}_1), \vec{c}_\alpha(\vec{1}_1) = w_\alpha \vec{C}_\alpha(\vec{1}_1). \tag{3.9}$$

In this form the $\overleftrightarrow{c}_{b_\alpha}$ is what is measurable in (3.1), the normalization constant being an artifice of scaling the natural mode. For nondegenerate modes this dyad is characterized by a single two-component (transverse) complex vector. This is to be compared to the usual case of a backscattering dyad as a symmetric (due to reciprocity) 2×2 matrix characterized by three complex numbers.

If there is a modal degeneracy, the situation is a bit more complicated. This occurs for a body of revolution (C_∞ symmetry) which, if it has a symmetry plane containing the axis, gives a twofold degeneracy with symmetric and antisymmetric parts with respect to the symmetry plane P through the observer. Then (3.9) generalizes to (five real numbers)

$$\begin{aligned}\overleftrightarrow{c}_{b_\alpha}\left(\vec{1}_1\right) &= \vec{c}_{sy,\alpha'}\left(\vec{1}_1\right)\vec{c}_{sy,\alpha'}\left(\vec{1}_1\right) + \vec{c}_{as,\alpha'}\left(\vec{1}_1\right)\vec{c}_{as,\alpha'}\left(\vec{1}_1\right)\\ &= c_{b_{sy,\alpha'}}\left(\vec{1}_1\right)\vec{1}_{sy}\left(\vec{1}_1\right)\vec{1}_{sy}\left(\vec{1}_1\right)\\ &\quad + c_{b_{as,\alpha'}}\left(\vec{1}_1\right)\vec{1}_{as}\left(\vec{1}_1\right)\vec{1}_{as}\left(\vec{1}_1\right).\end{aligned} \tag{3.10}$$

In terms of h,v components the 2×2 matrix has (transverse components only)

$$\begin{aligned}\det\left(\overleftrightarrow{c}_{b_\alpha}\left(\vec{1}_1\right)\right) &= \det\left(\left(c_{b_{n,m}}^{(\alpha)}\left(\vec{1}_1\right)\right)\right)\\ &= c_{b_{h,h}}^{\alpha}\left(\vec{1}_1\right)c_{b_{v,v}}^{\alpha}\left(\vec{1}_1\right) - c_{b_{v,v}}^{(\alpha)^2}\left(\vec{1}_1\right)\\ &= c_{b_{sy,\alpha'}}^{\alpha}\left(\vec{1}_1\right)c_{b_{as,\alpha'}}^{\alpha}\left(\vec{1}_1\right)\\ \operatorname{tr}\left(\overleftrightarrow{c}_{b_\alpha}\left(\vec{1}_1\right)\right) &= \operatorname{tr}\left(\left(c_{b_{n,m}}^{(\alpha)}\left(\vec{1}_1\right)\right)\right)\\ &= c_{b_{h,h}}^{(\alpha)}\left(\vec{1}_1\right) + c_{b_{v,v}}^{(\alpha)}\left(\vec{1}_1\right)\\ &= c_{b_{sy,\alpha'}}\left(\vec{1}_1\right) + c_{b_{as,\alpha'}}\left(\vec{1}_1\right)\end{aligned} \tag{3.11}$$

from which both eigenvalues are readily determined. The normalized eigenvectors are real unit vectors.

The various types of scattering residue dyadics treated in [57] are summarized in Table 1. Note that further reductions occur for cases where s_α is on the negative real axis of the s plane due to the real-valued nature of measurable parameters there.

In the context of the far field, the entire function contribution is further complicated due to the time derivative (or multiplication by s) in going from currents to far fields. This emphasizes the high-frequency or fast-time-change (early-time plus possibly other times) part of the scattered field, where the entire function should contribute most. Appropriate choice of the incident-field waveform $F(t)$ should suppress this somewhat, say by beginning the waveform as a ramp function. This requires further investigation. Note also that as $s \to \infty$ the far-field approximation breaks down, further complicating matters. In any event, the pole terms contain the information discussed here, so our concern is being able to find these in the experimental data.

