Mon. 1-9-02

Math 119A Winter '02 - Introduction

Class: Math 119A "Intro to ODE"
Prof: Blake Temple (GR & Shock-wave theory)
Off: 3148 MSB Off Hrs: MWF 2-3 & by appt.
Text: Strogatz - Nonlinear Dynamics & Chaos
Ch 1, 2, 3, 5, 6, 7, 8

TA: Edward Tavernetti, Mathematics

Course Webpage - (Info / Homework / Solutions)

http://www.math.ucdavis.edu/~temple/MAT119A/

Supplemental TA: David Melgin

Q: why is Calculus so important? Laws of

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Ans: The Laws of Science (physics, biology, chemistry, geology, engineering) usually come to us stated in terms of <u>rates</u> <u>of</u> <u>change</u>

<u>Derivative</u>: $f'(t) = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$

= "rate of change of f wrt t"

- To make a prediction —

- Start with <u>Law</u>
- Express it in an equation involving derivatives
- Integrate or solve the equation to get soln, make a graph/prediction

Ex 1 Law: A population $x(t)$ grows at a rate proportional to t

Equation: $\frac{dx}{dt} = ct$

Solution: $x(t) - x(t_0) = \int_0^t ct\,dt = \frac{c}{2}t^2$

Ex 2 Law: A population $x(t)$ grows at a rate proportional to its size

Equation: $\boxed{\frac{dx}{dt} = cx}$

Solution: $\frac{dx}{x} = ct$ $\quad \int_{x_0}^{x} \frac{dx}{x} = ct$

$\ln\frac{|x|}{|x_0|} = ct \qquad x(t) = x_0 e^{ct}$

Ex 3 Law: A population grows at a rate proportional to its size squared

Equation: $\frac{dx}{dt} = cx^2$

Solution: $\frac{dx}{x^2} = c\,dt$

$$\int_{x_0}^{x} \frac{dx}{x^2} = c(t-t_0)$$

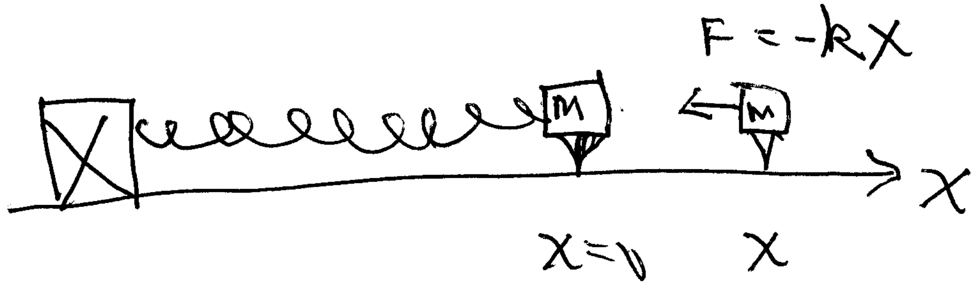
$$-x^{-1}\Big]_{x_0}^{x} = c(t-t_0)$$

$$-\frac{1}{x} + \frac{1}{x_0} = c(t-t_0)$$

$$\frac{1}{x} = \frac{1}{x_0} - c(t-t_0) \qquad \boxed{x(t) = \frac{1}{\frac{1}{x_0} - c(t-t_0)}}$$

Ex 4: Newtons Law: $\vec{F} = m\vec{a}$

Position of mass m is $x(t)$



$a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$ "Newton's Law comes to us stated in terms of a rate of change"

Law: (Spring) "The force is proportional to minus the displacement"

Equation: $m\ddot{x} = -kx$

$$\boxed{\ddot{x} + \frac{k}{m}x = 0}$$

Solution:

$x_1(t) = A\cos\sqrt{\frac{k}{m}}\,t$

$x_2(t) = B\sin\sqrt{\frac{k}{m}}\,t$

The "most important" ODE'S —

(1) $\dot{x} = kx$ (exponential growth)

(2) $\dot{x} = kx^2$ (Ricotti)

(3) $\ddot{x} + kx = 0$ (Harmonic Oscillator)

Defn: An ODE is an equation that involves an unknown function $x(t)$ of one independent variable t and its derivatives $\dot{x}(t), \ddot{x}(t), \dddot{x}(t), \dots, x^{(n)}(t)$

- highest order derivative appearing is the order
- main point: No "partial derivatives"

Subject of 119A : Theory of ODE's

Comments –

1. Complicated ODE's cannot be solved in closed form $\rightsquigarrow$

* Theory about qualitative behavior

* Simple examples like (1) - (3) are fundamental

2. Linear vs Nonlinear

Homogeneous vs Non-Homogeneous

"Linear Homogeneous" superposition holds