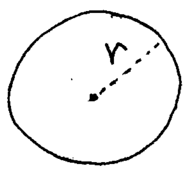
Section 2.8

2.) $Y = X^2 - 3X \rightarrow \frac{dY}{dt} = 2X \cdot \frac{dX}{dt} - 3 \cdot \frac{dX}{dt} = (2X-3) \cdot \frac{dX}{dt}$

a.) $\frac{dY}{dt} = (2 \cdot 3 - 3) \cdot (2) = 6$

b.) $5 = (2 \cdot 1 - 3) \cdot \frac{dX}{dt} \longrightarrow \frac{dX}{dt} = -5$

5.)



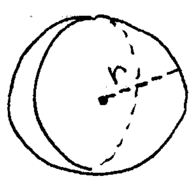
$A = \pi r^2 \rightarrow \frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$

and $\frac{dr}{dt} = 2$ in./min.

a.) $\frac{dA}{dt} = \pi \cdot 2(6)(2) = 24\pi$ in.^{2}/min.

b.) $\frac{dA}{dt} = \pi \cdot 2(24)(2) = 96\pi$ in.^{2}/min.

6.)



$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

and $\frac{dr}{dt} = 2$ in./min.

a.) $\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}(6)^2 \cdot (2) = 288\pi$ in.^{3}/min.

b.) $\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}(24)^2 \cdot (2) = 4608\pi$ in.^{3}/min.

9.) $V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$ and

$\frac{dV}{dt} = 20$ ft.^{3}/min.

a.) $20 = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}(1)^2 \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{5}{\pi}$ ft./min.

b.) $20 = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}(2)^2 \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{5}{4\pi}$ ft./min.

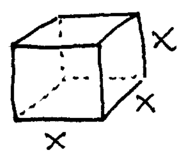
10.) $V = \frac{1}{3}\pi r^2 h$ and $h = 3r$ so $V = \pi r^3 \rightarrow$

$\frac{dV}{dt} = \pi \cdot 3r^2 \cdot \frac{dr}{dt}$ and $\frac{dr}{dt} = 2$ in./min.

a.) $\frac{dV}{dt} = \pi \cdot 3(6)^2 \cdot (2) = 216\pi$ in.^{3}/min.

b.) $\frac{dV}{dt} = \pi \cdot 3(24)^2(2) = 3456\pi$ in.^{3}/min.

13.)



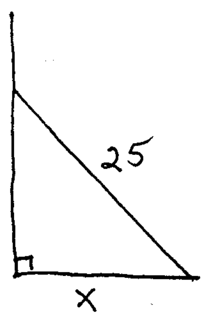
$V = x^3 \rightarrow \frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$

and $\frac{dx}{dt} = 3$ cm./sec.

a.) $\frac{dV}{dt} = 3(1)^2(3) = 9$ cm.^{3}/sec.

b.) $\frac{dV}{dt} = 3(10)^2(3) = 900$ cm.^{3}/sec.

17.)

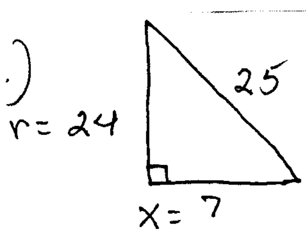


$x^2 + r^2 = 25^2$ and $\frac{dx}{dt} = 2$ ft./sec

find $\frac{dr}{dt}$:

$2x \cdot \frac{dx}{dt} + 2r\frac{dr}{dt} = 0 \rightarrow$

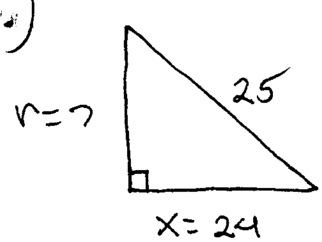
a.)



$2(7)(2) + 2(24)\frac{dr}{dt} = 0 \rightarrow$

$\frac{dr}{dt} = \frac{-28}{48} = \frac{-7}{12}$ ft./sec.

c.)



$2(24)(2) + 2(7)\frac{dr}{dt} = 0 \rightarrow$

$\frac{dr}{dt} = \frac{-96}{14} = -\frac{48}{7}$ ft./sec.