IV. Natural-Resonance-Based Target Discrimination

The SEM exposed in Sections II and III suggests that the late-time scattered field of a target, interrogated by pulsed EM radiation, can be represented as a sum of natural-resonance modes. Since the excitation-independent natural frequencies depend upon the detailed size and shape of the target, then the full complement of those frequencies is unique to a specific target and provides a potential basis for its identification. A prominent early effort to approximate the transient and impulse responses of a target was that of [2]. Here the emphasis was on the early-time (profile function) and late-time ramp response (polarizability), while the presence of a resonance region was recognized. This was followed by attempts [8], [9], [11], [12], [64] to identify and discriminate targets by examination of the natural-frequency content in their pulse-response waveforms. Other efforts on target imaging [10], [16], [20] were based upon the broad-band transient responses of those targets. These methods are limited by the low energy content in the late-time transient responses of practical low-Q targets.

Identification of targets based upon their natural resonances precipitated extensive research on the extraction of natural frequencies from measured target pulse responses. The first such efforts [13], [18] were based upon Prony's method, but in the practical low signal-to-noise environment only one or several modes could be reliably extracted using that inherently ill-conditioned algorithm. Various improvements to Prony's method included [26], where an effort was made to identify and exclude nonphysical "curve-fitting" poles. Finally, efforts to overcome the ill-conditioned nature of natural-frequency extraction from noisy measured data [35], [54] exploited the use of multiple data sets.

Various discriminant waveforms, synthesized to identify a specific target response from among an ensemble of such returns, have emerged. These are linear time-domain filters which, when convolved with the target responses to which they are matched, annihilate preselected natural-frequency content of those responses. The excitation-independent natural frequencies of the relevant target can be measured in the laboratory using scale-model targets in an optimal low-noise environment. The first such synthesized signal was Kennaugh's K-pulse [28], defined as that waveform of minimal duration which would "kill" all the natural modes

Table 1 Properties of scattering residue dyadic

Geometrical properties	Scattering residue dyadic $\overleftrightarrow{C}_{f_\alpha}\left(\vec{1}_r, \vec{1}_1\right)$(bistatic) properties of $\vec{C}_{r_\alpha}\left(\vec{1}_r\right)$ $\left(\vec{C}_\alpha\left(\vec{1}_1\right) \text{ similar }\right)$	Backscattering residue dyadic $\overleftrightarrow{c}_{b_\alpha}\left(\vec{1}_1\right)$(monostatic)
Nondegenerate modes, no special symmetry	$\vec{C}_{r_\alpha}\left(\vec{1}_r\right)$: 2 complex numbers -excitation independent $\vec{C}_{r_\alpha}\left(\vec{1}_r\right)/\lvert\vec{C}_{r_\alpha}\rvert_{\max}$= scatterer polarization -3 real numbers	$\overleftrightarrow{c}_{b_\alpha}\left(\vec{1}_1\right) = \vec{c}_\alpha\left(\vec{1}_1\right)\vec{c}_\alpha\left(\vec{1}_1\right)$ -2 complex numbers Test by $\det\left(\overleftrightarrow{c}_{b_\alpha}\left(\vec{1}_1\right)\right) = 0$ (2×2 sense)
Nondegenerate modes, symmetry plane P through observer	$\vec{C}_{r_\alpha}\left(\vec{1}_r\right)$: symmetric ($//\, P$) or antisymmetric ($\perp\ P$) - complex number times real unit vector (3 real numbers)	$\overleftrightarrow{c}_{b_\alpha}\left(\vec{1}_1\right) = c_{b_\alpha}\vec{1}_{sy}\vec{1}_{sy}$ or $c_{b_\alpha}\vec{1}_{as}\vec{1}_{as}$ - 3 real numbers
Body of revolution with symmetry plane P containing axix: $C_{\infty a}$	$\overleftrightarrow{C}_{f_\alpha}\left(\vec{1}_r, \vec{1}_1\right) = \vec{C}_{r_{sy,\alpha'}}\left(\vec{1}_r\right)\vec{C}_{sy,\alpha'}\left(\vec{1}_1\right) + \vec{C}_{r_{as,\alpha'}}\left(\vec{1}_r\right)\vec{C}_{as,\alpha'}\left(\vec{1}_1\right)$	$\overleftrightarrow{c}_{b_\alpha}\left(\vec{1}_1\right) = \vec{c}_{sy,\alpha'}\left(\vec{1}_1\right)\vec{c}_{sy,\alpha'}\left(\vec{1}_1\right) + \vec{c}_{as,\alpha'}\left(\vec{1}_1\right)\vec{c}_{as,\alpha'}\left(\vec{1}_1\right)$ - 5 real numbers Test by $\det\left(\overleftrightarrow{c}_{b_\alpha}\left(\vec{1}_1\right)\right) \neq 0$ (2×2 sense)
Body with C_N symmetry, $N \geq 3$, axis through observer		$\overleftrightarrow{c}_{b_\alpha}\left(\vec{1}_1\right) = c_{b_\alpha}\overleftrightarrow{1}_1$ - 1 complex number

