MATH 116
Introduction

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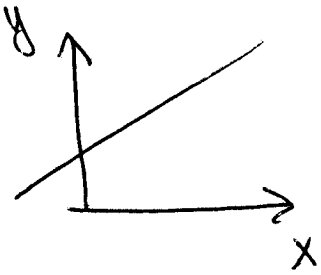
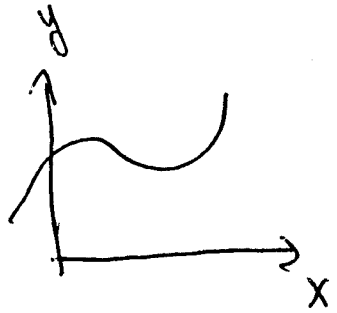
- Differential Geometry ≡ "the study of curvature"
- Main problem: what is curvature?
  - Curves:



st. line is not curved

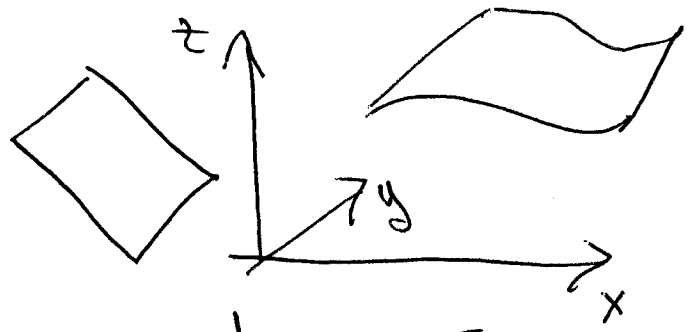


curvature should have something to do with 2nd derivative

Problem: rotate the curve & 2nd derivative changes: Q: what is the curvature.

⇒ Frennet-Servet formulas

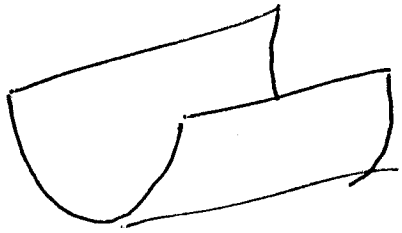
- surfaces



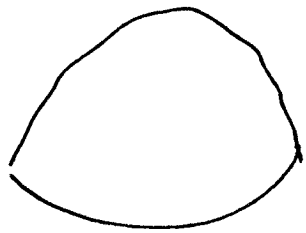
- a plane has no curvature r

~~cylinder has no curvature "just a rolle~~

Compare: $\frac{1}{2}$ cylinder with a hemisphere



cf



we can unfold ~~a~~ cylinder into flat plane

$\frac{1}{2}$ sphere is fundamentally different.

Conclude: sphere is curved
cylinder is flat

Q: what definition of curvature is correct.

⇒ Bigger problem: 3-space.

We live in 3-space: how can we tell whether we live in a 3-dimensional plane or in a curved 3-d space?

Note: in 2-d, we can look at it inside a bigger space and tell – not so for 3-space.

~~⇒ Problem:~~

⇒ Central Problem: If you lived on a sphere (an ant), how could you tell by properties of the 2-d sphere alone that your world is curved? ← Must be based on lengths ⇒ geodesics are relevant

– Intrinsic defn of curvature for 2-d surfaces first proposed by Gauss – Topic of this class.

History

- 306 BC: Ptolemy I inherits Egypt from Alex. the Gt.; Establishes Museum in Alexandria.

- 300 BC: Euclid publishes Elements Plane geom.
  - 11 postulate
  - 2000 year effort to prove 11-post from other 4 post'c.

- 1827: Gauss: first understood curvature as an intrinsic property of surfaces

- 1832: Lobachevski & Bolyai; constructed plane geometries in which 1st 4 axioms held, but fifth failed;
  - spaces of constant curvature!

- Major Problem 1827 - 1854 ; what is the meaning of Curvature in dimensions higher than 2 ?

Ans: (Greatest paper in history of geometry)

Riemann 1854 : "On the assumptions that lie at the foundations of geometry"

- Inaugural lecture Gottingen (Habilitation lecture) Story
- Riemann Curvature Tensor R^i_{jkl}

- 1916 Einstein "On the gen. th. of rel"
  - "Gravitational forces" are really "Spacetime Curv
  - Geometry & Curvature fundamental to physics (basis for Yang-Mills fields describing elementary particles)
  - Eins Eq : $G = 8\pi T$ ← mass = stress energy tensor

replaced $F = G\frac{M_p M_s}{r^2}$