Section 8.4

13.) $y = e^{x^2} \sec x \xrightarrow{D} y' = e^{x^2} \cdot \sec x \tan x + 2x e^{x^2} \cdot \sec x$

14.) $y = e^{-x} \sin x \xrightarrow{D} y' = e^{-x} \cdot \cos x - e^{-x} \cdot \sin x$

15.) $y = \cos 3x + \sin^2 x \xrightarrow{D} y' = {}^{-}\sin 3x \cdot (3) + 2 \sin x \cdot \cos x$

16.) $y = \csc^2 x - \cos 2x \xrightarrow{D}$
$y' = 2 \csc x \cdot {}^{-}\csc x \cot x - {}^{-}\sin 2x \cdot (2) \rightarrow$

17.) $y = \sin \pi x \xrightarrow{D} y' = \cos \pi x \cdot (\pi)$

18.) $y = \frac{1}{2} \csc 2x \xrightarrow{D} y' = \frac{1}{2} \cdot {}^{-}\csc 2x \cot 2x \cdot (2)$

19.) $y = x \sin\left(\frac{1}{x}\right) \xrightarrow{D} y' = x \cdot \cos\left(\frac{1}{x}\right) \cdot \left(\frac{-1}{x^2}\right) + (1) \cdot \sin\left(\frac{1}{x}\right)$

20.) $y = x^2 \sin\left(\frac{1}{x}\right) \xrightarrow{D} y' = x^2 \cdot \cos\left(\frac{1}{x}\right) \cdot \left(\frac{-1}{x^2}\right) + (2x) \cdot \sin\left(\frac{1}{x}\right)$

21.) $y = 3 \tan(4x) \xrightarrow{D} y' = 3 \cdot \sec^2(4x) \cdot (4)$

22.) $y = \tan(e^x) \xrightarrow{D} y' = \sec^2(e^x) \cdot e^x$

23.) $y = 2 \tan^2(4x) \xrightarrow{D} y' = 2 \cdot 2 \tan(4x) \cdot \sec^2(4x) \cdot (4)$

24.) $y = -\sin^4(2x) \xrightarrow{D} y' = -4 \sin^3(2x) \cdot \cos(2x) \cdot (2)$

25.) $y = e^{2x} \sin(2x) \xrightarrow{D} y' = e^{2x} \cdot \cos(2x) \cdot (2) + 2e^{2x} \cdot \sin(2x)$

26.) $y = e^{-x}\cos\left(\frac{x}{2}\right) \xrightarrow{D} y' = e^{-x} \cdot -\sin\left(\frac{x}{2}\right)\cdot\left(\frac{1}{2}\right) - e^{-x}\cdot\cos\left(x/2\right)$

28.) $y = \frac{1}{4}\sin^2(2x) \xrightarrow{D} y' = \frac{1}{4}\cdot 2\sin(2x)\cdot\cos(2x)\cdot(2)$

31.) $y = \ln|\sin x| \xrightarrow{D} y' = \frac{1}{\sin x}\cdot\cos x$

36.) $y = \frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x \xrightarrow{D}$

$y' = \frac{1}{7}\cdot 7\sec^6 x\cdot\sec x\tan x - \frac{1}{5}\cdot 5\sec^4 x\cdot\sec x\tan x$

41.) $y = \sin(4x) \xrightarrow{D} y' = \cos(4x)\cdot 4$ and

$x=\pi,\ y=0$ so slope $m = 4\cos(4\pi) = 4(1) = 4$

and line is $y - y_1 = m(x - x_1) \longrightarrow$

$y - 0 = 4(x - \pi) \longrightarrow \underline{y = 4x - 4\pi}$.