in the resulting target response. More recent discrimination waveforms [44], [49] are the (extinction) E-pulse and (single-mode) S-pulse, which are detailed below. The E-pulse is synthesized to annihilate, when convolved with a band-limited late-time target pulse response, all natural modes present in that response. The S-pulse is an E-pulse synthesized to annihilate all but one natural mode of a target, so when it is convolved with that target response a single natural mode emerges. Characteristics of the K-pulse and the E-pulse have recently [52] been compared. A similar discriminant pulse [55], based upon natural-mode annihilation, has been conceptualized using a different synthesis scheme. Parametric modeling methods have also been exploited [25] to identify targets from their transient electromagnetic returns. Discriminant waveforms for any number of targets can be synthesized, based upon natural frequencies measured in the laboratory, and stored in disk files for subsequent convolution with a measured target return to discriminate that target. The most recent efforts on discriminant waveform synthesis [50], [53], [60], [65] have resulted in methods to construct those signals directly from measured target response data, without *a priori* knowledge of the natural frequencies. Each of those techniques ultimately yields the natural frequencies from zeros of the corresponding discriminant signal spectrum. Synthesis of the E-pulse is detailed below, as well as its implementation for natural-mode extraction and target discrimination.

Synthesis conditions for an E-pulse waveform can be easily established. It has been shown that the scattered field response of a conducting object can be written in the late-time as a sum of damped sinusoids

$$r(t) = \sum_{n=1}^{N} a_n e^{\sigma_n t}\cos(\omega_n t + \phi_n), \quad t > t_L \tag{4.1}$$

where T_L is the beginning of the late-time response, a_n and ϕ_n are the aspect dependent amplitude and phase of the nth mode, $s = \sigma + j\omega$, and only N modes are assumed excited by the incident field waveform. The convolution of an E-pulse waveform $e(t)$ having duration T_e with the above response is given by

$$c(t) = \sum_{n=1}^{N} a_n |E(s_n)| e^{\sigma_n t}\cos(\omega_n t + \psi_n) \tag{4.2}$$

where ψ_n is dependent on $e(t)$ and

$$E(s) = \{Le(t)\} = \int_0^{T_e} e(t)e^{-st}dt \tag{4.3}$$

is the Laplace transform of the E-pulse. Constructing an E-pulse to produce a null late-time convolved response, $c(t) = 0$, is seen to require

$$E(s_n) = E(s_n^*) = 0, \quad 1 \leq n \leq N. \tag{4.4}$$

A single-mode extraction signal necessitates the same except for $n \neq m$ to leave the mth mode "unextinguished" in the convolved response. The E-pulse is represented as

$$e(t) = e^f(t) + e^e(t) \tag{4.5}$$

where $e^f(t)$ is a forcing component which excites the target's response, and $e^e(t)$ is an extinction component which extinguishes the response due to $e^f(t)$. The forcing component is chosen freely, while the extinction component is expanded in a set of basis functions

$$e^e(t) = \sum_{m=1}^{M} \alpha_m f_m(t) \tag{4.6}$$

and the synthesis conditions are applied. For an E-pulse designed to extinguish all the modes of a target response, (4.4) results in a matrix equation for the basis function amplitudes

