Section 3.1

2.) $f(x) = x + 32x^{-2} \xrightarrow{D} f'(x) = 1 - 64x^{-3}$;

at $x = 2 \rightarrow f'(2) = 1 - 64(2)^{-3} = 1 - 64\left(\frac{1}{8}\right) = -7$,

at $x = 4 \rightarrow f'(4) = 1 - 64(4)^{-3} = 1 - 64\left(\frac{1}{64}\right) = 0$,

at $x = 8 \rightarrow f'(8) = 1 - 64(8)^{-3} = 1 - 64\left(\frac{1}{8}\right)^3 = \frac{7}{8}$

3.) $f(x) = (x+2)^{2/3} \xrightarrow{D} f'(x) = \frac{2}{3}(x+2)^{-1/3}$;

at $x = -3 \longrightarrow f'(-3) = \frac{2}{3}(-1)^{-1/3} = \frac{2}{3}(-1) = -\frac{2}{3}$,

at $x = -2 \longrightarrow f'(-2) = \frac{2}{3}(0)^{-1/3} = \frac{2}{3(0)^{1/3}}$ so

$f'(-2)$ DNE (corner)

at $x = -1 \longrightarrow f'(-1) = \frac{2}{3}(1)^{-1/3} = \frac{2}{3}(1) = \frac{2}{3}$

6.) $f(x) = \frac{x^3}{4} - 3x \xrightarrow{D} f'(x) = \frac{3}{4}x^2 - 3$

$= 3\left(\frac{x^2}{4} - 1\right) = 0 \longrightarrow x = 2,\ x = -2$:

$$\begin{array}{ccccc} + & 0 & - & 0 & + \end{array} \quad f'$$

rel. max. $\left\{ \begin{array}{l} x = -2 \\ y = 4 \end{array} \right.$ $\quad \left. \begin{array}{l} x = 2 \\ y = -4 \end{array} \right\}$ rel. min.

f is $\uparrow$ for $x < -2,\ x > 2$;

f is $\downarrow$ for $-2 < x < 2$.

8.) $f(x) = \frac{x^2}{x+1} \xrightarrow{D} f'(x) = \frac{(x+1)(2x) - x^2(1)}{(x+1)^2}$

$= \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2} = 0 \rightarrow$

$x = 0,\ x = -2$: NO

$$\begin{array}{ccccccc} + & 0 & - & \vdots & - & 0 & + \end{array} \quad f'$$

rel. max. $\left\{ \begin{array}{l} x = -2 \\ y = -4 \end{array} \right.$ $\quad x = -1 \quad$ $\left. \begin{array}{l} x = 0 \\ y = 0 \end{array} \right\}$ rel. min.

f is $\uparrow$ for $x<-2,\ x>0$;
f is $\downarrow$ for $-2<x<-1,\ -1<x<0$.

14.) $y=-x^2+2x \xrightarrow{D} y'=-2x+2=0 \rightarrow x=1$:

$$\begin{array}{ccc} + & 0 & - \\ \hline & x=1 & \\ & y=1 & \end{array} \quad y'$$

y is $\uparrow$ for $x<1$;
y is $\downarrow$ for $x>1$.

15.) $y=x^3-6x^2 \xrightarrow{D} y'=3x^2-12x=3x(x-4)=0$
$\rightarrow x=0,\ x=4$:

$$\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & x=0 & & x=4 & \\ & y=0 & & y=-32 & \end{array} \quad y'$$

rel. max. $\left\{\begin{array}{l} x=0 \\ y=0 \end{array}\right.$ $\quad$ $\left.\begin{array}{l} x=4 \\ y=-32 \end{array}\right\}$ rel. min.

y is $\uparrow$ for $x<0,\ x>4$;
y is $\downarrow$ for $0<x<4$.

