Section 8.4

1.) $y = \frac{1}{2} - 3\sin x \xrightarrow{D} y' = 0 - 3\cos x = -3\cos x$

2.) $y = 5 + \sin x \xrightarrow{D} y' = 0 + \cos x = \cos x$

3.) $y = x^2 - \cos x \xrightarrow{D} y' = 2x - -\sin x = 2x + \sin x$

4.) $g(t) = \pi \cos t - t^{-2} \xrightarrow{D} g'(t) = \pi \cdot -\sin t - -2t^{-3}$
$\longrightarrow g'(t) = -\pi \sin t + 2t^{-3}$

5.) $f(x) = 4\sqrt{x} + 3\cos x \xrightarrow{D} f'(x) = 4 \cdot \frac{1}{2} x^{-1/2} + 3 \cdot -\sin x$
$\longrightarrow f'(x) = \frac{2}{\sqrt{x}} - 3\sin x$

6.) $f(x) = \sin x + \cos x \xrightarrow{D} f'(x) = \cos x - \sin x$

7.) $f(t) = t^2 \cos t \xrightarrow{D} f'(t) = t^2 \cdot -\sin t + 2t \cdot \cos t$
$\longrightarrow f'(t) = 2t\cos t - t^2 \sin t$

8.) $f(x) = (x+1)\cos x \xrightarrow{D} f'(x) = (x+1) \cdot -\sin x + (1)\cos x$
$\longrightarrow f'(x) = \cos x - (x+1)\sin x$

9.) $g(t) = \frac{\cos t}{t} \xrightarrow{D} g'(t) = \frac{t \cdot -\sin t - \cos t \cdot (1)}{t^2}$
$\longrightarrow g'(t) = \frac{-t\sin t - \cos t}{t^2}$

10.) $f(x) = \frac{\sin x}{x} \xrightarrow{D} f'(x) = \frac{x\cos x - \sin x \cdot (1)}{x^2}$
$\longrightarrow f'(x) = \frac{x\cos x - \sin x}{x^2}$

11.) $y = \tan x + x^2 \xrightarrow{D} y' = \sec^2 x + 2x$

12.) $y = x + \cot x \xrightarrow{D} y' = 1 - \csc^2 x$

39.) $y = \tan x \xrightarrow{D} y' = \sec^2 x$ and $x = -\frac{\pi}{4}$, $y = -1$ so slope $m = \sec^2\left(-\frac{\pi}{4}\right)$

$= \frac{1}{\cos^2\left(-\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{1}{\frac{1}{2}} = 2$ and line is $y - y_1 = m(x - x_1) \rightarrow$

$y - (-1) = 2\left(x - \left(-\frac{\pi}{4}\right)\right) \rightarrow \underline{y + 1 = 2\left(x + \frac{\pi}{4}\right)}$.

40.) $y = \sec x \xrightarrow{D} y' = \sec x \tan x$ and

$x = \frac{\pi}{3}$, $y = 2$ so slope $m = \sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right)$

$= \frac{1}{\cos(\pi/3)} \cdot \frac{\sin(\pi/3)}{\cos(\pi/3)} = \frac{\sqrt{3}/2}{(1/2)^2} = \frac{\sqrt{3}}{2} \cdot \frac{4}{1} = 2\sqrt{3}$

and line is $y - y_1 = m(x - x_1) \rightarrow$

$\underline{y - 2 = 2\sqrt{3}\left(x - \pi/3\right)}$.

43.) $y = \frac{\cos x}{\sin x} \xrightarrow{D} y' = \frac{\sin x(-\sin x) - \cos x \cdot (\cos x)}{\sin^2 x}$

$\rightarrow y' = -\frac{(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x}$ and

$x = \frac{3\pi}{4}$, $y = -1$ so slope $m = \frac{-1}{\sin^2\left(\frac{3\pi}{4}\right)} = \frac{-1}{\left(+\frac{\sqrt{2}}{2}\right)^2}$

$= \frac{-1}{\frac{2}{4}} = -2$ and line is $y - y_1 = m(x - x_1) \rightarrow$

$y - (-1) = -2\left(x - \frac{3\pi}{4}\right) \rightarrow \underline{y + 1 = -2\left(x - \frac{3\pi}{4}\right)}$.

44.) $y = \sin x \cos x \xrightarrow{D} y' = \sin x \cdot -\sin x + \cos x \cdot \cos x$
$\rightarrow y' = \cos^2 x - \sin^2 x$ and $x = \frac{3\pi}{2}$, $y = 0$
so slope $m = \cos^2\left(\frac{3\pi}{2}\right) - \sin^2\left(\frac{3\pi}{2}\right)$
$= (0)^2 - (-1)^2 = -1$ and line is
$y - y_1 = m(x - x_1) \rightarrow y - 0 = -1\left(x - \frac{3\pi}{2}\right) \rightarrow$
$y = \frac{3\pi}{2} - x$.

SA16: Initially ($t=0$), 200 l. with 0 g. salt;
in come 5 l./min., 30 g./l., so 150 g./min.

$t = 20$ min.: $200 + 5(20) = 300$ l. with $150(20) = 3000$ g. salt,
so concentration is $\frac{3000 \text{ g.}}{300 \text{ l.}} = 10$ g./l.;
remove 250 l. leaving [50 l.] with
concentration 10 g./l., or [500 g.] salt;
resume filling tank.

$t = 40$ min.: $50 + 5(20) = 150$ l. with $500 + 150(20) = 3500$
g. of salt so

a.) 150 l. of solution b.) 3500 g. of salt

c.) concentration is $\frac{3500 \text{ g.}}{150 \text{ l.}} = 23.33$ g./l.

d.) Let x: # min. past 20 then concentration is

$28 = \frac{500 + 150x}{50 + 5x} \rightarrow 1400 + 140x = 500 + 150x \rightarrow x = 90$ min.
so total time is $20 + 90 = 110$ min.