$$\begin{bmatrix} F_1(s_1) & F_2(s_1) & \dots & F_M(s_1) \\ \vdots & \vdots & \vdots & \vdots \\ F_1(s_N) & F_2(s_N) & \dots & F_M(s_N) \\ F_1(s_1^*) & F_2(s_1^*) & \dots & F_M(s_1^*) \\ \vdots & \vdots & \vdots & \vdots \\ F_1(s_N^*) & F_2(s_N^*) & \dots & F_M(s_N^*) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} = - \begin{bmatrix} E^f(s_1) \\ \vdots \\ E^f(s_N) \\ E^f(s_1^*) \\ \vdots \\ E^f(s_N^*) \end{bmatrix} \tag{4.7}$$

where $F_m(s) = L\{f_m(t)\}, E_f(s) = L\{e^f(t)\}$ and $M = 2N$ is chosen to make the matrix square. Note that if a dc offset artifact is present in the measured response the E-pulse can be synthesized to remove the dc by demanding, in addition to the above requirements, $E(s = 0) = 0$.

The matrix equation (4.7) has a solution for any choice of E-pulse duration. However, for some choices of T_e the determinant of the matrix vanishes, and (4.7) has a solution only if $e^f(t) = 0$. This type of E-pulse is termed a "natural" E-pulse, while all others are called "forced" E-pulses.

A variety of basis functions have been used in the expansion (4.6), including δ-functions [44], [51], Fourier cosines [39], damped sinusoids [33], and polynomials [42], [43]. While each choice has its own important motivation, perhaps the most versatile expansion is in terms of subsectional basis functions [49]

$$f_m(t) = \begin{cases} g(t - [m-1]\Delta), & (m-1)\Delta \le t \le m\Delta \\ 0, & \text{elsewhere} \end{cases} \tag{4.8}$$

so that $T_e = 2N\Delta$ and

$$F_m(s) = Z^M F_1(s) e^{s\Delta}, \quad Z = e^{-s\Delta} \tag{4.9}$$

giving a matrix of the Vandermonde type. The determinant of this matrix is zero when

$$\Delta = \frac{p\pi}{\omega_k}, \quad p = 1, 2, 3, \cdots, \qquad 1 \le k \le N \tag{4.10}$$

revealing that the duration of a natural E-pulse is only dependent upon the imaginary part of one of the natural frequencies. The minimum natural E-pulse duration is just

$$T_e = 2N \frac{\pi}{\omega_{\max}} \tag{4.11}$$

where ω_{max} is the largest radian frequency among the modes.

Early researchers interested in experimentally determining natural frequencies concentrated on Prony's method [13], [21], [46] but soon found the technique to be highly sensitive to both random noise and estimates of the number of poles present in the data [22], [25]. In its basic form, Prony's method is inherently an ill-conditioned algorithm [1], but several recent improvements have made the scheme more robust [62] while techniques have also been devised for estimating pole content [37], [61]. A variety of other techniques for resonance extraction have been introduced, including the pencil-of-function methods [34], [48], [59] and several nonlinear [38], [47], [58] and combined linear–nonlinear [7], [14] least square approaches. In addition, Ksienski [41] has outlined the benefits of using multiple data sets, while Baum has stressed the importance of incorporating *a priori* information about the scatterer [36].

Particularly suited for radar target applications are a group of resonance extraction techniques which synthesize the discriminant waveform directly from the measured data, and provide the natural resonance frequencies as a by-product of the algorithm. Several authors have developed algorithms around this approach [50], [60], [65] and typical is the E-pulse mode extraction scheme described as follows.

Let $r_k(t)$ represent the scattered field, current or charge response of a target to an interrogating waveform, measured at aspect angle k, $k = 1, \cdots, K$. The convolution of an E-pulse for the target with the measured response will be zero at each aspect angle. Writing the convolution in the time domain and using the expansion (4.6) gives

$$\sum_{m=1}^{2N} \alpha_m \int_0^{T_e} f_m(t') r_k(t - t') dt' = - \int_0^{T_e} e^f(t') r_k(t - t') dt'$$
$$k = 1, 2, \cdots, K, \qquad t > T_{L_k} + Te \tag{4.12}$$

