Section 2.7

1.) $5xy = 1 \xrightarrow{D} 5x \cdot y' + 5 \cdot y = 0 \rightarrow y' = \frac{-5y}{5x} \rightarrow y' = \frac{-y}{x}$

4.) $4x^2y - \frac{3}{y} = 0 \rightarrow 4x^2y^2 - 3 = 0 \xrightarrow{D}$

$4x^2 \cdot 2yy' + 8x \cdot y^2 = 0 \rightarrow y' = \frac{-8xy^2}{8x^2y} \rightarrow y' = \frac{-y}{x}$

6.) $xy^2 + 4xy = 10 \xrightarrow{D} x \cdot 2yy' + 1 \cdot y^2 + 4x \cdot y' + 4 \cdot y = 0 \rightarrow$

$(2xy + 4x)y' = -4y - y^2 \longrightarrow y' = \frac{-4y - y^2}{2xy + 4x}$

10.) $\frac{xy - y^2}{y - x} = 1 \longrightarrow xy - y^2 = y - x \xrightarrow{D}$

$xy' + 1 \cdot y - 2yy' = y' - 1 \rightarrow xy' - 2yy' - y' = -y - 1 \rightarrow$

$(x - 2y - 1)y' = -y - 1 \longrightarrow y' = \frac{-y-1}{x-2y-1}$

15.) $Y + XY = 4 \rightarrow Y' + XY' + Y = 0 \rightarrow$

$(1+X)Y' = -Y \rightarrow Y' = \frac{-Y}{1+X}$ at $(-5,-1) \longrightarrow$

$Y' = \frac{-(-1)}{1-5} = \frac{-1}{4}$

18.) $X^2Y + Y^2X = -2 \xrightarrow{D} X^2Y' + 2XY + Y^2(1) + 2YY' \cdot X = 0 \rightarrow$

$(X^2 + 2XY)Y' = -2XY - Y^2 \rightarrow Y' = \frac{-2XY - Y^2}{X^2 + 2XY}$ at $(2,-1) \rightarrow$

$Y' = \frac{4-1}{4-4} = \frac{3}{0}$ is undefined.

22.) $(XY)^{1/2} = X - 2Y \xrightarrow{D} \frac{1}{2}(XY)^{-1/2} \cdot [XY' + Y \cdot (1)] = 1 - 2Y' \rightarrow$

$XY' + Y = 2(XY)^{1/2}[1 - 2Y'] \rightarrow XY' + Y = 2(XY)^{1/2} - 4(XY)^{1/2}Y' \rightarrow$

$[X + 4(XY)^{1/2}]Y' = 2(XY)^{1/2} - Y \longrightarrow$

$Y' = \frac{2(XY)^{1/2} - Y}{X + 4(XY)^{1/2}}$ at $(4,1) \rightarrow Y' = \frac{2\sqrt{4} - 1}{4 + 4\sqrt{4}} = \frac{3}{12} = \frac{1}{4}$

24.) $(X+Y)^3 = X^3+Y^3 \xrightarrow{D} \cancel{3}(X+Y)^2\cdot(1+Y') = \cancel{3}X^2+\cancel{3}Y^2\cdot Y' \rightarrow$

$(X+Y)^2+(X+Y)^2Y' = X^2+Y^2Y' \rightarrow (X+Y)^2Y'-Y^2Y' = X^2-(X+Y)^2 \rightarrow$

$[(X+Y)^2-Y^2]Y' = X^2-(X+Y)^2 \rightarrow Y' = \dfrac{X^2-(X+Y)^2}{(X+Y)^2-Y^2}$ at $(-1,1)$

$\rightarrow Y' = \dfrac{1-0}{0-1} = -1$.

30.) $X^2-Y^3=0 \xrightarrow{D} 2X-3Y^2Y' = 0 \rightarrow Y' = \dfrac{2X}{3Y^2}$ at $(-1,1)$

$\rightarrow Y' = \dfrac{-2}{3}$.

