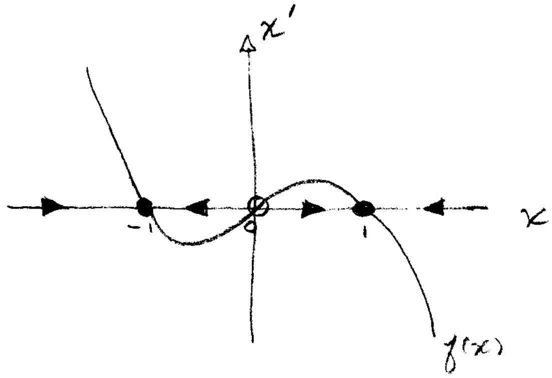
① 2.2.3 $x' = x - x^3 = f(x)$



FIXED PTS.

$x^* = -1$, $x^* = 1$ } STABLE

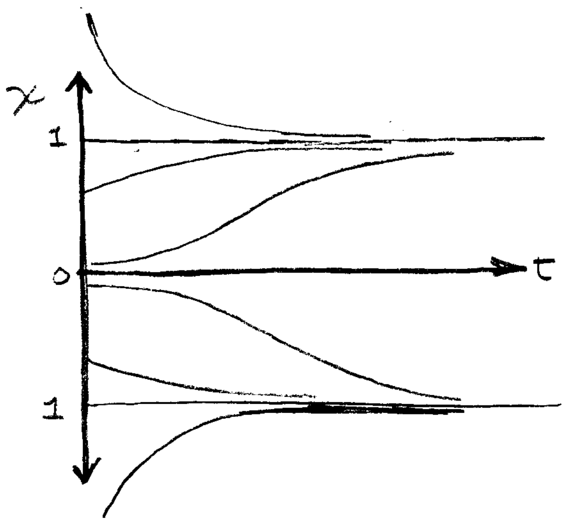
$x^* = 0$ UNSTABLE

NOTE

$f'(x) = 1 - 3x^2$

$f'(\pm 1) = -2 < 0$

$f'(0) = 1 > 0$



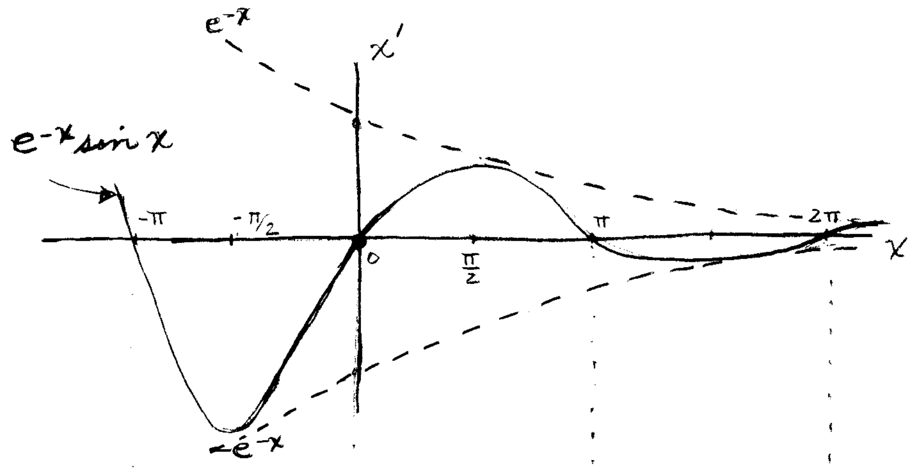
solve analytically by partial fractions

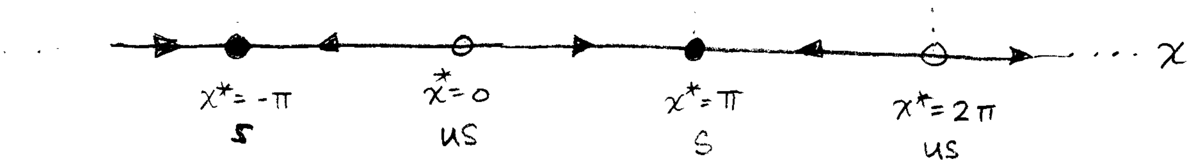
$$\int \frac{dx}{x - x^3} = t + c$$

② 2.2.4 $\quad x' = e^{-x} \sin x$

amplitude

$e^{-x} \neq 0$





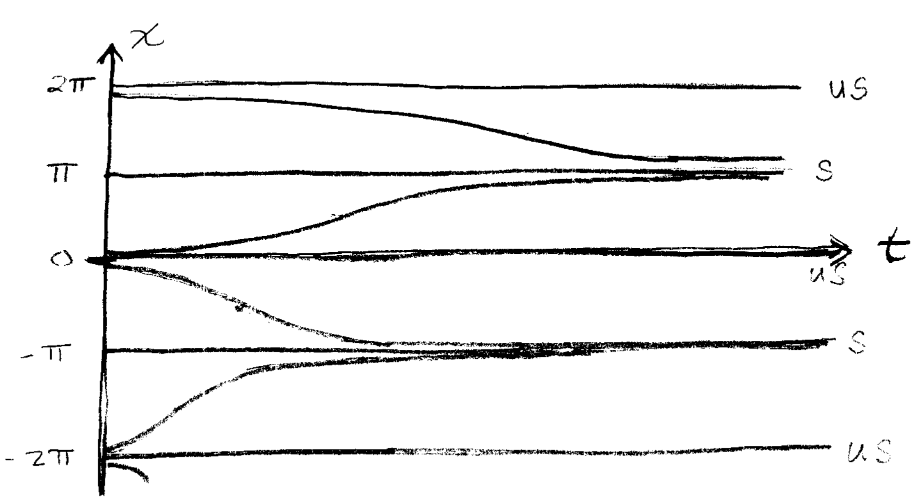
FIXED PTS $\quad e^{-x^*} \sin x^* = 0$

$\sin x^* = 0$

$x^* = k\pi, \quad k \in \mathbb{Z}$

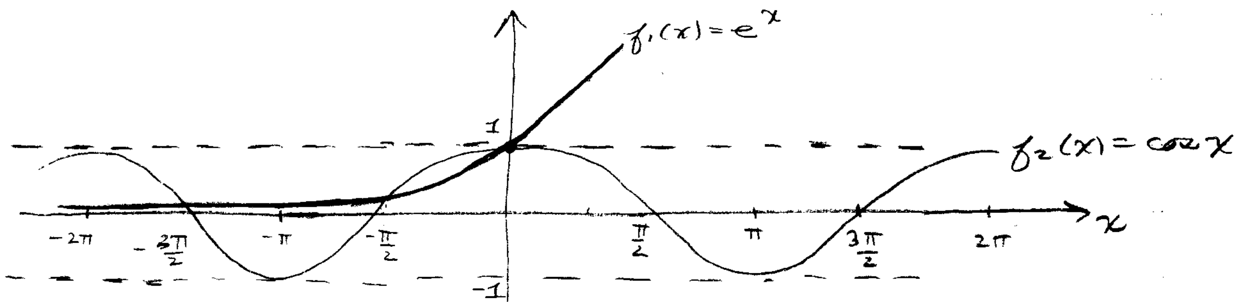
k odd, x^* is STABLE

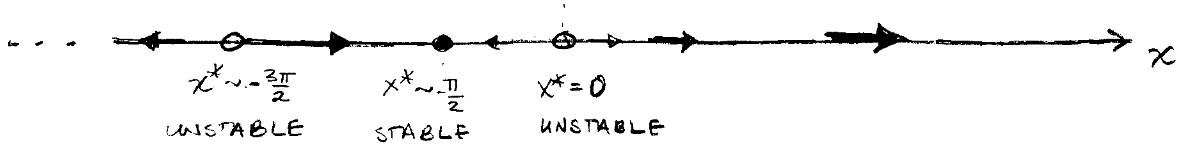
k even, x^* is UNSTABLE

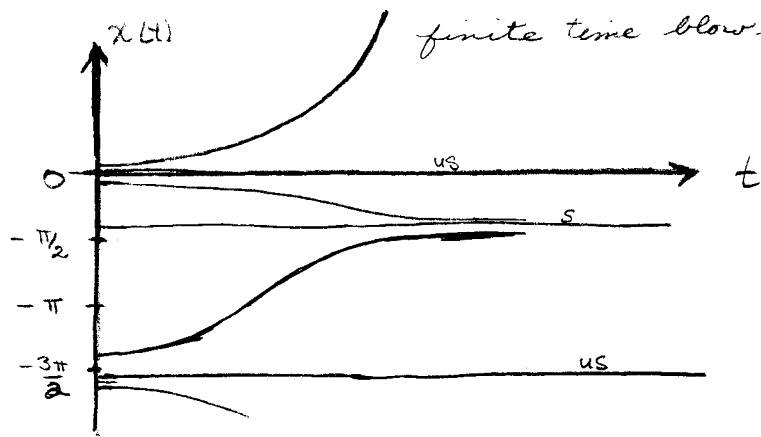


③ **2.2.7**

$$x' = e^x - \cos x$$
$$= f_1(x) - f_2(x)$$





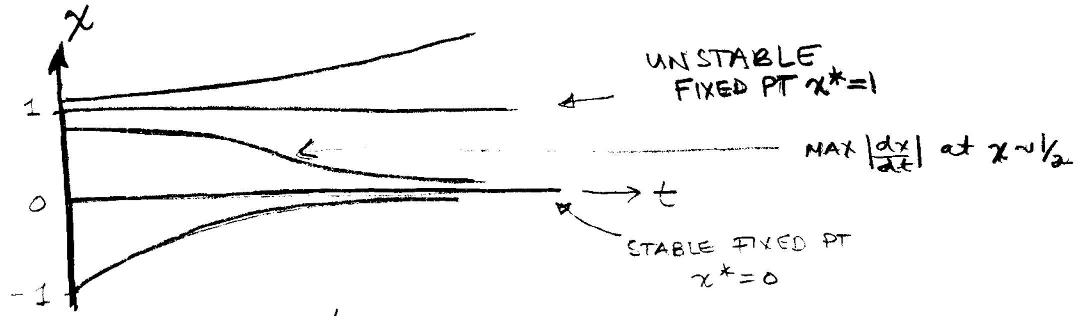


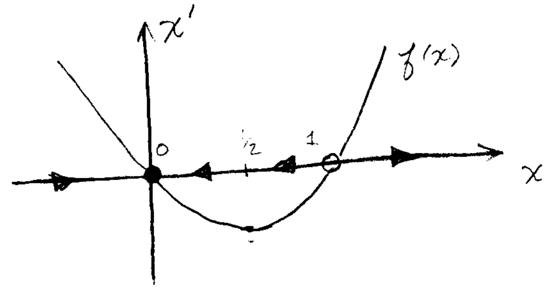
finite time blowup! ie. for large x

$x' \simeq e^x$

$\Rightarrow x(t) = \ln \frac{1}{c-t}$