46.) $y = (\sin x)^{1/2} \xrightarrow{D} y' = \frac{1}{2}(\sin x)^{-1/2}\cdot\cos x$

and $x = \frac{\pi}{6},\ y = \frac{\sqrt{2}}{2}$ so slope

$m = \frac{1}{2}\cdot\frac{\cos(\pi/6)}{\sqrt{\sin(\pi/6)}} = \frac{1}{2}\cdot\frac{\sqrt{3}/2}{\sqrt{1/2}} = \frac{\sqrt{3}/2}{2/\sqrt{2}} = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4}$

and line is $y - y_1 = m(x - x_1) \longrightarrow$

$\underline{y - \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4}\left(x - \pi/6\right)}$.

Section 2.5

17.) $y = (2x-7)^3 \xrightarrow{D} y' = 3(2x-7)^2 \cdot (2)$

22.) $f(x) = (4x-x^2)^3 \xrightarrow{D} f'(x) = 3(4x-x^2)^2 \cdot (4-2x)$

23.) $f(x) = (x^2-9)^{2/3} \xrightarrow{D} f'(x) = \frac{2}{3}(x^2-9)^{-1/3} \cdot (2x)$

30.) $y = 2(4-x^2)^{1/2} \xrightarrow{D} y' = 2 \cdot \frac{1}{2}(4-x^2)^{-1/2} \cdot (-2x)$

38.) $f(x) = x \cdot \sqrt{x^2+5} \xrightarrow{D}$

$f'(x) = x \cdot \frac{1}{2}(x^2+5)^{-1/2} \cdot (2x) + (1) \cdot \sqrt{x^2+5} \longrightarrow$

$f'(x) = \dfrac{x^2}{\sqrt{x^2+5}} + \sqrt{x^2+5}$ and $x = 2$, $f(2) = 6$

so slope $m = \frac{4}{\sqrt{9}} + \sqrt{9} = \frac{4}{3} + 3 = \frac{13}{3}$ and

line is $y - y_1 = m(x - x_1) \longrightarrow \underline{y - 6 = \frac{13}{3}(x-2)}$.

39.) $f(x) = (x^2-2x+1)^{1/2} \xrightarrow{D} f'(x) = \frac{1}{2}(x^2-2x+1)^{-1/2} \cdot (2x-2)$

$\longrightarrow f'(x) = \dfrac{x-1}{\sqrt{x^2-2x+1}}$ and $x = 2$, $f(2) = 1$ so

slope $m = \frac{1}{\sqrt{1}} = 1$ and line is $y - y_1 = m(x - x_1)$

$\longrightarrow y - 1 = 1 \cdot (x-2) \longrightarrow \underline{y = x - 1}$.

49.) $f(x) = (x^2-3x)^{-2} \xrightarrow{D} f'(x) = -2(x^2-3x)^{-3} \cdot (2x-3)$

59.) $f(x) = \left(\dfrac{3-2x}{4x}\right)^{1/2} \xrightarrow{D}$

$$f'(x) = \frac{1}{2}\left(\frac{3-2x}{4x}\right)^{-1/2} \cdot \frac{4x \cdot (-2) - (3-2x) \cdot (4)}{(4x)^2}$$

64.) $y = \left(\frac{4x^2}{3-x}\right)^3 \xrightarrow{D}$

$y' = 3\left(\frac{4x^2}{3-x}\right)^2 \cdot \frac{(3-x)\cdot(8x) - (4x^2)\cdot(-1)}{(3-x)^2}$

67.) $f(t) = (t^2-9)\sqrt{t+2} \xrightarrow{D}$

$f'(t) = (t^2-9)\cdot\frac{1}{2}(t+2)^{-1/2} + (2t)\cdot\sqrt{t+2}$ and

$t=-1$, $f(-1) = -8$ so slope

$m = (-8)(\frac{1}{2})(1)^{-1/2} - 2\sqrt{1} = -4-2 = -6$

and line is $y-(-8) = -6\,(x-(-1)) \rightarrow$

$y+8 = -6\,(x+1) = -6x-6 \rightarrow \underline{y = -6x-14}$.

