Section 2.3

1.) a.) ARC $\approx \dfrac{115-64}{5-0} = \$\dfrac{10.2}{\text{yr.}}$ (bill.)

b.) ARC $\approx \dfrac{153-115}{10-5} = \dfrac{\$7.6}{\text{yr.}}$ (bill.)

c.) ARC $\approx \dfrac{184-150}{15-10} = \dfrac{\$6.8}{\text{yr.}}$ (bill.)

d.) ARC $\approx \dfrac{265-184}{20-15} = \$\dfrac{16.2}{\text{yr.}}$ (bill.)

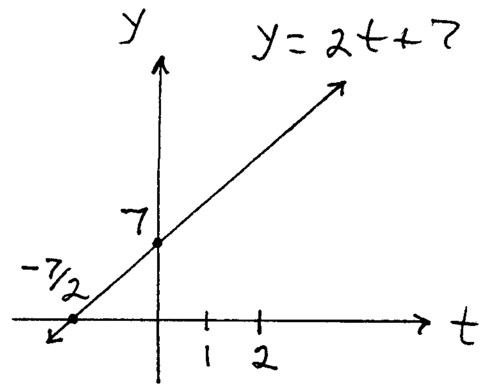
e.) ARC $\approx \dfrac{275-64}{22-0} \approx \$\dfrac{9.6}{\text{yr.}}$ (bill.)

f.) ARC $\approx \dfrac{275-153}{22-10} \approx \dfrac{\$10.17}{\text{yr.}}$ (bill.)

3.) $f(t) = 2t+7 \xrightarrow{D} f'(t) = 2$;

ARC $= \dfrac{f(2)-f(1)}{2-1} = \dfrac{11-9}{1} = 2$;

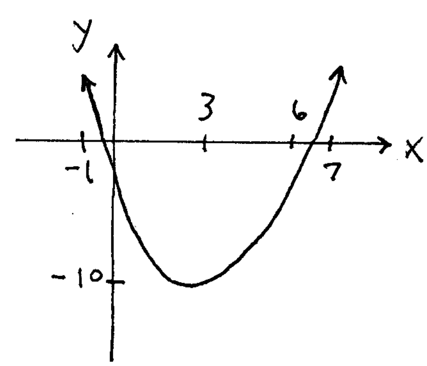
IRC : $f'(1) = 2$, $f'(2) = 2$

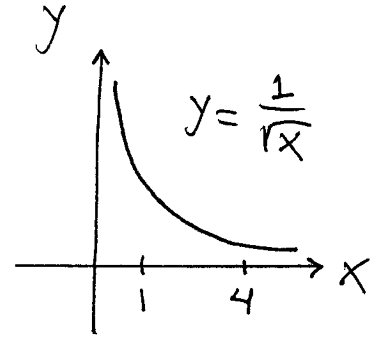


6.) $f(x) = x^2 - 6x - 1 \xrightarrow{D} f'(x) = 2x - 6$;

ARC $= \dfrac{f(3)-f(-1)}{3-(-1)} = \dfrac{-10-6}{4} = -4$;

IRC: $f'(3) = 0$, $f'(-1) = -8$





8.) $f(x) = \frac{1}{\sqrt{x}} = x^{-1/2} \xrightarrow{D}$

$f'(x) = -\frac{1}{2}x^{-3/2} = \frac{-1}{2x^{3/2}}$;

$ARC = \frac{f(4)-f(1)}{4-1} = \frac{\frac{1}{2}-1}{3} = \frac{-1}{2}\cdot\frac{1}{3} = \frac{-1}{6}$;

IRC: $f'(1) = \frac{-1}{2}$, $f'(4) = \frac{-1}{2(4)^{3/2}} = \frac{-1}{2\cdot 8} = \frac{-1}{16}$

12.) a.) ARC appears to be greatest from $t = 1$ hr. to $t = 2$ hr. :

$ARC \approx \frac{550-250}{2-1} = 300 \frac{mg.}{hr.}$

b.) $M'(4) \approx \frac{600-1000}{7-2} = -\frac{400}{5} = -80 \frac{mg}{hr.}$

(slope of tangent line at $t = 4$ hr.) ;

the ARC on the interval $[1.5, 6]$ is

$ARC \approx \frac{0-360}{6-1.5} = \frac{-360}{4.5} = -80 \frac{mg.}{hr.}$

14.) $H = 330v^{1/2} - 33v + 344.85$

a.) $\frac{dH}{dv}$ has units $\frac{\left(\frac{kc}{m^2}\right)}{(hr.)} \div \left(\frac{m.}{s.}\right)$ and is

a measure of the rate of heat loss relative to the wind speed

b.) $H' = 330\cdot\frac{1}{2}v^{-1/2} - 33 = \frac{165}{\sqrt{v}} - 33$;

$H'(2) = \frac{165}{\sqrt{2}} - 33 \approx 83.67$ $(kc)(s)/(m.^3)(hr.)$,

$H'(5) = \frac{165}{\sqrt{5}} - 33 \approx 40.79$ $(kc)(s)/(m.^3)(hr.)$

15.) $s(t) = -16t^2 + 555 \xrightarrow{D} s'(t) = -32t$

a.) Ave. Vel. = $\dfrac{s(3)-s(2)}{3-2} = 411 - 491 = -80$ ft./sec.

b.) Inst. Vel. : $s'(2) = -64$ ft./sec., $s'(3) = -96$ ft./sec.

c.) hit ground: $s(t) = 0 \rightarrow -16t^2 + 550 = 0 \rightarrow$

$t^2 = \dfrac{555}{16} \rightarrow t \approx 5.89$ sec.

d.) $s'(5.89) = -188.47$ ft./sec.

