Section 2.8

16.) $Y = (1+x^2)^{-1} \rightarrow \dfrac{dY}{dt} = -(1+x^2)^{-2} \cdot 2x \cdot \dfrac{dx}{dt}$,

$\dfrac{dx}{dt} = 2$ cm./min. so $\dfrac{dY}{dt} = -(1+x^2)^{-2} \cdot 2x \cdot 2$ or

$\dfrac{dY}{dt} = \dfrac{-4x}{(1+x^2)^2}$;

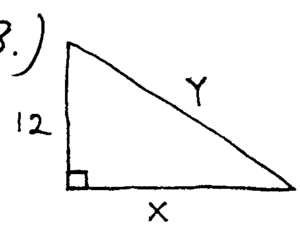
a.) If $x=-2$ then $\dfrac{dY}{dt} = \dfrac{-4(-2)}{(1+4)^2} = \dfrac{8}{25}$ cm./min.

b.) If $x=2$ then $\dfrac{dY}{dt} = \dfrac{-4(2)}{(1+4)^2} = \dfrac{-8}{25}$ cm./min.

c.) If $x=0$ then $\dfrac{dY}{dt} = \dfrac{-4(0)}{(1+0)^2} = 0$ cm./min.

d.) If $x=10$ then $\dfrac{dY}{dt} = \dfrac{-4(10)}{(1+100)^2} = \dfrac{-40}{10{,}201}$ cm./min.

18.)

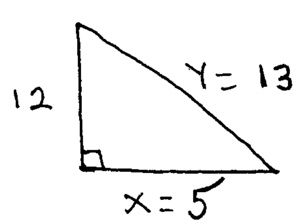


$x^2 + 12^2 = Y^2$ and $\dfrac{dY}{dt} = -4$ ft./sec.

find $\dfrac{dx}{dt}$:

$\cancel{2}x \cdot \dfrac{dx}{dt} = \cancel{2}Y \cdot \dfrac{dY}{dt} \rightarrow$

a.)



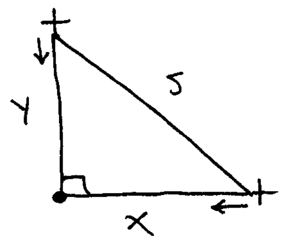
$(5)\dfrac{dx}{dt} = (13)(-4) \rightarrow$

$\dfrac{dx}{dt} = \dfrac{(13)(-4)}{5} = -\dfrac{52}{5}$ ft./sec.

b.) $\dfrac{dx}{dt} = \dfrac{Y}{x}\left(\dfrac{dY}{dt}\right) = \dfrac{-4Y}{x}$ and as $x \rightarrow 0$, $Y \rightarrow 12$

so $\dfrac{dx}{dt}$ approaches $\dfrac{-48}{0} = -\infty$ ft./sec.

19.)

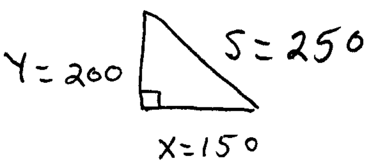


$$x^2 + Y^2 = S^2 \rightarrow$$

$$\cancel{2}x \frac{dx}{dt} + \cancel{2}Y \frac{dY}{dt} = \cancel{2}S \frac{dS}{dt} \; ;$$

a.) Find $\frac{dS}{dt}$ when $x = 150$ mi., $\frac{dx}{dt} = -450$ mph., $Y = 200$ mi., and $\frac{dY}{dt} = -600$ mph. :

$$(150)(-450) + (200)(-600) = (250) \cdot \frac{dS}{dt} \rightarrow$$



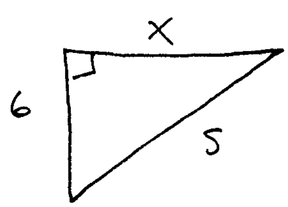
$$\frac{dS}{dt} = -750 \text{ mph.}$$

b.) assuming that the planes would "meet" at the "corner", we have

$D = RT \rightarrow 200 = 600T \rightarrow T = \frac{1}{3}$ hr

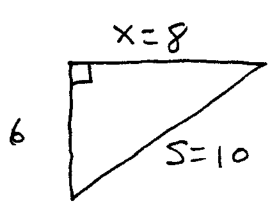
or $T = 20$ min.