70.) $y = \frac{x}{\sqrt{25+x^2}} \xrightarrow{D} y' = \frac{\sqrt{25+x^2}\cdot(1) - x\cdot\frac{1}{2}(25+x^2)^{-1/2}(2x)}{25+x^2}$

and $x=0$, $y=0$ so slope $m = \frac{5-0}{25} = 1/5$

and line is $y-0 = \frac{1}{5}(x-0) \rightarrow \underline{y = \frac{1}{5}x}$.

73.) $N = 400 - 1200\,(t^2+2)^{-2} \longrightarrow$

$\frac{dN}{dt} = 2400\,(t^2+2)^{-3}\cdot 2t = \frac{4800t}{(t^2+2)^3}$

t	0	1	2	3	4	days
$\frac{dN}{dt}$	0	177.8	44.4	10.8	3.3	$\frac{\text{bacteria}}{\text{day}}$

The growth rate for the number of bacteria is decreasing.

74.) FACT: "A is inversely proportional to B" means $A = \frac{k}{B}$ for some constant k. Then ...

a.) $V = \frac{k}{\sqrt{t+1}}$ and $t=0$, $V = \$10{,}000$ so

$$10{,}000 = \frac{k}{\sqrt{0+1}} \rightarrow k = 10{,}000 \rightarrow \boxed{V = 10{,}000\,(t+1)^{-1/2}} ;$$

b.) c.) rate of depreciation is

$$\frac{dV}{dt} = -5000\,(t+1)^{-3/2} \quad \text{so}$$

$t=1 \rightarrow \frac{dV}{dt} = -\$1767.77/\text{yr.}$ and

$t=3 \rightarrow \frac{dV}{dt} = -\$625/\text{yr.}$

Math 16A
Kouba
Worksheet 5

1. Differentiate. Do not simplify answers.

   a. $y = \sin(x^3)$

   b. $y = \sin^3 x$ (which can also be written $(\sin x)^3$)

   c. $y = \sin^3(x^3)$

   d. $f(x) = \cos(3 + \sqrt{x})\ \sin^2(5x)$

   e. $g(x) = \cos(\sin^4(7x^3))$

   f. $y = \dfrac{\csc^3(5x)}{\cos^2(3x)}$

2. Find an equation of the line tangent to the graph of $y = \dfrac{\sin 2x}{\cos 3x + \tan x}$ at $x = \pi/6$.

3. Find the slope of the line perpendicular to the graph of $y = \sin(\pi/3 \tan 2x)$ at $x = \pi/8$.

4. Solve $f'(x) = 0$ for x, where $0 \le x \le 2\pi$.

   a. $f(x) = (1/2)\sin 2x + \sin x$

   b. $f(x) = \cos^2 x + \sin x$

Worksheet 5

1.) a.) $Y' = \cos(x^3) \cdot 3x^2$

b.) $Y' = 3\sin^2 x \cdot \cos x$

c.) $Y' = 3\sin^2(x^3) \cdot \cos(x^3) \cdot 3x^2$

d.) $f'(x) = \cos(3+\sqrt{x}) \cdot 2\sin(5x) \cdot \cos(5x) \cdot 5$
$+ {}^{-}\sin(3+\sqrt{x}) \cdot \frac{1}{2}x^{-1/2} \cdot \sin^2(5x)$

e.) $g'(x) = -\sin(\sin^4(7x^3)) \cdot 4\sin^3(7x^3) \cdot \cos(7x^3) \cdot 21x^2$

f.) $$Y' = \frac{\cos^2(3x) \cdot 3\csc^2(5x) \cdot {}^{-}\csc(5x)\cot(5x) \cdot 5 - \csc^3(5x) \cdot 2\cos(3x) \cdot {}^{-}\sin(3x) \cdot 3}{\cos^4(3x)}$$