18.) $f(x)=\sqrt{4-x^2} \xrightarrow{D} f'(x)=\frac{1}{2}(4-x^2)^{-1/2}\cdot(-2x)$
$=\dfrac{-x}{\sqrt{4-x^2}}=0 \rightarrow x=0$:

$$\begin{array}{ccccccc} \text{NO} & & + & 0 & - & & \text{NO} \\ \hline & x=-2 & & x=0 & & x=2 & \\ & y=0 & & & & y=0 & \end{array} \quad f'$$

rel. min. $\left\{\begin{array}{l} x=-2 \\ y=0 \end{array}\right.$ $\quad x=0 \quad$ $\left.\begin{array}{l} x=2 \\ y=0 \end{array}\right\}$ rel. min.

f is $\uparrow$ for $-2<x<0$;
f is $\downarrow$ for $0<x<2$.

22.) $y=x^3-3x+2 \xrightarrow{D} y'=3x^2-3=3(x-1)(x+1)=0 \rightarrow$
$x=1,\ x=-1$:

$$\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & x=-1 & & x=1 & \\ & y=4 & & y=0 & \end{array} \quad y'$$

rel. max. $\left\{\begin{array}{l} x=-1 \\ y=4 \end{array}\right.$ $\quad$ $\left.\begin{array}{l} x=1 \\ y=0 \end{array}\right\}$ rel. min.

y is $\uparrow$ for $x<-1,\ x>1$;
y is $\downarrow$ for $-1<x<1$.

23.) $f(x) = x\sqrt{x+1} \xrightarrow{D} f'(x) = x\cdot\frac{1}{2}(x+1)^{-1/2} + (1)\cdot\sqrt{x+1}$

$$= \frac{x}{2\sqrt{x+1}} + \frac{\sqrt{x+1}}{1} = \frac{x+2(x+1)}{2\sqrt{x+1}} = \frac{3x+2}{2\sqrt{x+1}} = 0 \rightarrow$$

$x = -2/3$:

NO $\quad - \quad 0 \quad + \quad f'$

$x=-1 \qquad \left.\begin{array}{l} x = -2/3 \\ y = \frac{-2}{3\sqrt{3}} \end{array}\right\}$ rel. min.

f is $\uparrow$ for $x > -2/3$;
f is $\downarrow$ for $-1 < x < -2/3$.

25.) $f(x) = x^4 - 2x^3 \xrightarrow{D} f'(x) = 4x^3 - 6x^2 = 2x^2(2x-3) = 0$
$\longrightarrow x = 0,\ x = 3/2$:

$- \quad 0 \quad - \quad 0 \quad + \quad f'$

$x = 0 \qquad \left.\begin{array}{l} x = 3/2 \\ y = -27/16 \end{array}\right\}$ rel. min.

y is $\uparrow$ for $x > 3/2$;
y is $\downarrow$ for $x < 0$,
$0 < x < 3/2$.

27.) $f(x) = \frac{x}{x^2+4} \xrightarrow{D} f'(x) = \frac{(x^2+4)(1) - x\cdot(2x)}{(x^2+4)^2}$

$$= \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2} = \frac{(2-x)(2+x)}{(x^2+4)^2} = 0 \rightarrow$$

$x = 2,\ x = -2$:

$- \quad 0 \quad + \quad 0 \quad - \quad f'$

rel. min. $\left\{\begin{array}{l} x = -2 \\ y = -1/4 \end{array}\right. \qquad \left.\begin{array}{l} x = 2 \\ y = 1/4 \end{array}\right\}$ rel. max.

f is $\uparrow$ for $-2 < x < 2$;
f is $\downarrow$ for $x < -2$,
$x > 2$.

35.) $C = \frac{10}{x} + \frac{10x}{x+3} \xrightarrow{D} C' = \frac{-10}{x^2} + \frac{(x+3)(10) - (10x)(1)}{(x+3)^2}$

$$= \frac{-10}{x^2} + \frac{\cancel{10x} + 30 - \cancel{10x}}{(x+3)^2} = \frac{-10(x^2+6x+9) + 30x^2}{x^2(x+3)^2}$$

$$= \frac{-10x^2 - 60x - 90 + 30x^2}{x^2(x+3)^2} = \frac{20x^2 - 60x - 90}{x^2(x+3)^2}$$

$$= \frac{10(2x^2 - 6x - 9)}{x^2(x+3)^2} = 0 \rightarrow 2x^2 - 6x - 9 = 0 \rightarrow$$

$$x = \frac{6 \pm \sqrt{36 + 72}}{4} = \frac{6 \pm 6\sqrt{3}}{4} = \frac{3 \pm 3\sqrt{3}}{2} \rightarrow$$

$$x = \frac{3 + 3\sqrt{3}}{2} \approx 4.1 \text{ cars} :$$

C': $-$ 0 $+$

$x = 1$ $\quad$ $\left.\begin{array}{l} x \approx 4.1 \\ \text{cost } C \approx \$821 \end{array}\right\}$ rel. min.

