Section 2.6

1.) $f(x) = 5 - 4x \xrightarrow{D} f'(x) = -4 \xrightarrow{D} f''(x) = 0$

4.) $f(x) = 3x^2 + 4x \xrightarrow{D} f'(x) = 6x + 4 \xrightarrow{D} f''(x) = 6$

7.) $f(t) = \frac{3}{4}t^{-2} \xrightarrow{D} f'(t) = \frac{3}{4}(-2)t^{-3} = -\frac{3}{2}t^{-3} \xrightarrow{D}$
$f''(t) = \frac{9}{2}t^{-4}$

11.) $f(x) = \frac{x+1}{x-1} \xrightarrow{D} f'(x) = \frac{(x-1)(1)-(x+1)(1)}{(x-1)^2}$
$\rightarrow f'(x) = \frac{\cancel{x}-1-\cancel{x}-1}{(x-1)^2} = \frac{-2}{(x-1)^2} = -2(x-1)^{-2}$
$\xrightarrow{D} f''(x) = 4(x-1)^{-3}$

13.) $y = x^4 + 4x^3 + 8x^2 \xrightarrow{D} y' = 4x^3 + 12x^2 + 16x \xrightarrow{D}$
$y'' = 12x^2 + 24x + 16$

16.) $f(x) = x^4 - 2x^3 \xrightarrow{D} f'(x) = 4x^3 - 6x^2 \xrightarrow{D}$
$f''(x) = 12x^2 - 12x \xrightarrow{D} f'''(x) = 24x - 12$

19.) $f(x) = \frac{3}{16}x^{-2} \xrightarrow{D} f'(x) = -\frac{6}{16}x^{-3} = -\frac{3}{8}x^{-3} \xrightarrow{D}$
$f''(x) = \frac{9}{8}x^{-4} \xrightarrow{D} f'''(x) = -\frac{36}{8}x^{-5} = -\frac{9}{2}x^{-5}$

20.) $f(x) = x^{-1} \xrightarrow{D} f'(x) = -x^{-2} \xrightarrow{D} f''(x) = 2x^{-3}$
$\xrightarrow{D} f'''(x) = -6x^{-4}$

21.) $g(t) = 5t^4 + 10t^2 + 3 \xrightarrow{D} g'(t) = 20t^3 + 20t \xrightarrow{D}$
$g''(t) = 60t^2 + 20$ so $g''(2) = 60(4) + 20 = 260$

24.) $f(t) = \sqrt{2t+3} \xrightarrow{D} f'(t) = \frac{1}{2}(2t+3)^{-1/2} \cdot (2)$

$= (2t+3)^{-1/2} \xrightarrow{D} f''(t) = -\frac{1}{2}(2t+3)^{-3/2} \cdot (2)$

$= -(2t+3)^{-3/2} \xrightarrow{D} f'''(t) = \frac{3}{2}(2t+3)^{-5/2} \cdot (2)$

$\rightarrow f'''(t) = 3(2t+3)^{-5/2}$ ~~so~~

$f'''(\frac{1}{2}) = 3(4)^{-5/2} = 3 \cdot \frac{1}{(4^{1/2})^5} = 3 \cdot \frac{1}{2^5} = \frac{3}{32}$

34.) $f(x) = 3x^3 - 9x + 1 \xrightarrow{D} f'(x) = 9x^2 - 9 = 9(x-1)(x+1) = 0$

$\rightarrow x = 1$, $x = -1$; then

$f''(x) = 18x = 0 \rightarrow x = 0$.

35.) $f(x) = (x+3)(x-4)(x+5) \xrightarrow{D}$ (TPR)

$f'(x) = (1)(x-4)(x+5) + (x+3)(1)(x+5) + (x+3)(x-4)(1)$

$= x^2 + x - 20 + x^2 + 8x + 15 + x^2 - x - 12$

$= 3x^2 + 8x - 17 = 0 \rightarrow$

$$x = \frac{-8 \pm \sqrt{64 - 4(3)(-17)}}{2(3)} = \frac{-8 \pm \sqrt{268}}{6}$$

$$\rightarrow x = \frac{-8 \pm 2\sqrt{67}}{6} = \frac{-4 \pm \sqrt{67}}{3} \text{ ; then}$$

$f''(x) = 6x + 8 = 0 \rightarrow x = -8/6 \rightarrow x = -4/3$.

38.) $f(x) = x\sqrt{4-x^2} \xrightarrow{D}$

$f'(x) = x \cdot \frac{1}{2}(4-x^2)^{-1/2} \cdot (-2x) + (1) \cdot (4-x^2)^{1/2}$

$$= \frac{-x^2}{(4-x^2)^{1/2}} + \frac{(4-x^2)^{1/2}}{1} = \frac{-x^2 + (4-x^2)}{(4-x^2)^{1/2}}$$

$$= \frac{4-2x^2}{(4-x^2)^{1/2}} = 0 \rightarrow 4 - 2x^2 = 2(2-x^2) = 0 \rightarrow$$