34.) $4Y^2-X^2 = 7 \xrightarrow{D} 8YY'-2X = 0 \rightarrow Y' = \dfrac{2X}{8Y} = \dfrac{X}{4Y}$

at $(3,2) \rightarrow Y' = \dfrac{3}{8}$.

36.) $X^2+Y^2 = 9 \rightarrow 2X+2YY' = 0 \rightarrow Y' = {}^{-X}/_{Y}$,

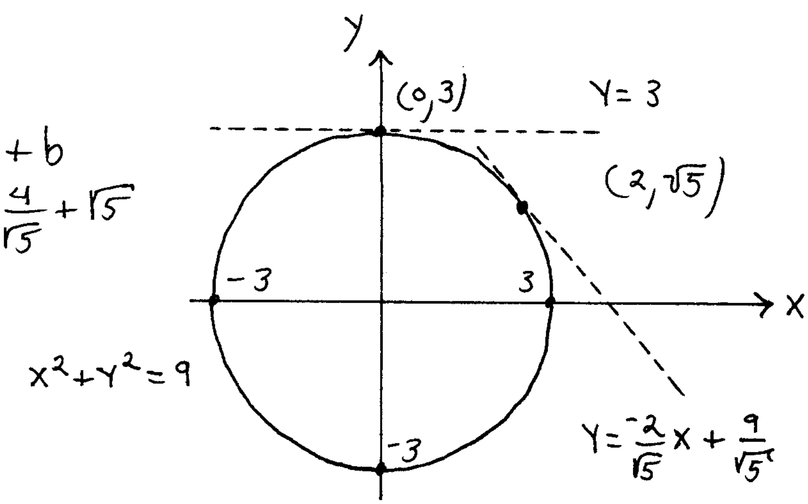
<u>at $(0,3)$</u>: $Y' = \frac{0}{3} = 0$ so tangent line is $Y=3$;

<u>at $(2,\sqrt{5})$</u>: $Y' = \dfrac{-2}{\sqrt{5}}$ so

tangent line is $Y = mX+b$

$\rightarrow \sqrt{5} = \dfrac{-2}{\sqrt{5}}(2)+b \rightarrow b = \dfrac{4}{\sqrt{5}}+\sqrt{5}$

$\rightarrow b = \dfrac{9}{\sqrt{5}} \rightarrow Y = \dfrac{-2}{\sqrt{5}}X+\dfrac{9}{\sqrt{5}}$



40.) $Y^2 = \dfrac{X^3}{4-X} \rightarrow$

$4Y^2-XY^2 = X^3 \xrightarrow{D} 8YY'-(X\cdot 2YY'+Y^2) = 3X^2 \rightarrow$

$(8Y-2XY)Y' = 3X^2+Y^2 \rightarrow Y' = \dfrac{3X^2+Y^2}{8Y-2XY}$,

<u>at $(2,2)$</u>: $Y'=2 \rightarrow Y = mX+b \rightarrow 2 = 2(2)+b \rightarrow b = -2 \rightarrow$

tangent line is $Y = 2X-2$;

<u>at $(2,-2)$</u>: $Y' = -2 \rightarrow Y = mX+b \rightarrow -2 = -2(2)+b \rightarrow b = 2 \rightarrow$

tangent line is $Y = -2X+2$.

45.) $100\,X^{0.75}\,Y^{0.25} = 135{,}540$

a.) Differentiate implicitly :

$100\,X^{0.75}\cdot(0.25)\,Y^{-0.75}\,Y' + 100\,(0.75)\,X^{-0.25}\cdot Y^{0.25} = 0$,

