# Section 2.4

1.) $f(x) = x^2(3x^3 - 1) \xrightarrow{D}$
$f'(x) = x^2(9x^2) + 2x \cdot (3x^3 - 1)$ then
$f'(1) = 9 + 4 = 13$

3.) $f(x) = \frac{1}{3}(2x^3 - 4) \xrightarrow{D}$
$f'(x) = \frac{1}{3}(6x^2)$ then $f'(0) = 0$

6.) $g(x) = (x^2 - 2x + 1)(x^3 - 1) \xrightarrow{D}$
$g'(x) = (x^2 - 2x + 1)(3x^2) + (2x - 2)(x^3 - 1)$ then
$g'(1) = (0)(3) + (0)(0) = 0$

8.) $h(x) = \dfrac{x^2}{x+3} \xrightarrow{D} h'(x) = \dfrac{(x+3)(2x) - x^2 \cdot (1)}{(x+3)^2}$

then $h'(-1) = \dfrac{-4 - 1}{(2)^2} = \dfrac{-5}{4}$

12.) $f(x) = \dfrac{x+1}{x-1} \xrightarrow{D} f'(x) = \dfrac{(x-1)(1) - (x+1)(1)}{(x-1)^2}$

then $f'(2) = \dfrac{1 - 3}{(1)^2} = -2$ .

25.) $g(t) = (2t^3 - 1)^2 = (2t^3 - 1)(2t^3 - 1) \xrightarrow{D}$
$g'(t) = (2t^3 - 1) \cdot (6t^2) + (6t^2) \cdot (2t^3 - 1)$

30.) $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1} \xrightarrow{D}$

$$f'(x) = \frac{(x^2 - 1) \cdot (3x^2 + 3) - (x^3 + 3x + 2)(2x)}{(x^2 - 1)^2}$$

32.) $f(x) = (x^5 - 3x)(x^{-2}) \xrightarrow{D}$
$f'(x) = (x^5 - 3x)(-2x^{-3}) + (5x^4 - 3)(x^{-2})$

33.) $f(x) = x\left(1 - \frac{2}{x+1}\right) = x - \frac{2x}{x+1} \xrightarrow{D}$
$f'(x) = 1 - \frac{(x+1)(2) - 2x\cdot(1)}{(x+1)^2}$

37.) $g(x) = \left(\frac{x-3}{x+4}\right)(x^2 + 2x + 1) \xrightarrow{D}$

$g'(x) = \left(\frac{x-3}{x+4}\right)(2x+2) + \frac{(x+4)(1) - (x-3)(1)}{(x+4)^2}(x^2 + 2x + 1)$

38.) $f(x) = (3x^3 + 4x)(x-5)(x+1) \xrightarrow{D}$
(TRIPLE PRODUCT RULE :
$D(fgh) = f'gh + fg'h + fgh'$ )
$f'(x) = (9x^2 + 4)(x-5)(x+1)$
$+ (3x^3 + 4x)(1)(x+1) + (3x^3 + 4x)(x-5)(1)$

45.) $f(x) = \frac{x^2}{x-1} \xrightarrow{D} f'(x) = \frac{(x-1)(2x) - x^2(1)}{(x-1)^2}$
$= \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2} = 0$

$\rightarrow x(x-2) = 0 \longrightarrow x = 0, \; x = 2$.

46.) $f(x) = \frac{x^2}{x^2+1} \xrightarrow{D} f'(x) = \frac{(x^2+1)(2x) - x^2(2x)}{(x^2+1)^2}$

$$= \frac{2x \cdot [(\cancel{x^2}+1) - \cancel{x^2}]}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2} = 0 \rightarrow$$

$$2x = 0 \rightarrow x = 0.$$

47.) $$f(x) = \frac{x^4}{x^3+1} \xrightarrow{D} f'(x) = \frac{(x^3+1)(4x^3) - x^4 \cdot (3x^2)}{(x^3+1)^2}$$

$$= \frac{x^3 \cdot [4(x^3+1) - 3x^3]}{(x^3+1)^2} = \frac{x^3(4x^3+4-3x^3)}{(x^3+1)^2}$$

$$= \frac{x^3(x^3+4)}{(x^3+1)^2} = 0 \rightarrow x=0, \quad x = -4^{1/3}$$

48.) $$f(x) = \frac{x^4+3}{x^2+1} \xrightarrow{D} f'(x) = \frac{(x^2+1)(4x^3) - (x^4+3)(2x)}{(x^2+1)^2}$$