④ <u>2.2.9</u>





$$\Rightarrow \quad x' = f(x) = x(x-1) = x^2 - x$$

⑤ 2.2.13

$$m\dot{v} = mg - kv^2 \qquad m, k, g > 0$$

$$\frac{m}{k}\frac{dv}{dt} = \frac{mg}{k} - v^2$$

a)

$$\int \frac{dv}{\frac{mg}{k} - v^2} = \frac{k}{m}(t + c) \qquad \text{let } a^2 = \frac{mg}{k}$$

$$\frac{1}{2a}\ln\left(\frac{v+a}{v-a}\right) = \frac{k}{m}(t+c)$$

$$v(t) = a\left(\frac{e^{2\frac{k}{m}a(t+c)} + 1}{e^{2\frac{k}{m}a(t+c)} - 1}\right) = a\tanh\left(a\frac{k}{m}(t+c)\right)$$

using $v(0) = 0 \Rightarrow c = 0.$

$$v(t) = \sqrt{\frac{mg}{k}}\tanh\left(\sqrt{\frac{mg}{k}}\frac{k}{m}t\right)$$

b) terminal velocity

$$V_T = \lim_{t\to\infty} v(t) = \sqrt{\frac{mg}{k}}$$

c) average velocity

$$V_{avg} = \frac{31400\text{ ft} - 2100\text{ ft}}{116\text{ sec}} = 252.6\text{ ft/sec}$$

5 cont'd

d) $$v(t) = V_T \tanh\left(\frac{g}{V_T} t\right)$$ ie. $\sqrt{\frac{mg}{k}}\,\frac{k}{m} = \sqrt{\frac{k}{m}}$ $= \frac{g}{V_T}$

$$\frac{ds}{dt} = v \quad , \; s(0) = 0$$

$$s(t) = \int V_T \tanh\left(\frac{g}{V_T} t\right) dt$$

