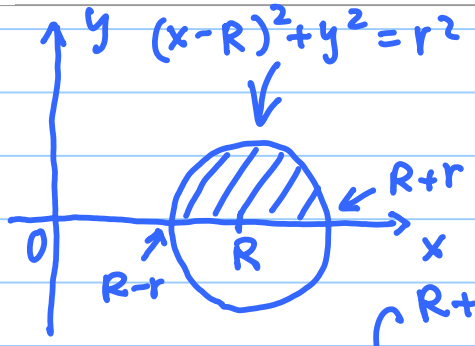
Volume of a Torus

Note Title 10/18/2011



The total volume = 2 × volume of ◠ part

So, $V = 2\int_{R-r}^{R+r} 2\pi \times \sqrt{r^2-(x-R)^2}\,dx$

Set $x-R = r\cos\theta \Leftrightarrow x = R+r\cos\theta$

Then $dx = -r\sin\theta\,d\theta$

Also, $\sqrt{r^2-(x-R)^2} = \sqrt{r^2-r^2\cos^2\theta}$

$= \sqrt{r^2\sin^2\theta}$

$= r\sin\theta$ $\quad 0\le\theta\le\pi$

So $V = 4\pi\int_{\pi}^{0}(R+r\cos\theta)\,r\sin\theta\cdot(-r\sin\theta)\,d\theta$

$= 4\pi r^2\int_0^{\pi}(R+r\cos\theta)\sin^2\theta\,d\theta$

$= 4\pi r^2\left[R\int_0^{\pi}\sin^2\theta\,d\theta + r\int_0^{\pi}\cos\theta\sin^2\theta\,d\theta\right]$

$= 4\pi r^2\left[R\int_0^{\pi}\frac{1-\cos 2\theta}{2}\,d\theta + r\int_0^{\pi}\cos\theta\sin^2\theta\,d\theta\right]$

$= 4\pi r^2\left[R\left[\frac{\theta-\frac{1}{2}\sin 2\theta}{2}\right]_0^{\pi} + r\left[\frac{1}{3}\sin^3\theta\right]_0^{\pi}\right]$

$= 4\pi r^2\left[\frac{R}{2}(\pi-0-\frac{1}{2}(0-0)) + r(0-0)\right]$

$= \underline{2\pi^2 r^2 R}$