C is $\uparrow$ for $x > 4.1$;
C is $\downarrow$ for $1 < x < 4.1$.

40.) $P = 2.36x - \frac{1}{25{,}000}x^2 - 3500 \xrightarrow{D}$

$P' = 2.36 - \frac{1}{12{,}500}x \rightarrow x = 29{,}500$ bags :

P': $+$ 0 $-$

$x = 0$ $\quad$ rel. max. $\left\{\begin{array}{l} x = 29{,}500 \\ \text{profit } P = \$31{,}310 \end{array}\right.$ $\quad$ $x = 50{,}000$

P is $\uparrow$ for $0 < x < 29{,}500$;
P is $\downarrow$ for $29{,}500 < x < 50{,}000$.

Section 3.2

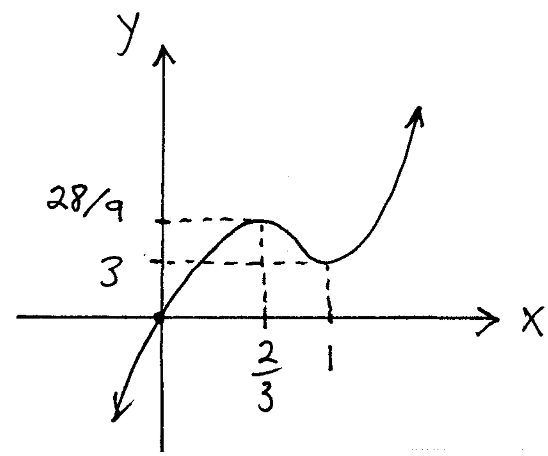
5.) $g(x) = 6x^3 - 15x^2 + 12x \xrightarrow{D}$

$g'(x) = 18x^2 - 30x + 12 = 6(3x^2 - 5x + 2)$

$= 6(3x-2)(x-1) = 0 \longrightarrow x = 2/3,\ x = 1$:

$+ \quad 0 \quad - \quad 0 \quad +$ $\quad g'$

rel. max. $\left\{ \begin{array}{l} x = 2/3 \\ y = 28/9 \end{array} \right.$ $\qquad \left. \begin{array}{l} x = 1 \\ y = 3 \end{array} \right\}$ rel. min.



g is $\uparrow$ for $x < \frac{2}{3}$, $x > 1$;

g is $\downarrow$ for $\frac{2}{3} < x < 1$;

$x = 0$: $y = 0$

$y = 0$: $x(6x^2 - 15x + 12) = 0 \rightarrow x = 0$

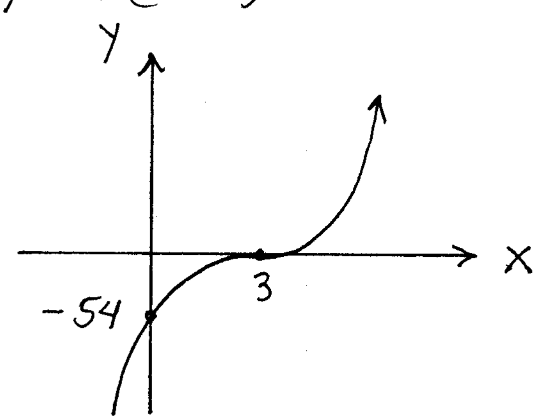
8.) $h(x) = 2(x-3)^3 \xrightarrow{D} h'(x) = 6(x-3)^2 = 0 \longrightarrow$

$x = 3$:

$+ \quad 0 \quad +$ $\quad h'$

$x = 3$

$y = 0$



h is $\uparrow$ for $x < 3$, $x > 3$;

$x = 0$: $y = -54$,

$y = 0$: $x = 3$.