let $X = 1500$ and $Y = 1000$ then

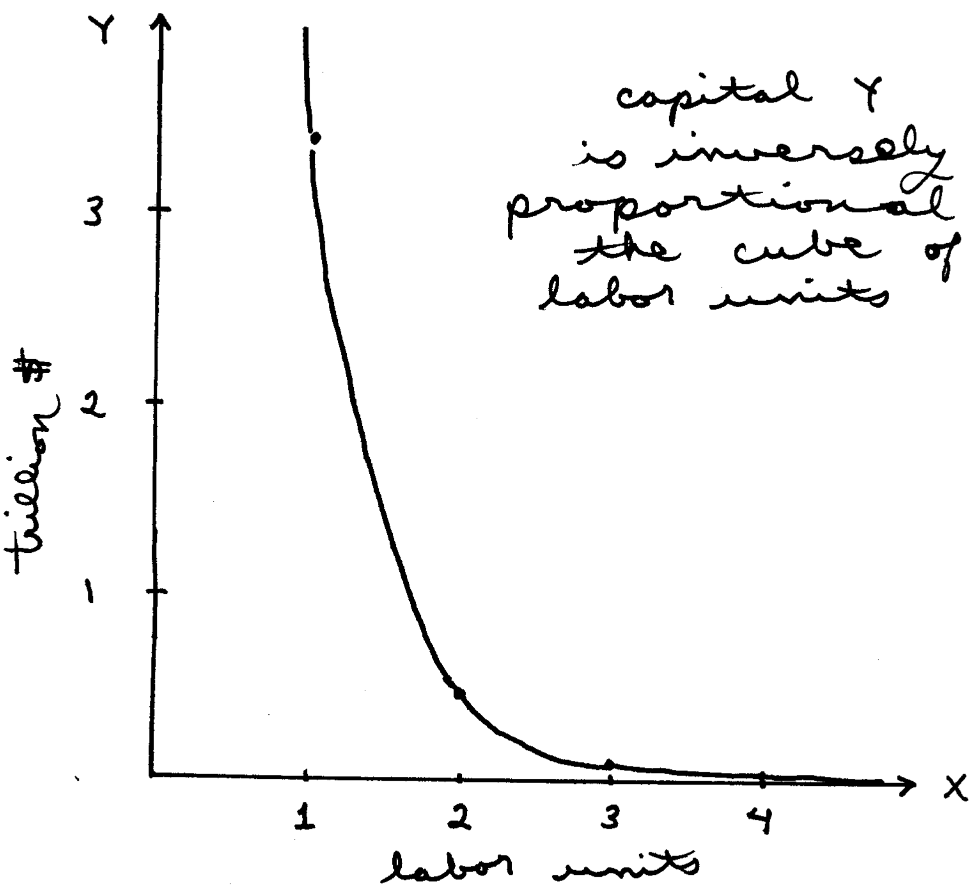
$(6025.7)(0.0056)\,Y' + (12.05)(5.623) = 0 \longrightarrow$

$Y' = -2.01$ \$/labor units

b.) $100\,X^{3/4}\,Y^{1/4} = 135{,}540 \longrightarrow$

$Y^{1/4} = \dfrac{1355.4}{X^{3/4}} \longrightarrow Y = \left(\dfrac{1355.4}{X^{3/4}}\right)^4 \longrightarrow$

$Y = \dfrac{(3.375)(10^{12})}{X^3} = \dfrac{3.375}{X^3}$ trillion \$



capital Y is inversely proportional to the cube of labor units X

Section 8.4

47.) $\sin x + \cos 2Y = 1 \rightarrow$

$\cos X - 2\sin 2Y \cdot Y' = 0 \rightarrow$

$Y' = \dfrac{\cos X}{2\sin 2Y}$ at $\left(\frac{\pi}{2}, \frac{\pi}{4}\right) \rightarrow$

$Y' = \dfrac{\cos \frac{\pi}{2}}{2\sin \frac{\pi}{2}} = \dfrac{0}{2 \cdot 1} = 0$

48.) $\tan(X+Y) = X \rightarrow$

$\sec^2(X+Y) \cdot (1+Y') = 1 \rightarrow$

$\sec^2(X+Y) + \sec^2(X+Y) \cdot Y' = 1 \rightarrow$

$Y' = \dfrac{1 - \sec^2(X+Y)}{\sec^2(X+Y)}$ at $(0,0) \rightarrow$

$Y' = \dfrac{1 - \sec^2(0)}{\sec^2(0)} = \dfrac{1 - \frac{1}{\cos^2(0)}}{\frac{1}{\cos^2(0)}} = \dfrac{1 - \frac{1}{1}}{\frac{1}{1}} = 0.$