2.) $$Y = \frac{\sin 2x}{\cos 3x + \tan x}$$

$$\rightarrow Y' = \frac{(\cos 3x + \tan x) \cdot 2\cos 2x - \sin 2x \cdot ({}^{-}3\sin 3x + \sec^2 x)}{(\cos 3x + \tan x)^2},$$

at $x = \frac{\pi}{6}$, $Y = \dfrac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{2} + \tan\frac{\pi}{6}} = \dfrac{\frac{\sqrt{3}}{2}}{0 + \frac{1}{\sqrt{3}}} = \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{3}}{1} = \dfrac{3}{2}$,

and slope $Y' = \dfrac{(\cos\frac{\pi}{2} + \tan\frac{\pi}{6}) \cdot 2\cos\frac{\pi}{3} - \sin\frac{\pi}{3}({}^{-}3\sin\frac{\pi}{2} + \sec^2\frac{\pi}{6})}{(\cos\frac{\pi}{2} + \tan\frac{\pi}{6})^2}$

$$= \frac{(0 + \frac{1}{\sqrt{3}})2 \cdot \frac{1}{2} - \frac{\sqrt{3}}{2}(-3 \cdot 1 + (\frac{2}{\sqrt{3}})^2)}{(0 + \frac{1}{\sqrt{3}})^2} = \left(\frac{1}{\sqrt{3}} - \frac{-5}{2\sqrt{3}}\right) \cdot 3$$

$= \sqrt{3} + \frac{5}{2}\sqrt{3} = \frac{7}{2}\sqrt{3}$ so tangent line is

$$Y - 3/2 = \frac{7}{2}\sqrt{3}\left(x - \frac{\pi}{6}\right).$$

3.) $Y = \sin\left(\frac{\pi}{3}\tan 2x\right) \rightarrow$

$Y' = \cos\left(\frac{\pi}{3}\tan 2x\right) \cdot \frac{\pi}{3}\sec^2 2x \cdot 2$, at $x = \frac{\pi}{8}$

$Y' = \cos\left(\frac{\pi}{3}\tan\frac{\pi}{4}\right) \cdot \frac{\pi}{3}\sec^2\frac{\pi}{4} \cdot 2$

$= \frac{2\pi}{3}\cos\left(\frac{\pi}{3} \cdot 1\right)\left(\sqrt{2}\right)^2 = \frac{4\pi}{3} \cdot \frac{1}{2} = \frac{2\pi}{3}$ so

slope of line $\perp$ is $m = \frac{-3}{2\pi}$.

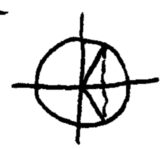
4.) a.) $f(x) = \frac{1}{2}\sin 2x + \sin x \rightarrow$

$f'(x) = \frac{1}{\cancel{2}} \cdot \cancel{2}\cos 2x + \cos x = \left(2\cos^2 x - 1\right) + \cos x = 0 \rightarrow$

$(2\cos x - 1)(\cos x + 1) = 0$

$\downarrow \qquad \downarrow$

$\cos x = \frac{1}{2} \qquad \cos x = -1 \longrightarrow x = \pi$

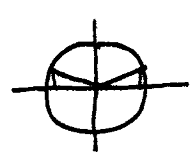
$\hookrightarrow$  $x = \frac{\pi}{3}, \frac{5\pi}{3}$

b.) $f(x) = \cos^2 x + \sin x \longrightarrow$

$f'(x) = -2\cos x \cdot \sin x + \cos x = 0 \longrightarrow$

$\cos x\,(1 - 2\sin x) = 0$

$\downarrow$

$\sin x = \frac{1}{2}$  $\rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\downarrow$

$\cos x = 0 \longrightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

<u>Supplemental Trig</u>

ST6.) $Y = \sin^3\left(\tan^2(3x)\right)$

$Y' = 3\sin^2\left(\tan^2(3x)\right) \cdot \cos\left(\tan^2(3x)\right) \cdot 2\tan(3x) \cdot \sec^2(3x) \cdot 3$