4 $\boxed{\text{Serret-Frenet}}$ (Summary)

- $\gamma(s)$ a curve, s arclength, $\gamma'(s) \neq 0$
- $\boxed{\gamma'(s) = T(s)}$, $\|T(s)\| = 1$ because $s \equiv$ arclength

  <u>defines</u> $T(s)$

- $\|T(s)\| = 1 \Rightarrow T'(s) \perp T(s)$

  $\boxed{T'(s) = \kappa(s) N(s)}$ $\|N(s)\| = 1$ <u>defines</u> $\kappa(s), N(s)$

- $\boxed{B(s) = T(s) \times N(s)}$ defines $B(s)$

  $\|B(s)\| = \|T(s)\| \|N(s)\| \sin(\theta) = 1$

(1) $B'(s) \perp B(s)$

$$0 = B(s) \times T(s) \Rightarrow 0 = \frac{d}{ds}\big(B(s) \times T(s)\big) = B' \times T + \cancel{B \times T'}$$

$T' \parallel N \perp B$

(2) $B'(s) \perp T(s)$

(1) + (2) $\Rightarrow B'(s) \parallel N(s)$

- $\boxed{B'(s) = -\tau(s) N(s)}$ defines $\tau(s)$

<u>Equations</u>:

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix} \quad (*)$$

Knowing $\kappa(s)$ & $\tau(s)$ you can integrate up to get $T(s), N(s), B(s)$ $\Rightarrow$ all geometric information is encoded in $\kappa(s), \tau(s)$.

<u>Exam</u> <u>Question</u>: Derive $(*)$