$$= \frac{2x \cdot [(x^2+1)(2x^2) - (x^4+3)]}{(x^2+1)^2}$$

$$= \frac{2x \cdot [2x^4 + 2x^2 - x^4 - 3]}{(x^2+1)^2}$$

$$= \frac{2x \cdot [x^4 + 2x^2 - 3]}{(x^2+1)^2}$$

$$= \frac{2x(x^2-1)(x^2+3)}{(x^2+1)^2} = \frac{2x(x-1)(x+1)(x^2+3)}{(x^2+1)^2} = 0$$

$$\rightarrow x=0, \quad x=1, \quad x=-1.$$

55.) $$f(t) = \frac{t^2-t+1}{t^2+1} \xrightarrow{D}$$

$$f'(t) = \frac{(t^2+1)(2t-1) - (t^2-t+1)(2t)}{(t^2+1)^2}$$

$$= \frac{2t^3 - t^2 + 2t - 1 - (2t^3 - 2t^2 + 2t)}{(t^2+1)^2}$$

$$= \frac{\cancel{2t^3} - t^2 + \cancel{2t} - 1 - \cancel{2t^3} + 2t^2 - \cancel{2t}}{(t^2+1)^2}$$

$$= \frac{t^2 - 1}{(t^2+1)^2} \longrightarrow \boxed{f(t) = \frac{t^2-1}{(t^2+1)^2}} \quad ;$$

a.) $$f'\left(\tfrac{1}{2}\right) = \frac{\frac{1}{4} - 1}{\left(\frac{1}{4}+1\right)^2} = \frac{-3/4}{\frac{25}{16}} = \frac{-3}{\cancel{4}} \cdot \frac{\cancel{16}^{\,4}}{25}$$

$$= \frac{-12}{25} = -0.48 = -48\,\%/\text{week} \;;$$

b.) $$f'(2) = \frac{3}{(5)^2} = \frac{3}{25} = 0.12 = 12\,\%/\text{week} \;;$$

c.) $$f'(8) = \frac{63}{(65)^2} \approx 1.49\,\%/\text{week} \;.$$

57.) $$P = 500 + \frac{2000t}{50 + t^2} \xrightarrow{D}$$

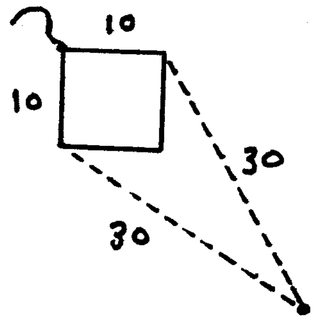
$$P' = \frac{(50+t^2)(2000) - (2000t)(2t)}{(50+t^2)^2}$$

$$= \frac{2000 \cdot [50 + t^2 - 2t^2]}{(50+t^2)^2}$$

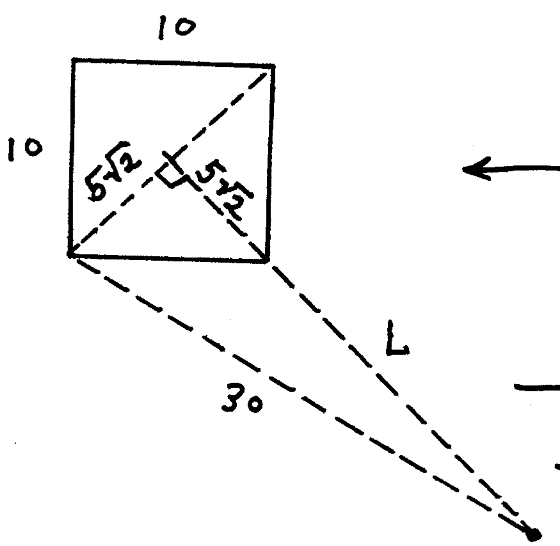
$$= \frac{2000 \cdot [50 - t^2]}{(50+t^2)^2} \quad ;$$

$$P'(2) = \frac{2000(46)}{(54)^2} \approx 31.55 \text{ bac./hr.}$$

$\boxed{SA17}$:




← closest point

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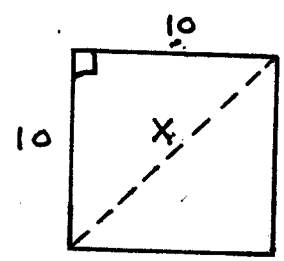


$x^2 = 10^2 + 10^2 \rightarrow$

$x^2 = 200 \rightarrow$

$x = 10\sqrt{2}$

$\rightarrow (5\sqrt{2})^2 + (5\sqrt{2} + L)^2 = 30^2$

$\rightarrow 50 + (5\sqrt{2} + L)^2 = 900$

$\rightarrow (5\sqrt{2} + L)^2 = 850$

$\rightarrow 5\sqrt{2} + L = \sqrt{850}$

$\rightarrow L = \sqrt{850} - 5\sqrt{2} \approx 22.1$ so

closest point is 22.1 ft. from corner of shed