20.)



$$x^2 + 6^2 = S^2 \rightarrow$$

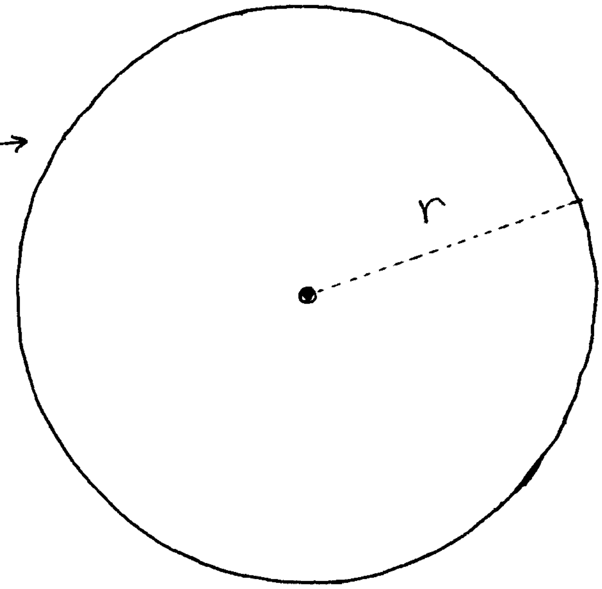
$$\cancel{2}x \cdot \frac{dx}{dt} = \cancel{2}S \cdot \frac{dS}{dt} \rightarrow$$



$$(8) \frac{dx}{dt} = (10)(-240) \rightarrow$$

$$\frac{dx}{dt} = -300 \text{ mph.}$$

23.) thickness $\longrightarrow$ is 0.08 ft. of spreading oil slick ; volume of slick (cylinder) is



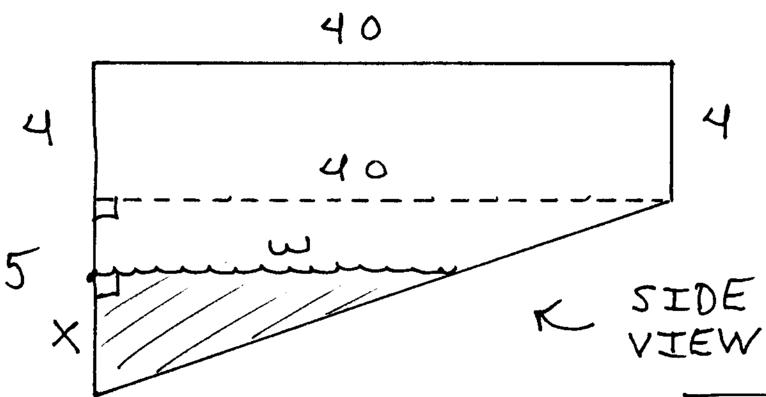
$$V = \pi r^2 h$$
$$= \pi r^2 (0.08) = (0.08)\pi r^2 \quad ;$$

find $\frac{dV}{dt}$ when $r = 750$ ft. and $\frac{dr}{dt} = 0.5$ ft./min. :

$$V = (0.08)\pi r^2 \longrightarrow$$
$$\frac{dV}{dt} = (0.08)\pi \cdot 2r \cdot \frac{dr}{dt}$$
$$= (0.08)\pi \cdot 2(750)(0.5)$$
$$= 60\pi \quad \text{ft.}^3/\text{min.}$$