where T_{L_k} is the beginning of late-time for the kth measurement and N is the number of modes expected. Matching both sides of the equation at discrete times t_ℓ, $\ell = 1, 2, \cdots, L$, yields a matrix equation for the E-pulse amplitudes $\{\alpha_m\}$. Generally the product KL is chosen to be greater than $2N$, so that the matrix equation is overdetermined, and a solution is obtained using least squares and the singular-value decomposition. Once the E-pulse waveform is determined, the natural frequencies in the measured response can be determined by solving for the roots $\{s_n\}$ to $E(s) = 0$. That is, if the convolution of $r_k(t)$ and $e(t)$ is zero, $E(s)$ must be zero at the complex frequencies comprising $r_k(t)$. If subsectional basis functions are used in the E-pulse expansion, and the forcing function is chosen to be an identical subsectional function, then by (4.9) the solutions to $E(s) = 0$ are merely the roots of a polynomial equation

$$\sum_{m=1}^{2N+1} \alpha_m Z^m = 0 \tag{4.13}$$

where α_{2N+1} is the amplitude of the forcing pulse.

The above scheme is found to be quite insensitive to both the presence of random Gaussian noise and to estimates of the modal content of the measured data, if the proper E-pulse duration is used. See [53] for typical results using measured data. Incorporating multiple aspect data is important, as the modal amplitudes $\{\alpha_n\}$ are highly aspect dependent—some modes may not be excited at certain aspects.

Empirical results show that if T_e is chosen to be less than the minimum natural E-pulse duration (4.11) the resulting E-pulse is highly oscillatory with a majority of its energy above $\omega_{\max}$ [42] and poor results are obtained in the presence of random noise. It is tempting to solve (4.12) in the least squares sense and choose the E-pulse duration which produces a minimum error, but this approach is often misleading. For certain values of T_e a good solution to (4.12) may produce solutions to (4.13) which are poor approximations to the actual resonant frequencies present in $r_k(t)$. This dilemma can be resolved by choosing the value of T_e which results in the solution to (4.13) which best reproduces the measured data; i.e., T_e is chosen to minimize

$$\epsilon = \sum_k \epsilon_k = \sum_k \frac{1}{\varepsilon_k} \|r_k(t) - \hat{r}_k(t)\|^2 \tag{4.14}$$

where ε_k is the late-time energy in $r_k(t)$, $\hat{r}_k(t)$ is the reconstructed waveform

$$\hat{r}_k(t) = \sum_{n=1}^{N} \hat{a}_{nk} e^{\hat{\sigma}_n t} \cos(\hat{\omega}_n t + \hat{\phi}_{nk}) \tag{4.15}$$

and the norm is over the late-time $t > T_{L_k} + T_e$. Here $\{\hat{s}_n = \hat{\sigma}_n + j\hat{\omega}_n\}$ are the solutions to (4.13) while $\{a_{nk}, \phi_{nk}\}$ minimize ϵ_k with T_e and $\{\hat{s}_n\}$ fixed.

Interestingly, an E-pulse duration found in this manner very often approaches that of a natural E-pulse (4.10), suggesting that the natural E-pulse is optimum for target discrimination in the presence of random noise.

V. Experimental Validation of the E-Pulse Method

The E-pulse radar target discrimination scheme has been successfully demonstrated on numerous occasions using measurements taken on a ground plane range [44], [49], [63]. Recently, a time-domain anechoic chamber has been implemented at Michigan State University for the purpose of demonstrating the E-pulse technique in a free-field environment. The chamber allows a simulation of the free-space radar environment where realistic scale-model targets can be illuminated at arbitrary aspect and polarization.

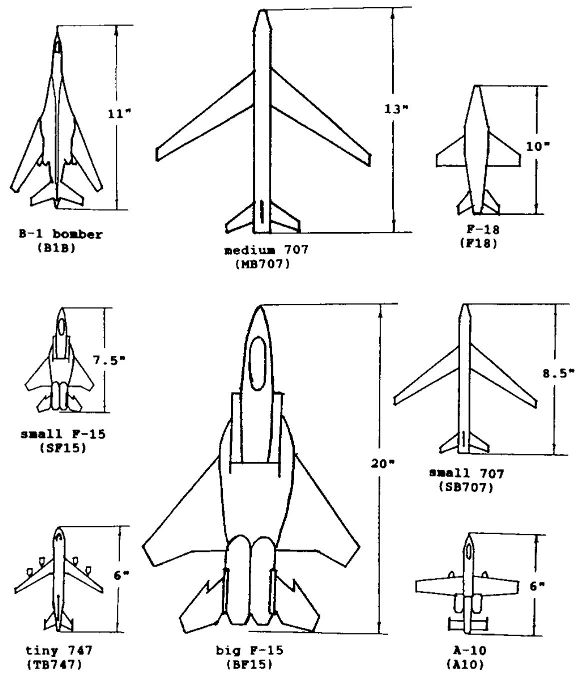


Fig. 2. Eight target models used in discrimination experiments in the free-field chamber scattering range.