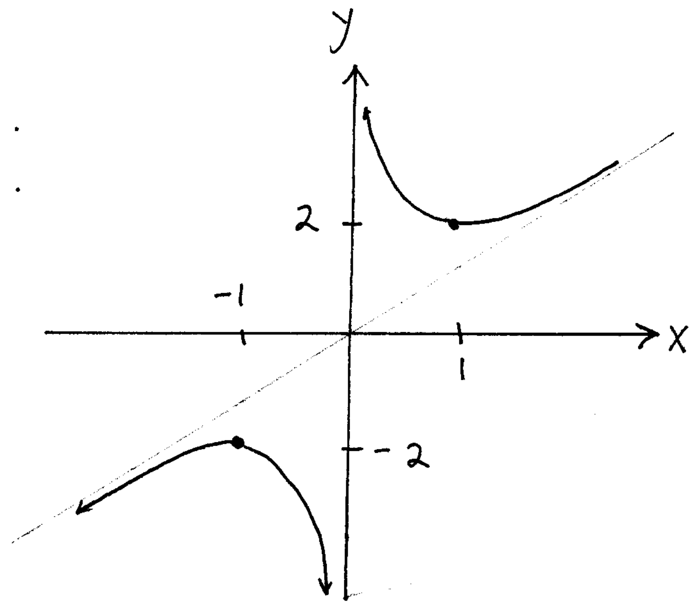
16.) $f(x) = x + \frac{1}{x} \xrightarrow{D} f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$

$= \frac{(x-1)(x+1)}{x^2} = 0 \longrightarrow x = 1,\ x = -1$:

NO

$$\begin{array}{ccccccc} + & 0 & - & \vdots & - & 0 & + \end{array} \quad f'$$

rel. max $\begin{cases} x=-1 \\ y=-2 \end{cases}$ $\quad x=0 \quad$ $\left.\begin{matrix} x=1 \\ y=2 \end{matrix}\right\}$ rel. min.



f is $\uparrow$ for $x<-1, x>1$;
f is $\downarrow$ for $-1<x<0$,
$0<x<1$;

$x=0$: (NO)
$y=0$: $0=x+\frac{1}{x} \to$
$x=-1/x \to x^2=-1$ (NO);

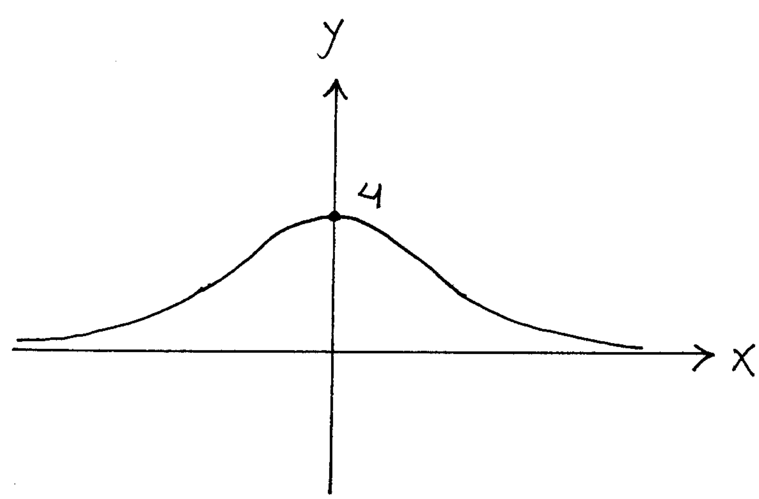
$$\left.\begin{matrix} \lim\limits_{x\to 0^+} \left(x+\frac{1}{x}\right) = 0+\frac{1}{0^+} = +\infty \\ \lim\limits_{x\to 0^-} \left(x+\frac{1}{x}\right) = 0+\frac{1}{0^-} = -\infty \end{matrix}\right\} \text{V.A. is } x=0$$

18.) $h(x) = \frac{4}{x^2+1} \xrightarrow{D} h'(x) = -4(x^2+1)^{-2}\cdot(2x) = \frac{-8x}{(x^2+1)^2} = 0$

$\to x=0$:

$$\begin{array}{ccc} + & 0 & - \end{array} \quad h'$$

abs. max. $\begin{cases} x=0 \\ y=4 \end{cases}$



h is $\uparrow$ for $x<0$;
h is $\downarrow$ for $x>0$;