$$= \frac{V_T^2}{g} \ln\left(\cosh \frac{gt}{V_T}\right)$$

$t_f = 116$ sec $\qquad s(t_f) = (31400 - 2100)$ ft $= 29300$

$g = 32.2$ ft/sec^2

$$29300 = \frac{V_T^2}{32.2} \ln\left(\cosh\left(\frac{32.2 \times 116}{V_T}\right)\right)$$

$$\Rightarrow V_T \simeq 265.686 \text{ ft/sec.}$$

note: velocity at 2100 ft ie. at $t = 116$ sec.

$$V(116) = V_T \tanh\left(\frac{32.2}{V_T} 116\right)$$

$$\simeq 265.686 \tanh\left(\frac{32.2 \times 116}{265.686}\right) \simeq V_T$$

7 2.4.7 $x' = ax - x^3 = f(x)$

- LINEAR STABILITY ANALYSIS

FIXED PTS $ax^* - x^{*3} = 0$

$x^* = 0$ AND $x^* = \pm\sqrt{a}$ WHEN $a > 0$.

STABILITY

$f'(x) = a - 3x^2$

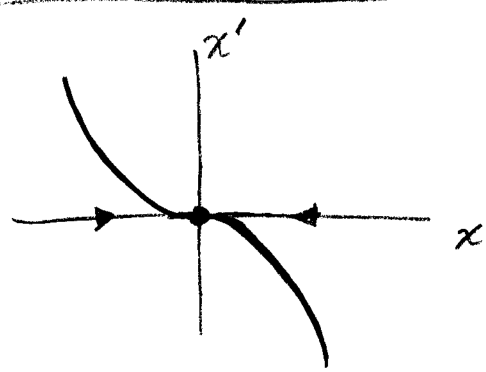
(i) $x^* = 0$, $f'(0) = a \Rightarrow$
- $x^* = 0$ is STABLE for $a < 0$.
- $x^* = 0$ is UNSTABLE for $a > 0$.
- NO CONCLUSION FOR $a = 0$.

(ii) $x^* = \pm\sqrt{a}$, $a > 0$, $f'(\pm\sqrt{a}) = a - 3a = -2a < 0$

- $x^* = \pm\sqrt{a}$ are STABLE when they exist ($a>0$).

- GRAPHICAL STABILITY ANALYSIS FOR $a = 0$.

$x' = -x^3$



$x^* = 0$ is STABLE.