Math 16A
Kouba
Worksheet 6

1.) Find the derivative $dy/dx = y'$ for each of the following. You need not simplify your answers.

a.) $x y^3 + x^2 y = x - y$

b.) $y^2 (x + y)^3 = x^2$

c.) $\sin(3x + 2y) = \tan(x^3)$

d.) $y \sec(y^2 + 1) = \cos(x + y)$

e.) $(x - \csc(3y))^5 = 7 + y$

2. Find the slope of the line which is tangent to the graph of $xy + x^2 + y^3 = 8$ at $x = 0$.

3. Find the concavity of the graph $x^2 y + y^3 = x + 1$ at $x = 0$.

Worksheet 6

1.) a.) $XY^3 + X^2Y = X - Y \rightarrow$

$X \cdot 3Y^2 \cdot Y' + Y^3 + X^2Y' + 2XY = 1 - Y' \rightarrow$

$3XY^2 Y' + X^2 Y' + Y' = 1 - 2XY - Y^3 \rightarrow$

$(3XY^2 + X^2 + 1) Y' = 1 - 2XY - Y^3 \rightarrow$

$$Y' = \frac{1 - 2XY - Y^3}{3XY^2 + X^2 + 1} .$$

b.) $Y^2 (X+Y)^3 = X^2 \rightarrow$

$Y^2 \cdot 3(X+Y)^2 (1+Y') + 2YY'(X+Y)^3 = 2X \rightarrow$

$3Y^2(X+Y)^2 + 3Y^2(X+Y)^2 Y' + 2Y(X+Y)^3 Y' = 2X \rightarrow$

$[3Y^2(X+Y)^2 + 2Y(X+Y)^3] Y' = 2X - 3Y^2(X+Y)^2 \rightarrow$

$$Y' = \frac{2X - 3Y^2(X+Y)^2}{3Y^2(X+Y)^2 + 2Y(X+Y)^3} .$$

c.) $\sin(3X + 2Y) = \tan(X^3) \rightarrow$

$\cos(3X+2Y) \cdot (3 + 2Y') = \sec^2(X^3) \cdot 3X^2 \rightarrow$

$3\cos(3X+2Y) + 2\cos(3X+2Y) Y' = 3X^2 \sec^2(X^3) \rightarrow$

$2\cos(3X+2Y) Y' = 3X^2 \sec^2(X^3) - 3\cos(3X+2Y) \rightarrow$

$$Y' = \frac{3X^2 \sec^2(X^3) - 3\cos(3X+2Y)}{2\cos(3X+2Y)} .$$

d.) $Y \cdot \sec(Y^2+1) = \cos(X+Y) \rightarrow$

$Y \cdot \sec(Y^2+1)\tan(Y^2+1) \cdot 2YY' + Y'\sec(Y^2+1)$

$= -\sin(X+Y) \cdot (1+Y') \rightarrow$

$$2Y^2 \sec(Y^2+1)\tan(Y^2+1)\cdot Y' + \sec(Y^2+1)\cdot Y'$$
$$= -\sin(X+Y) - \sin(X+Y)\cdot Y' \rightarrow$$

$$2Y^2 \sec(Y^2+1)\tan(Y^2+1)\, Y' + \sec(Y^2+1)\cdot Y'$$
$$+ \sin(X+Y)\cdot Y' = -\sin(X+Y) \rightarrow$$

$$\left[2Y^2 \sec(Y^2+1)\tan(Y^2+1) + \sec(Y^2+1) + \sin(X+Y)\right]\cdot Y'$$
$$= -\sin(X+Y) \rightarrow$$

$$Y' = \frac{-\sin(X+Y)}{2Y^2\sec(Y^2+1)\tan(Y^2+1) + \sec(Y^2+1) + \sin(X+Y)}$$

e.) $(X - \csc(3Y))^5 = 7 + Y \rightarrow$

$$5(X-\csc(3Y))^4\cdot\left[1 + \csc(3Y)\cot(3Y)\cdot 3Y'\right] = Y' \rightarrow$$

$$5(X-\csc(3Y))^4 + 15(X-\csc(3Y))^4\csc(3Y)\cot(3Y)\cdot Y' = Y' \rightarrow$$

$$15(X-\csc(3Y))^4\csc(3Y)\cot(3Y)\cdot Y' - Y' = -5(X-\csc(3Y))^4 \rightarrow$$

$$\left[15(X-\csc(3Y))^4\csc(3Y)\cot(3Y) - 1\right]Y' = -5(X-\csc(3Y))^4 \rightarrow$$