The chamber is 7.32-m long by 3.66-m wide by 3.66-m high and is lined with a 30-cm pyramid absorber. A pulse generator provides a one-half nanosecond duration pulse to an American Electronic Laboratories model H-1734 wide-band horn (0.5–6 GHz) which has been resistively loaded to reduce inherent oscillations, and the field scattered from a radar target is received by an identical horn. A waveform processing oscilloscope is used to acquire the received signal and the data are then passed to a microcomputer for processing and analysis.

Accurate discrimination among eight different target models at a variety of aspects has been demonstrated using the free-field range. The targets, shown in Fig. 2, include simple aluminum models as well as detailed cast-metal models, and range in fuselage length of from 6 to 18 in. Figure 3 shows the responses of the big F-15 and A-10 target models measured at a 45° aspect angle (0° aspect is nose-on to the horns), with both the early- and late-time portions of the responses indicated. Note that the late-time period begins at different times for the two targets, due to their dissimilar sizes. E-pulse waveforms have been constructed to eliminate all the modes of each target using the E-pulse mode extraction scheme with measurements from five different aspect angles. These waveforms are shown in Fig. 4.

Discrimination between the big F-15 and the A-10 can be accomplished by convolving the E-pulses with the measured responses, and observing which E-pulse produces the smallest late-time output. First assume the 45° response

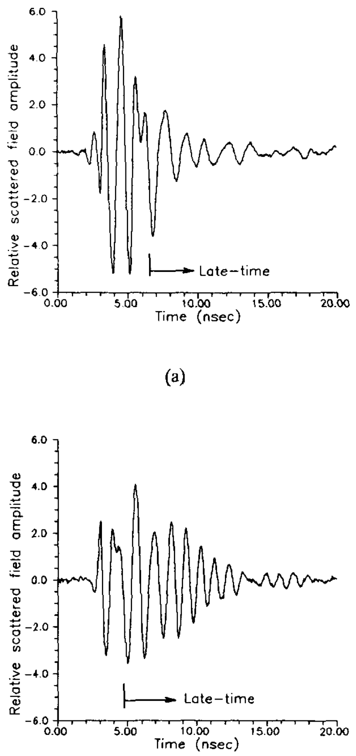


Fig. 3. Scattered field pulse responses of (a) big F-15 and (b) A-10 target models measured at 45° aspect.

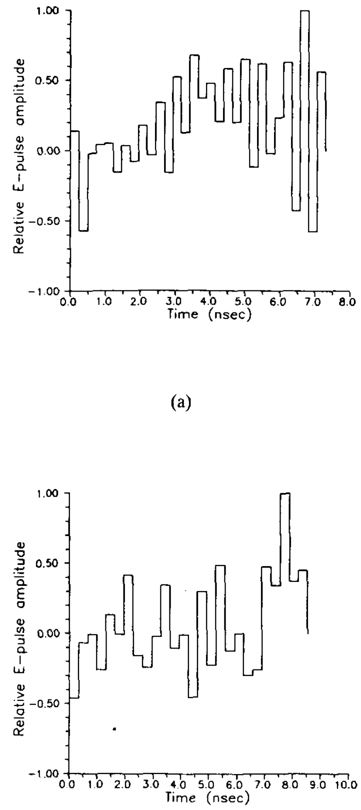


Fig. 4. E-pulses constructed to eliminate the modes of the (a) big F-15 and (b) A-10.

of the big F-15 is from an unknown target. Figure 5(a) shows the convolutions of the two E-pulses with this response. Clearly the big F-15 E-pulse produces the smaller late-time signal, and thus the response is identified as coming from a big F-15 aircraft. For the complementary situation, assume the 45° response of the A-10 is from an unknown target. Figure 5(b) shows the convolutions of the E-pulses with this response. In this case the A-10 E-pulse produces the smaller late-time signal, indicating the response is from an A-10 aircraft.