Chapter 2 Review

99.) $\frac{dV}{dt} = 10 \text{ ft.}^3/\text{min.}$, find $\frac{dx}{dt}$ when $x = 4$ ft. :



by similar triangles

$$\frac{40}{w} = \frac{5}{x}$$

$\longrightarrow 5w = 40x \longrightarrow \boxed{w = 8x}$; the volume of the in the pool is

$$V = (\text{area of } \text{▰})(20)$$
$$= \tfrac{1}{2}(\text{base})(\text{height})(20)$$
$$= 10(x)(w)$$
$$= 10(x)(8x) \quad \text{or} \quad \boxed{V = 80x^2} \ ;$$

$$\xrightarrow{D} \frac{dV}{dt} = 80 \cdot 2x \cdot \frac{dx}{dt}$$

$$\longrightarrow 10 = 80 \cdot 2(4) \cdot \frac{dx}{dt}$$

$$\longrightarrow \frac{dx}{dt} = \frac{10}{640} = \frac{1}{64} \text{ ft./min.}$$

Math 16A
Kouba
Worksheet 7

1.) A woman 6 feet tall is walking away from a street lamp which is 10 feet high at the rate of 3 ft./sec. How fast is the tip of her shadow moving away from her feet when she is

a.) 10 feet from the base of the street lamp ?

b.) 100 feet from the base of the street lamp ?

2.) A small helium-filled balloon sits 20 feet from the base of a street lamp which is 10 feet high. The balloon is released and rises vertically at the constant rate of 2 ft./sec. How fast is the tip of the balloon's shadow moving away from the base of the street lamp when the balloon is

a.) 4 feet above the ground ?

b.) 8 feet above the ground ?

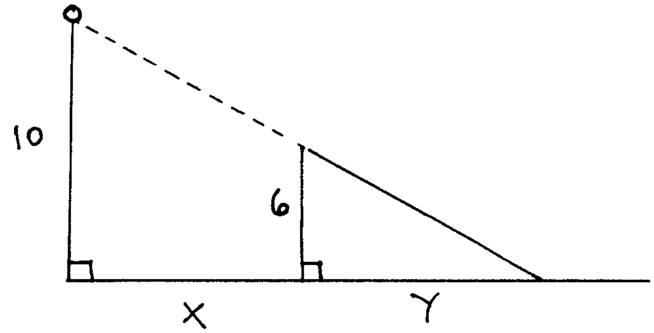
c.) 9.5 feet above the ground ?

3.) The volume of a cube is changing at the constant rate of 15 ft.^{3}/min. At what rate is the surface area of the cube changing when

a.) the surface area is 24 ft.^{2} ?

b.) the surface area is 150 ft.^{2} ?

Worksheet 7



1.) $\frac{dX}{dt} = 3$ ft./sec.

$$\frac{10}{X+Y} = \frac{6}{Y} \rightarrow$$

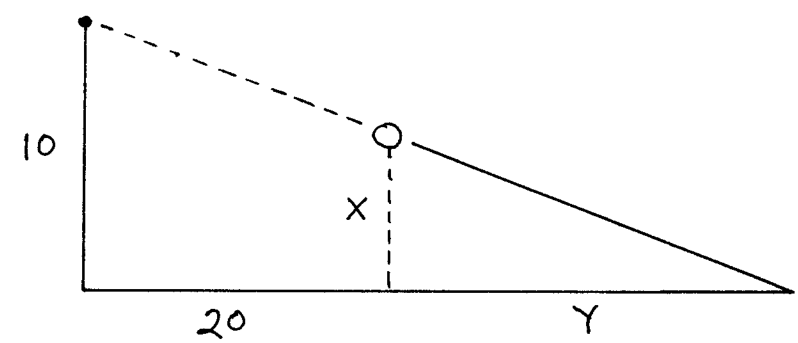
$$10Y = 6X + 6Y \rightarrow 4Y = 6X \rightarrow \boxed{Y = \frac{3}{2}X} \; ;$$

a.) Find $\frac{dY}{dt}$ when $X = 10$ ft. :

$$\frac{dY}{dt} = \frac{3}{2} \cdot \frac{dX}{dt} = \frac{3}{2} \cdot 3 = \frac{9}{2} \text{ ft./sec.}$$

b.) Find $\frac{dY}{dt}$ when $X = 100$ ft. :

$$\frac{dY}{dt} = \frac{3}{2} \cdot \frac{dX}{dt} = \frac{3}{2} \cdot 3 = \frac{9}{2} \text{ ft./sec.}$$



