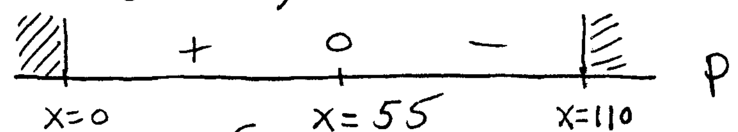
Section 3.4

1.) $x, y > 0$ and $x + y = 110 \rightarrow y = 110 - x$,

maximize $P = xy \rightarrow P = x(110 - x) = 110x - x^2 \rightarrow$

$P' = 110 - 2x = 0 \rightarrow$

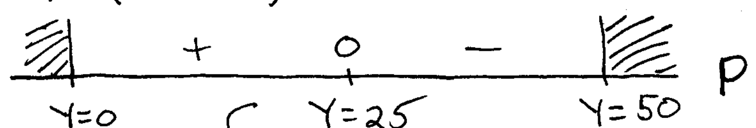


abs. max. $\begin{cases} x = 55 \\ y = 55 \\ P = 3025 \end{cases}$

4.) $x, y > 0$ and $x + 2y = 100 \rightarrow x = 100 - 2y$,

maximize $P = xy \rightarrow P = (100 - 2y)y = 100y - 2y^2 \rightarrow$

$P' = 100 - 4y = 0 \rightarrow$



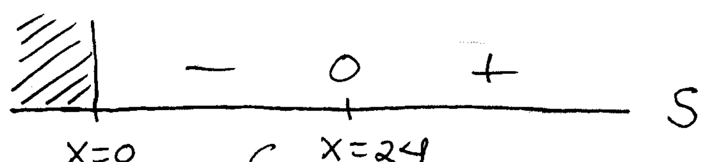
abs. max. $\begin{cases} y = 25 \\ x = 50 \\ P = 1250 \end{cases}$

6.) $x, y > 0$ and $xy = 192 \rightarrow y = \frac{192}{x}$,

minimize $S = x + 3y \rightarrow S = x + 3\left(\frac{192}{x}\right) = x + \frac{576}{x} \rightarrow$

$S' = 1 - \frac{576}{x^2} = \frac{x^2 - 576}{x^2} = 0$

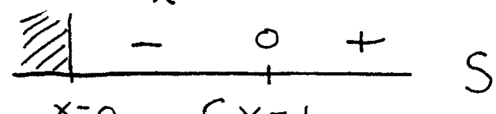
$\rightarrow x^2 - 576 = 0 \rightarrow x = 24$



abs. min. $\begin{cases} y = 8 \\ S = 48 \end{cases}$

7.) $x > 0$ minimize $S = x + \frac{1}{x} \rightarrow$

$S' = 1 - x^{-2} = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2} = 0$



abs. min. $\begin{cases} x = 1 \\ S = 2 \end{cases}$

8.) $x - y = 50 \rightarrow x = y + 50$, minimize

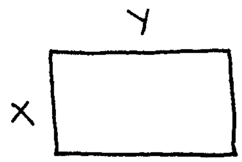
$P = xy = (y+50)y = y^2 + 50y \rightarrow$

$P' = 2y + 50 = 0 \rightarrow$

$$\begin{array}{ccc} - & 0 & + \end{array} \quad P'$$

abs. min. $\begin{cases} y = -25 \\ x = 25 \\ P = -625 \end{cases}$

9.)



$2x + 2y = 100 \rightarrow y = 50 - x$,

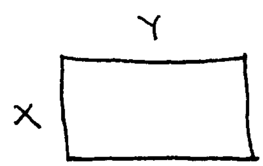
maximize area

$A = xy = x(50-x) = 50x - x^2 \rightarrow A' = 50 - 2x = 0 \rightarrow$

$$\begin{array}{ccccc} | & + & 0 & - & | \\ x=0 & & & & x=50 \end{array} \quad A'$$

abs. max $\begin{cases} x = 25 \text{ ft.} \\ y = 25 \text{ ft.} \\ A = 625 \text{ ft.}^2 \end{cases}$

11.)



$xy = 64 \rightarrow y = \frac{64}{x}$,

minimize perimeter

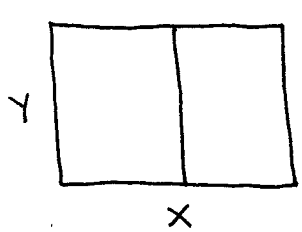
$P = 2x + 2y = 2x + \frac{128}{x} \rightarrow P' = 2 - \frac{128}{x^2} = 0 \rightarrow$

$$\begin{array}{cccc} | & - & 0 & + \\ x=0 & & x = 8 \text{ ft.} & \end{array} \quad P'$$

$y = 8$ ft.

$P = 32$ ft. $\}$ abs. min.

13.)

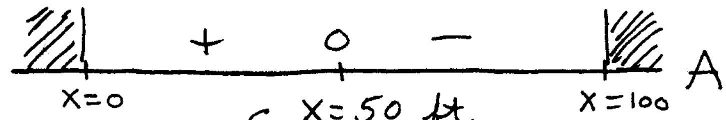


$2x + 3y = 200 \rightarrow y = \frac{200 - 2x}{3}$,

maximize area

$A = xy = x\left(\frac{200-2x}{3}\right) = \frac{200}{3}x - \frac{2}{3}x^2 \rightarrow$

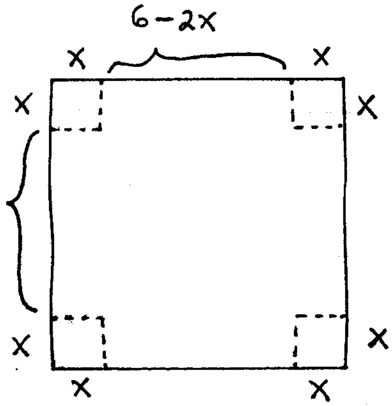
$$A' = \frac{200}{3} - \frac{4}{3}x = 0$$



$x = 50$ ft.

abs. max. $\left\{ Y = \frac{100}{3} \text{ ft.}, \quad A = \frac{5000}{3} \text{ ft.}^2 \right.$

18.)