As the number of prospective targets becomes large, a visual inspection of the convolved outputs becomes more subjective, and eventually impractical. A scheme has, therefore, been devised to automate the discrimination decision. Ideally, if the E-pulse convolutions were uncorrupted, the energy ratio

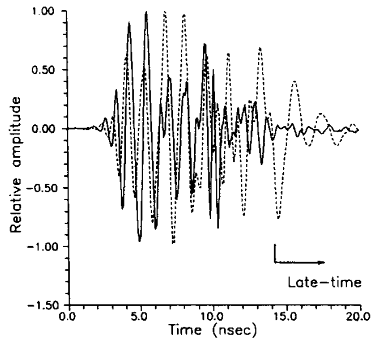
$$E = \frac{\int_{T_{LES}}^{T_{LEE}} c^2(t)dt}{\int_0^{T_e} e^2(t)dt} \tag{5.1}$$

would be zero only for the correct E-pulse. Here $c(t)$ is the convolution of the E-pulse $e(t)$ with the measured response, and T_{LES} is the earliest time at which the unknown target convolution is CERTAIN to be a unique series of natural modes

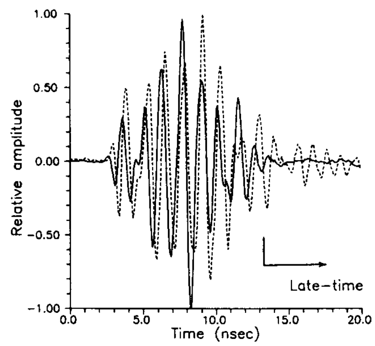
$$T_{LES} = T_e + 2T_r \tag{5.2}$$

where T_r is the one-way transit time of the largest dimension of the target corresponding to the E-pulse. (The largest dimension must be used since the target aspect is unknown.) The end of the energy window, T_{LEE}, is chosen so that the window width, $T_{LEE} - T_{LES}$, is the same for all convolutions.

To show that successful discrimination is possible regardless of target aspect, E-pulses for the eight targets have been convolved with the responses of the big F-15 measured at five different aspect angles from 0° (nose-on) to 90° (broadside). The energy ratio (5.1) has been plotted as a function of aspect angle in Fig. 6 for each expected target. It is obvious that for all aspects tested the big F-15 produces the smallest late-time convolved response, with a minimum 10-dB difference in late-time energy. Thus the big F-15 is identified from among all the possible targets at each aspect angle. Finally, discrimination among all eight targets can be demonstrated when any of the eight is the unknown target. Table 2 shows the energy ratios (5.1) obtained by assuming that each target in turn is the unknown target and convolving the E-pulses for each of the eight expected targets with the response of the unknown target. Here the target responses were all measured at 45° aspect. Accurate discrimination for each target is indicated by the minimum energy ratio being due to the E-pulse of the unknown



(a)



(b)

Fig. 5. (a) Convolution of the big F-15 E-pulse (solid line) and the A-10 E-pulse (dashed line) with the 45° response of the big F-15. (b) Convolution of the big F-15 E-pulse (dashed line) and the A-10 E-pulse (solid line) with the 45° response of the A-10.

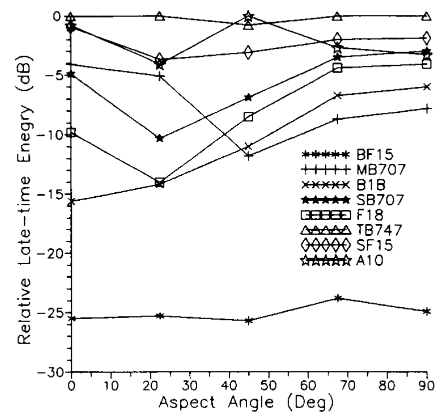


Fig. 6. Late-time energy from convolution of eight target E-pulses with responses of big F-15 measured at various aspect angles.

target. For example, the convolution of the F-18 E-pulse with the F-18 response produces a late-time energy 29.8 dB below that produced by the convolution of the medium 707 E-pulse with the F-18 response, and 15.3 dB below that produced by the convolution of the B-1 bomber E-pulse with the F-18 response.