16.) a.) Ave. Vel. = $\dfrac{0.75 \text{ km.}}{20 \text{ sec.}} \cdot \dfrac{1000 \text{ m.}}{1 \text{ km.}} = 37.5$ m./sec.

b.) Ave. Vel. = $\dfrac{750+750 \text{ m.}}{20+25 \text{ sec.}} = 33\frac{1}{3}$ m./sec.

32.) $P = 22t^2 + 52t + 10{,}000$

a.) $P(0) = 10{,}000$ people,
$P(10) = 12{,}720$ people,
$P(15) = 15{,}730$ people,
$P(20) = 19{,}840$ people,
$P(25) = 25{,}050$ people

b.) $\dfrac{dP}{dt} = 44t + 52$ people / yr.

c.) $P'(0) = 52$ people / yr.,
$P'(10) = 492$ people / yr.,
$P'(15) = 712$ people / yr.,
$P'(20) = 932$ people / yr.,
$P'(25) = 1152$ people / yr.

33.) $T = -0.0375t^2 + 0.3t + 100.4 \xrightarrow{D}$

$T' = -0.075t + 0.3$

c.) $T(0) = 100.4$ °F,

$T(4) = 101$ °F,

$T(8) = 100.4$ °F,

$T(12) = 98.6$ °F

d.) $\frac{dT}{dt}$ has units $\frac{°F}{hr.}$ and measures the rate at which temperature changes.

e.) $T'(0) = 0.3$ °F/hr.,

$T'(4) = 0.0$ °F/hr.,

$T'(8) = -0.3$ °F/hr.,

$T'(12) = -0.6$ °F/hr.

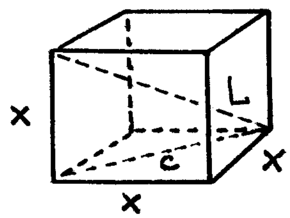
43.) $\frac{5}{16}'' = 100$ mi.

a.) $ARC \approx \left(\frac{1\,5/8 \cancel{\text{in.}}}{110 \text{ yr.}}\right)\left(\frac{100 \text{ mi.}}{5/16 \cancel{\text{in.}}}\right) \approx 4.73$ mi./yr.

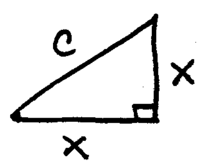
b.) $ARC \approx \left(\frac{1\,1/8 \text{ in.}}{100 \text{ yr.}}\right)\left(\frac{100 \text{ mi.}}{5/16 \text{ in.}}\right) = 3.6$ mi./yr.

c.) $ARC \approx \left(\frac{2\,3/4 \text{ in.}}{210 \text{ yr.}}\right)\left(\frac{100 \text{ mi.}}{5/16 \text{ in.}}\right) \approx 4.19$ mi./yr.

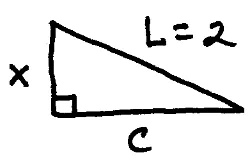
$\boxed{SA6}$:



Surface area of sphere is $S = 4\pi r^2 = 4\pi$, so radius of sphere is $r = 1$ ft. ; the diagonal of the cube corresponds to a diameter of the spere so that $L = 2$ ft :



$$x^2 + x^2 = c^2 \rightarrow 2x^2 = c^2$$



$$x^2 + c^2 = 2^2 \rightarrow x^2 + 2x^2 = 4 \rightarrow$$
$$3x^2 = 4 \rightarrow x^2 = \frac{4}{3} \rightarrow$$
$$x = \frac{2}{\sqrt{3}} \approx 1.15 \text{ ft.}$$

$\boxed{SA12}$: Let R: # of rabbits, M: # of mice, B: # of birds of prey, then $R = c \cdot \frac{M}{B^2}$

a.) new $R = c \cdot \frac{(1.4M)}{(0.95B)^2} = 1.55\left(c\frac{M}{B^2}\right) = 1.55(\text{old } R)$

so number of rabbits increases by 55%

b.) new $R = c \cdot \frac{(0.75M)}{(1.12B)^2} = 0.60\left(c\frac{M}{B^2}\right) = 0.60(\text{old } R)$

so number of rabbits decreases by 40% each year :

	# yrs.	# rabbits	
	1	$(0.6)R$	
i.)	2	$(0.6)^2R = 0.36R$	so decreases by 64% in 2 yrs.
	3	$(0.6)^3R$	
	4	$(0.6)^4R$	
ii.)	5	$(0.6)^5R = 0.08R$	so decreases by 92% in 5 yrs.