Mon Feb 6, 2002

(2) Dimensional Analysis -

- The values of physical quantities must be measured in terms of a chosen unit of time (s) length (m) mass (kg).

$$F = ma$$

The magnitude of F, m, a, change = "rescale" with change of units. To determine how they scale, we find their dimensions -

$$[F] = \text{"dimensions of } F\text{"}$$

$$[F] = [ma] = [m][a] = [m]\left[\frac{dx}{dt^2}\right]$$

$$[m] = M \qquad \left[\frac{dx}{dy}\right] = \frac{[x]}{[y]}$$

$$[a] = \frac{L}{T^2}$$

M, L, T are fundamental dimensions

- <u>Conclude</u>: $[F] = \frac{ML}{T^2}$ = "dimensions of mass times accel"

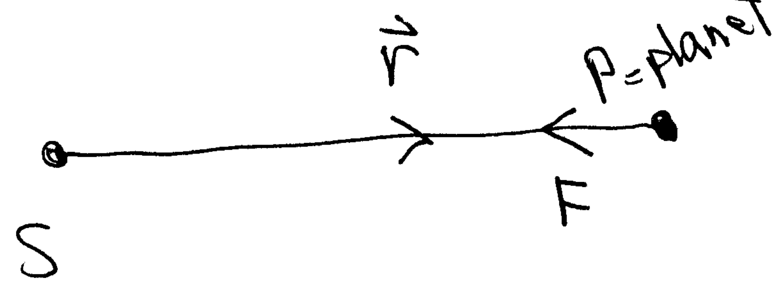
- <u>Ex</u>: If ~~the new~~ a change of dimensions doubles the mass, triples the length and five times the time, then it will rescale F by $\frac{2 \times 3}{5^2}$

"Dimensions keep track of rescalings under change of units"

- <u>Principle</u>: In every physical equation, the dimensions of each term are the same... $\Rightarrow$ not all equations can be physical equations!

otherwise - changing dimensions changes the equations of motion ✱

<u>Ex</u>: Newton's Law of Gravity —



" Gravity is an Inverse square force Law proportional to $M_E M_s$ "

$$\vec{F} = M_p \vec{a} = -G \frac{M_p M_s}{\|\vec{r}\|^2} \frac{\vec{r}}{\|\vec{r}\|}$$

" You have to have a dimensional constant here to make both sides have same dimension "

Q: What are the dimensions of the Gravitational constant G?

$$[\vec{F}] = [M_p \vec{a}] = \frac{ML}{T^2}$$

$$\left[G \frac{M_p M_s}{\|\vec{r}\|^2} \frac{\vec{r}}{\|\vec{r}\|}\right] = [G] \frac{[M_p][M_s]}{[\|\vec{r}\|]^2} \left[\frac{\vec{r}}{\|\vec{r}\|}\right] = [G] \frac{M^2}{L^2}$$

Conclude:

$$\frac{ML}{T^2} = [G]\frac{M^2}{L^2} \Rightarrow [G] = \frac{L^3}{MT^2}$$

"The theory implies $\exists$ a universal constant G, $[G] = \frac{L^3}{MT^2}$, to be measured"

- Note: $\cancel{M_p}\vec{a} = -G\frac{\cancel{M_p}M_s}{\|\vec{r}\|^2}\frac{\vec{r}}{\|\vec{r}\|}$

"The acceleration of the body is indep of the mass of planet"

"every object, feather & earth, will describe the same path thru grav. field" $\rightsquigarrow$ Equivalence Principle

$\rightsquigarrow$ Led Einstein to suspect that gravity was about the paths, not about forces $\rightsquigarrow$ GR

- Thus write: (The equation for a planet)

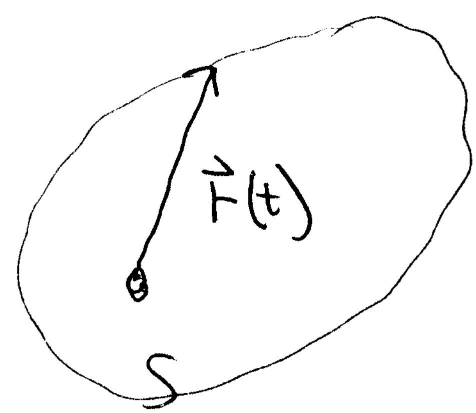
$$\vec{a} = -(GM_s)\frac{1}{\|\vec{r}\|^2}\frac{\vec{r}}{\|\vec{r}\|}$$

$$[GM_s] = [G][M_s] = \frac{L^3}{T^2}$$

ODE for planetary motion (Newton)

$$\boxed{\frac{d^2\vec{r}(t)}{dt^2} = -(GM_s)\frac{\vec{r}(t)}{\|\vec{r}(t)\|^3}} \qquad (*)$$

ODE for position $\vec{r}(t) = \overrightarrow{(x(t), y(t), z(t))}$



Kepler Laws – All derivable from (*)

(1) Planets move in ellipses about sun, with sun at focus of ellipse

(2) Planets sweep out equal area in equal time

(3) The mean distance from sun $= L_P$
Period $= T_P$ (depends on planet)

satisfies

$$\frac{L_P^3}{T_p^2} = \text{same } \forall \text{ planet}$$

<u>Note</u>: "Some thing independent of planet exists $\left(\frac{L_P^3}{T_p^2}\right)$ with same dimensions as the gravitational constant (GM_S)"

- Dimensional Analysis implies deep connections with solutions that enable you to guess answers ahead of time

Eg: since equations contain a universal constant $[GM_S] = \frac{L^3}{T^2}$, you might guess there is something independent of planet of these same dimensions —

$\Rightarrow$ get ideas before doing any real work in solving the equations.