HOMEWORK SOLUTIONS

(2.6.2) The point is that we can integrate $\int_t^{t+T} f(x)\frac{dx}{dt}dt$ two different ways to get a contradiction. The first way, recall $f(x) = -U'(x)$ where the antiderivative $U(x)$ is called the *potential*. Then since $(dx/dt)dt = dx$ we can use the substitution principle to get

$$\begin{aligned}\int_t^{t+T} f(x)\frac{dx}{dt}dt &= \int_t^{t+T} f(x)dx = \int_{x(t)}^{x(t+T)} \left(-U'(x)\right) dx \\ &= -[U(x(t+T) - U(x(t)] = 0, \qquad (1)\end{aligned}$$

because by assumption $x(t+T) = x(t)$.

On the other hand, $dx/dt = f(x)$, so

$$\int_t^{t+T} f(x)\frac{dx}{dt}dt = \int_t^{t+T} \left[f(x(t))\right]^2 dt > 0, \qquad (2)$$

because the integral of a positive function is positive. The two statements cannot both be true, so the assumption that the solution is periodic, (i.e., $x(t) = x(t+T)$), must be false. (Proof by contradiction!)