$\boxed{x = \sqrt{2},\ x = -\sqrt{2}}$; $\xrightarrow{D}$

$$f''(x) = \frac{(4-x^2)^{1/2}(-4x) - (4-2x^2)\cdot\frac{1}{2}(4-x^2)^{-1/2}\cdot(-2x)}{(4-x^2)}$$

$$= \frac{\frac{-4x(4-x^2)^{1/2}}{1} + \frac{x(4-2x^2)}{(4-x^2)^{1/2}}}{\frac{(4-x^2)}{1}} = \frac{-4x(4-x^2)+x(4-2x^2)}{(4-x^2)^{1/2}(4-x^2)}$$

$$= \frac{2x\left[-2(4-x^2)+(2-x^2)\right]}{(4-x^2)^{3/2}} = \frac{2x\left[x^2-6\right]}{(4-x^2)^{3/2}} = 0 \longrightarrow$$

$$2x\left[x^2-6\right]=0 \longrightarrow \boxed{x=0,\ x=\sqrt{6},\ x=-\sqrt{6}}$$

40.) $f(x) = \dfrac{x}{x^2+1} \rightarrow f'(x) = \dfrac{(x^2+1)(1)-x(2x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2} = 0$

$$\rightarrow 1-x^2=0 \rightarrow \boxed{x=1,\ x=-1} ;$$

$$f''(x) = \frac{(x^2+1)^2(-2x)-(1-x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$$

$$= \frac{-2x\,\cancel{(x^2+1)}\cdot\left[(x^2+1)+2(1-x^2)\right]}{(x^2+1)^{\cancel{4}\,3}} = \frac{-2x\left[3-x^2\right]}{(x^2+1)^3} = 0 \rightarrow$$

$$-2x\left[3-x^2\right]=0 \rightarrow x=0,\ x=\pm\sqrt{3}$$

FACT : Let $s(t)$ be height (feet) of object above ground at time t (seconds). Then it can be shown that

$$\boxed{s(t) = -16t^2 + v_0 t + s_0},$$

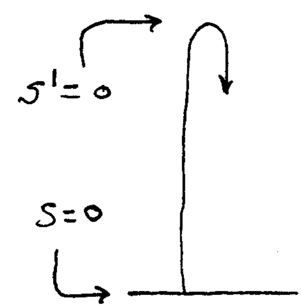
where v_0 is initial velocity and s_0 is initial height.

41.) a.) $v_0 = 144$ ft./sec. and $s_0 = 0$ ft. $\rightarrow$

$s(t) = -16t^2 + 144t$ is height at time t

b.) velocity : $s'(t) = -32t + 144$ and

acceleration : $s''(t) = -32$



c.) highest point : $s'(t) = 0 \rightarrow$

$-32t + 144 = 0 \rightarrow$ $t = 4.5$ sec.

and $s(4.5) = -16(4.5)^2 + 144(4.5)$

$= 324$ ft.

d.) hit ground : $s(t) = 0 \rightarrow 0 = -16t^2 + 144t \rightarrow$

$0 = 16t(-t+9) \rightarrow t = 0$ or $t = 9$ sec.,

$s'(9) = -144$ ft./sec. and velocity is 144 ft./sec., the same as initial speed.

42.) a.) $v_0 = 0$ ft./sec. and $s_0 = 1250$ ft. $\rightarrow$

$s(t) = -16t^2 + 0 \cdot t + 1250 = 1250 - 16t^2$

is height at time t

b.) velocity : $s'(t) = -32t$

acceleration : $s''(t) = -32$

c.) hit ground : $s(t) = 0 \rightarrow 1250 - 16t^2 = 0$

$\rightarrow t^2 = \frac{1250}{16} = 78.12 \rightarrow$ $t \approx 8.84$ sec.

d.) $s'(8.84) \approx -32(8.84) \approx -282.8$ ft./sec.

43.) velocity $s'(t) = \dfrac{90t}{t+10}$ ft./sec. and

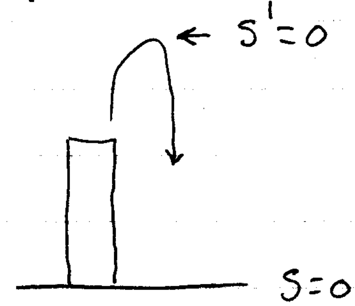
acceleration $s''(t) = \dfrac{(t+10)(90) - 90t(1)}{(t+10)^2} = \dfrac{900}{(t+10)^2}$ ft/sec.^{2}

t	:	0	10	20	30	40	50	60	sec.
s'	:	0	45	60	67.5	72	75	77.14	ft./sec.
s''	:	9	2.25	1	0.56	0.36	0.25	0.18	ft./sec.^{2}

Velocity is increasing ;
acceleration is decreasing ;
it appears car is approaching
a constant velocity (0 acceleration)

50.) a.) $V_0 = 48$ ft./sec. and $S_0 = 64$ ft. $\rightarrow$

$S(t) = -16t^2 + 48t + 64$ is height at time t



b.) velocity : $s'(t) = -32t + 48$

acceleration : $s''(t) = -32$

c.) hit ground : $s(t) = 0 \rightarrow$

$-16t^2 + 48t + 64 = -16(t^2 - 3t - 4)$

$= -16(t-4)(t+1) = 0 \rightarrow t = -1$ (no) or

$t = 4$ sec.

d.) $s'(t) = 0 \rightarrow -32t + 48 = 0 \rightarrow$

$t = \frac{48}{32} = \frac{3}{2}$ sec.

e.) $s\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 64 = 100$ ft.