Note Title 10/10/2011

$I = \int x^3 \sqrt{x^2+15}\, dx$, Let $u = \sqrt{x^2+15}$

Then $u^2 = x^2 + 15$

$$\frac{d}{dx}(u^2) = 2x$$

$$\Leftrightarrow \frac{d}{du}(u^2)\frac{du}{dx} = 2x$$

$$\Leftrightarrow \cancel{2}u \frac{du}{dx} = \cancel{2}x$$

$$\Leftrightarrow u\, du = x\, dx$$

So, $I = \int x^2 \cdot x \cdot \sqrt{x^2+15}\, dx$

$u = \sqrt{x^2+15}$

$u^2 = x^2+15$

$x^2 = u^2-15$

$$= \int \underset{u^2-15}{\underset{\|}{x^2}}\ \underset{u}{\underset{\|}{\sqrt{x^2+15}}}\ \underbrace{x\, dx}_{= u\, du}$$

$$= \int (u^2-15)\, u \cdot u\, du$$

$$= \int (u^2 - 15)\, u^2\, du$$

$$= \int (u^4 - 15u^2)\, du$$

$$= \frac{1}{5}u^5 - \frac{15}{3}u^3 + C$$

$\sqrt{x^2+15} = (x^2+15)^{\frac{1}{2}}$

$$= \frac{1}{5}(x^2+15)^{\frac{5}{2}} - 5(x^2+15)^{\frac{3}{2}} + C$$

$$= (x^2+15)^{\frac{3}{2}}\left[\frac{1}{5}(x^2+15)^{\frac{2}{2}} - 5\right] + C$$

$$= (x^2+15)^{\frac{3}{2}}\left[\frac{1}{5}x^2 + 3 - 5\right] + C$$

$$= (x^2+15)^{\frac{3}{2}}\left(\frac{1}{5}x^2 - 2\right) + C$$