$$Y' = \frac{-5(X-\csc(3Y))^4}{15(X-\csc(3Y))^4\csc(3Y)\cot(3Y) - 1}$$

2.) $XY + X^2 + Y^3 = 8 \xrightarrow{D} XY' + Y + 2X + 3Y^2Y' = 0 \rightarrow$

$(X + 3Y^2)Y' = -2X - Y \rightarrow Y' = \dfrac{-2X - Y}{X + 3Y^2}$,

at $X = 0$, $Y^3 = 8$ so $Y = 2$ and slope of tangent line is

$$Y' = \frac{0-2}{0+12} = \frac{-1}{6}.$$

3.) $X^2Y + Y^3 = X + 1 \xrightarrow{D} X^2Y' + 2XY + 3Y^2Y' = 1 \rightarrow$

$(X^2 + 3Y^2)Y' = 1 - 2XY$

$\rightarrow Y' = \dfrac{1 - 2XY}{X^2 + 3Y^2}$

at $X = 0$, $Y^3 = 1$

$\rightarrow Y = 1$ so that

$Y' = \dfrac{1}{3}$

differentiate again $\xrightarrow{D}$

$X^2Y'' + 2XY' + 2XY' + 2Y$

$+ 3Y^2Y'' + 6YY' \cdot Y' = 0$,

let $X = 0$, $Y = 1$, and $Y' = 1/3$

so that

$$(0)Y'' + (0)\left(\tfrac{1}{3}\right) + (0)\left(\tfrac{1}{3}\right) + 2(1) + 3(1)Y'' + 6(1)\left(\tfrac{1}{3}\right)^2 = 0$$

$$\rightarrow 3Y'' = -2 - \frac{2}{3} = \frac{-8}{3} \rightarrow Y'' = \frac{-8}{9} < 0$$

so graph is concave down at $X = 0$

Supplemental Trig

ST4.) $x^3 + \sin Y = Y^2 + 7X \xrightarrow{D}$

$3X^2 + \cos Y \cdot Y' = 2Y \cdot Y' + 7 \rightarrow$

$\cos Y \cdot Y' - 2Y \cdot Y' = 7 - 3X^2 \rightarrow$

$(\cos Y - 2Y) \cdot Y' = 7 - 3X^2 \rightarrow$

$Y' = \dfrac{7 - 3X^2}{\cos Y - 2Y}$; if $X = 0$, $Y = 0$ then

slope $m = Y' = \dfrac{7 - 0}{\cos 0 - 0} = \dfrac{7}{1} = 7$ and

tangent line is $\boxed{Y = 7X}$.

ST5.) $(X + \tan Y)^2 = 8 + \sin(XY) \xrightarrow{D}$

$2(X + \tan Y) \cdot (1 + \sec^2 Y \cdot Y') = \cos(XY) \cdot (X \cdot Y' + (1) \cdot Y) \rightarrow$

$2(X + X\sec^2 Y \cdot Y' + \tan Y + \tan Y \sec^2 Y \cdot Y') = X\cos(XY) \cdot Y' + Y\cos(XY) \rightarrow$

$2X + 2X\sec^2 Y \cdot Y' + 2\tan Y + 2\tan Y \sec^2 Y \cdot Y' = X\cos(XY) \cdot Y' + Y\cos(XY) \rightarrow$

$2X\sec^2 Y \cdot Y' + 2\tan Y \sec^2 Y \cdot Y' - X\cos(XY) \cdot Y' = Y\cos(XY) - 2X - 2\tan Y \rightarrow$

$(2X\sec^2 Y + 2\tan Y \sec^2 Y - X\cos(XY)) \cdot Y' = Y\cos(XY) - 2X - 2\tan Y \rightarrow$

$Y' = \dfrac{Y\cos(XY) - 2X - 2\tan Y}{2X\sec^2 Y + 2\tan Y \sec^2 Y - X\cos(XY)}$; if $X = 2$, $Y = \frac{\pi}{4}$

slope of tangent line is

$m = Y' = \dfrac{\frac{\pi}{4}\cos\frac{\pi}{2} - 4 - 2\tan\frac{\pi}{4}}{4\sec^2\frac{\pi}{4} + 2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4} - 2\cos\frac{\pi}{2}}$

$= \dfrac{\frac{\pi}{4}(0) - 4 - 2(1)}{4(\sqrt{2})^2 + 2(1)(\sqrt{2})^2 - 2(0)} = \dfrac{-6}{12} = \dfrac{-1}{2}$, and

slope of $\perp$ line is $\boxed{m = 2}$