2.) $\frac{dX}{dt} = 2$ ft./sec.

$$\frac{10}{20+Y} = \frac{X}{Y} \rightarrow$$

$$10Y = 20X + XY \rightarrow$$

$$10\frac{dY}{dt} = 20\frac{dX}{dt} + X \cdot \frac{dY}{dt} + \frac{dX}{dt} \cdot Y \rightarrow$$

$$(10 - X)\frac{dY}{dt} = (20 + Y)\frac{dX}{dt} \rightarrow$$

$$\frac{dY}{dt} = \frac{20+Y}{10-X} \cdot \frac{dX}{dt} \; ;$$

a.) Find $\frac{dY}{dt}$ when $X = 4$ ft. :

$10Y = 20X + XY \rightarrow 10Y = 20(4) + (4)Y \rightarrow$

$6Y = 80 \rightarrow Y = \frac{40}{3}$ ft so

$$\frac{dY}{dt} = \frac{20 + \frac{40}{3}}{10 - 4} \cdot (2) = \frac{100}{9} \text{ ft./sec.}$$

b.) Find $\frac{dY}{dt}$ when $X = 8$ ft. :

$10Y = 20X + XY \rightarrow 10Y = 20(8) + (8)Y$

$\rightarrow 2Y = 160 \rightarrow Y = 80$ ft. so

$$\frac{dY}{dt} = \frac{20 + 80}{10 - 8} \cdot (2) = 100 \text{ ft./sec.}$$

c.) Find $\frac{dY}{dt}$ when $X = 9\frac{1}{2}$ ft. :

$10Y = 20X + XY \rightarrow 10Y = 20\left(\frac{19}{2}\right) + \left(\frac{19}{2}\right)Y \rightarrow$

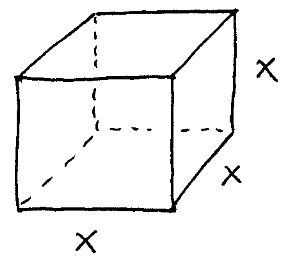
$\frac{1}{2}Y = 190 \rightarrow Y = 380$ ft. so

$$\frac{dY}{dt} = \frac{20 + 380}{10 - 9\frac{1}{2}} (2) = 1600 \text{ ft./sec.}$$

3.) $\frac{dV}{dt} = 15 \text{ ft.}^3/\text{min.}$

where volume

$V = x^3 \longrightarrow$



$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} \longrightarrow 15 = 3x^2 \frac{dx}{dt}$

$\longrightarrow \boxed{\frac{dx}{dt} = \frac{5}{x^2}}$;

surface area $S = 6x^2 \longrightarrow$

$\boxed{\frac{dS}{dt} = 12x \cdot \frac{dx}{dt} = 12x \cdot \frac{5}{x^2} = \frac{60}{x}}$;

a.) Find $\frac{dS}{dt}$ when $S = 24$:

$S = 24 \rightarrow 6x^2 = 24 \rightarrow x = 2$ ft. so

$\frac{dS}{dt} = \frac{60}{2} = 30 \text{ ft.}^2/\text{min.}$

b.) Find $\frac{dS}{dt}$ when $S = 150$:

$S = 150 \rightarrow 6x^2 = 150 \rightarrow x = 5$ ft. so

$\frac{dS}{dt} = \frac{60}{5} = 12 \text{ ft.}^2/\text{min.}$