$x=0$: $y=4$,
$y=0$: (NO);

$$\lim_{x\to\pm\infty} \frac{4}{x^2+1} = \text{"}\frac{4}{\infty}\text{"} = 0$$

$\to$ H.A. is $y=0$

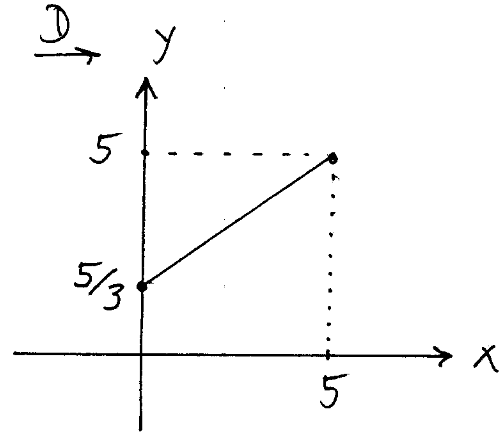
20.) $f(x) = \frac{1}{3}(2x+5)$ on $[0,5] \xrightarrow{D}$

$f'(x) = 2/3$

| + + + | f'

abs. min. $\left\{ \begin{array}{l} x=0 \\ y=5/3 \end{array} \right.$ $\quad$ $\left. \begin{array}{l} x=5 \\ y=5 \end{array} \right\}$ abs. max.

f is $\uparrow$ for $0<x<5$



21.) $f(x) = 5-2x^2$ on $[-1,2] \xrightarrow{D}$

$f'(x) = -4x = 0 \rightarrow x=0$:

| + 0 − | f'

$x=-1$	$x=0$	$x=2$
$\underbrace{y=3}$	$\underbrace{y=5}$	$\underbrace{y=-3}$
rel. min.	abs. max.	abs. min.

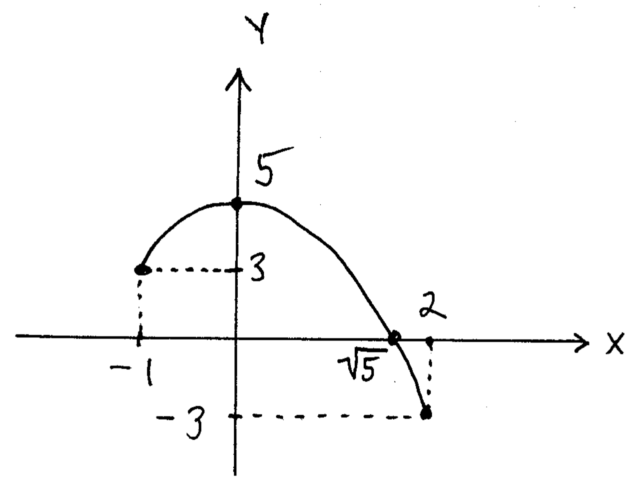
f is $\uparrow$ for $-1<x<0$;

f is $\downarrow$ for $0<x<2$;

$x=0: y=5$,

$y=0: 0=5-x^2 \rightarrow$

$x^2=5 \rightarrow x = \pm\sqrt{5}$



23.) $f(x) = x^3-3x^2$ on $[-1,4] \xrightarrow{D}$

$f'(x) = 3x^2-6x = 3x(x-2) = 0$

$\rightarrow x=0, x=2$:

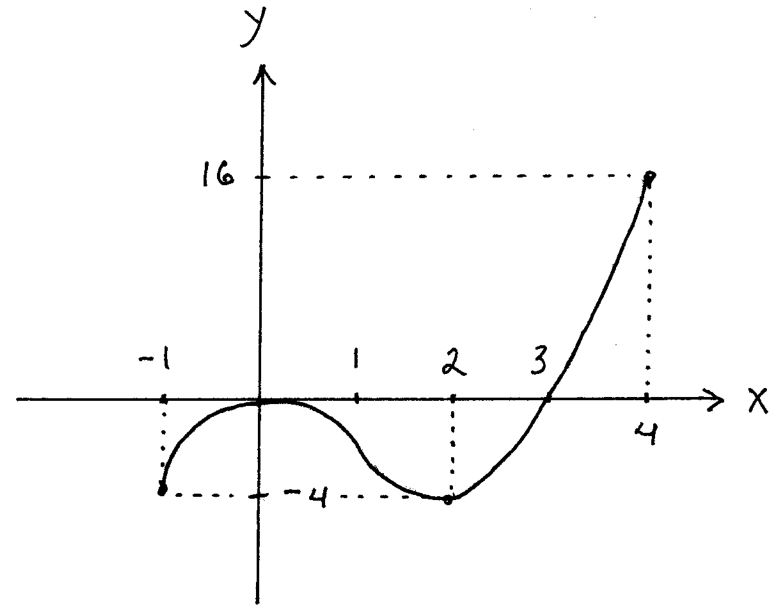
| + 0 − 0 + | f'

$x=-1$	$x=0$	$x=2$	$x=4$
$\underbrace{y=-4}$	$\underbrace{y=0}$	$\underbrace{y=-4}$	$\underbrace{y=16}$
abs. min.	rel. max.	abs. min	abs. max.

f is $\uparrow$ for $-1<x<0$,
$2<x<4$;

f is $\downarrow$ for $0<x<2$;

$x=0: \; y=0,$
$y=0: \; x^2(x-3)=0$
$\rightarrow x=0, x=3$



44.) Demand $x = c \cdot \frac{1}{p^3}$, where p: price; if $p=\$10$, $x=8$ so $8 = c \cdot \frac{1}{1000} \rightarrow c = 8000 \rightarrow$ demand $\boxed{x = \frac{8000}{p^3} \text{ or } p = \frac{20}{x^{1/3}}}$;

cost $C = 25 + 4x$ and revenue
$R = px = \left(\frac{20}{x^{1/3}}\right)x = 20\,x^{2/3}$ so profit

$$\boxed{P = R - C = 20x^{2/3} - (25+4x)}.$$

Find maximum profit:

$P' = \frac{40}{3}x^{-1/3} - 4 = 0 \rightarrow x^{-1/3} = \frac{3}{10} \rightarrow$

$x = (x^{-1/3})^{-3} = \left(\frac{3}{10}\right)^{-3} \approx 37$;

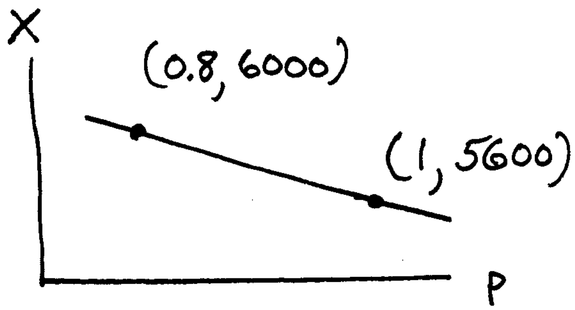
P': $+$ 0 $-$ at $x=37$, max.

The maximum profit occurs when

$x = 37$ units, $p = \$6$, and profit

$P = \$49.07$.

$\boxed{190:45}$ Let x: demand, p: price and if $x = 6000$, $p = \$.80$ and $x = 5600$, $p = \$1.00$ are points on a linear graph, then:



slope is

$$m = \frac{6000-5600}{0.8-1} = \frac{400}{-0.2} = {}^{-}2000$$

so demand function (line) is

$x - 5600 = {}^{-}2000(p-1)$ or $\boxed{p = \frac{19}{5} - \frac{x}{2000}}$;

cost $C = 5000 + 0.4x$ and revenue

$R = px = \frac{19}{5}x - \frac{x^2}{2000}$ so that profit

$$P = R - C = \frac{19}{5}x - \frac{x^2}{2000} - \left(5000 + \frac{2}{5}x\right)$$

or $\boxed{P = \frac{-x^2}{2000} + \frac{17}{5}x - 5000}$; determine

maximum profit $\longrightarrow$

$$P' = \frac{-x}{1000} + \frac{17}{5} = 0 \rightarrow x = 3400$$

so maximum profit occurs when

$+ \quad 0 \quad -$ $\quad P'$

$x = 3400$

$x = 3400$ cans, price $p = \$2.10$, and profit $P = \$780$