VI. Extensions and Implementation

The theoretical basis for the application of SEM concepts to aspect-independent target identification/discrimination is rather well established and the feasibility of the E-pulse scheme has been well verified in the laboratory with scale models of various aircraft as described in this paper. A possible radar system based on the E-pulse techniques has been discussed previously [44]. To advance this scheme to practical application, there remain some major tasks to be investigated: 1) the design of optimal transmitting and receiving antennas and associated optimal waveforms, 2) the generation of high power EM pulses, and 3) the refinement of the E-pulse synthesis technique.

Concerning the first task of the antenna and waveform design for this system, there is very little work done. Considering various aircraft targets, major resonant mode frequencies are of the order of a few to tens of megahertz, implying pulsewidths in the range of ten to a hundred or so nanoseconds to maximize energy content at these frequencies. To radiate and receive this pulse waveform, various antenna elements such as tapered and resistively loaded dipoles, radiating transmission line antennas and TEM horn antennas, etc., should be investigated. An array of these elements could be used to increase the signal strength and provide some beam steering capability. Two or more such arrays, sufficiently separated, might be used to give more accurate location of the target. Alternatively, one might combine such an array with a more traditional radar designed for high spatial resolution. It is also worthwhile to look into the possibility of modifying the antenna systems of existing radar systems such as over-the-horizon radars which utilize a similar frequency band.

There are other types of transmitted waveforms which may warrant consideration for the E-pulse radar discrimination scheme. Since the transmitted radar signal is intended to excite the resonant modes of the target, a waveform comprising a set of damped sinusoids resembling the target's resonance modes may be transmitted instead of a pulse. Another interesting waveform may be a set of CW sinusoids of different frequencies and finite durations to be transmitted simultaneously to excite a set of selected resonant modes of the target. Of course, the antenna design will be affected by the type of transmitted waveform.

Regarding the second task of the generation of high power EM pulses, there are indications that some types of high power EM pulse sources are already available, developed in other fields such as for the electromagnetic pulse.

Finally, the task of refining the E-pulse synthesis technique seems a never ending effort. Even though we and other researchers have studied this topic for many years, an optimal synthesis technique has yet to be developed. The synthesis technique includes an accurate extraction of the natural frequencies of the target from its measured pulse response, and the synthesis of an optimal E-pulse waveform

Table 2 Late-time energy in the convolutions of various E-pulses with responses of various targets at 45° aspects

E-Pulse / Target Response	BF15	MB707	B1B	SB707	F18	TB747	SF15	A-10
BF15	**-25.7 dB**	-11.8	-11.0	-6.9	-8.5	-0.8	-3.1	0
MB707	-20.0	**-32.0**	-16.9	-9.1	-9.6	-3.3	-4.8	0
B1B	-11.2	0	**-23.4**	-7.7	-7.0	-4.5	-3.1	-1.3
SB707	-16.7	-0.8	-20.1	**-25.1**	-8.0	0	-2.5	-3.8
F18	-7.1	0	-14.5	-2.6	**-29.8**	-11.8	-1.9	-4.6
TB747	-10.5	-0.1	-5.0	0	-6.0	**-21.5**	-3.0	-1.5
SF15	-4.5	-1.7	-6.2	-4.3	-5.5	-2.2	**-11.4**	0
A-10	-9.2	-2.9	-0.8	0	-4.9	-10.9	-6.8	**-17.6**

which provides the most sensitive discrimination capability as well as the most robust noise tolerance.

Two points can be made concerning the practical application of the E-pulse technique. 1) It is designed for noncooperative target recognition. When it is used for IFF purposes, some unfriendly targets (not in the radar's library) with unknown structure information can only be identified as unknown targets, 2) It will be affected by a shift in the target's resonance frequencies. However, the shift due to target motion is extremely small (in the order of v/c where v is the target speed and c the speed of light) and that due to target deformation (e.g., wing sweep angle) can be taken into account if the type of deformation is known beforehand.

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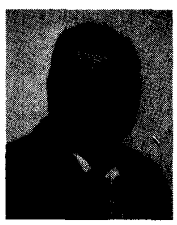

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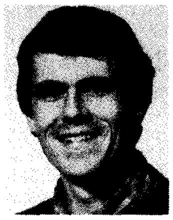

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