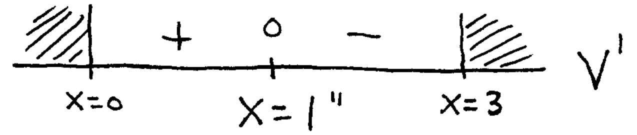


maximize volume

$$V = (6-2x)(6-2x)(x)$$
$$= (6-2x)^2 \cdot x \longrightarrow$$

$$V' = (6-2x)^2(1) + (2)(-2)(6-2x)\cdot x$$
$$= (6-2x)\cdot[6-2x-4x]$$
$$= (6-2x)[6-6x] = 0$$

$x = 3''$ ← impossible $\qquad x = 1''$



$x = 1''$

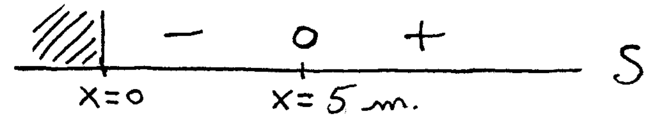
$V = 16$ in.^{3}

abs. max.

20.) $x^2 Y = 83\frac{1}{3} = \frac{250}{3} \longrightarrow Y = \frac{250}{3x^2}$, minimize surface area

$$S = x^2 + 3xY = x^2 + 3x\left(\frac{250}{3x^2}\right) = x^2 + \frac{250}{x} \longrightarrow$$

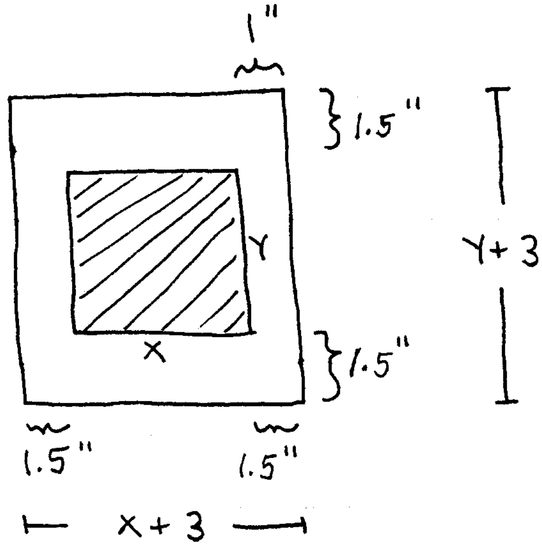
$$S' = 2x - \frac{250}{x^2} = \frac{2x^3 - 250}{x^2} = 0$$



$x = 5$ m.

abs. min. $\left\{ \begin{array}{l} Y = \frac{10}{3} \text{ m.} \\ S = 75 \text{ m.}^2 \end{array} \right.$

22.)



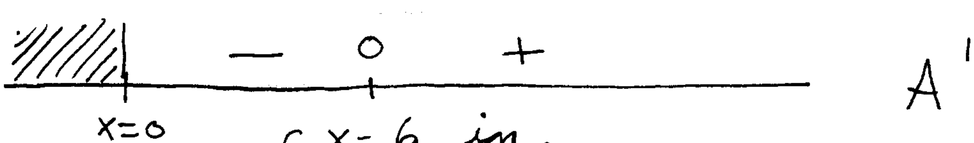
$XY = 36 \rightarrow Y = \dfrac{36}{X}$,

minimize total area of page

$$A = (X+3)(Y+3) = XY + 3X + 3Y + 9$$

$$= X \cdot \left(\frac{36}{X}\right) + 3X + 3\left(\frac{36}{X}\right) + 9 = 45 + 3X + \frac{108}{X} \rightarrow$$

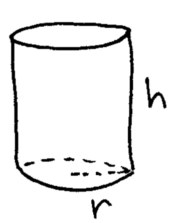
$$A' = 3 - \frac{108}{X^2} = 0 \rightarrow X = 6 \text{ in.}$$



abs. min. $\begin{cases} X = 6 \text{ in.} \\ Y = 6 \text{ in.} \\ A = (6+3)(6+3) = 81 \text{ in.}^2 \end{cases}$

dimensions of page: 9" × 9"

I.)



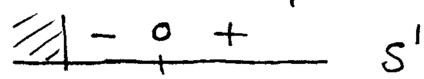
volume $64\pi = \pi r^2 h \rightarrow h = \dfrac{64}{r^2}$,

minimize surface area

$$S = \pi r^2 + 2\pi r h = \pi r^2 + 2\pi r\left(\frac{64}{r^2}\right) \rightarrow$$

$$S = \pi r^2 + \frac{128\pi}{r} \rightarrow S' = 2\pi r - \frac{128\pi}{r^2} = \frac{2\pi r^3 - 128\pi}{r^2}$$

$$= \frac{2\pi(r^3 - 64)}{r^2} = 0$$



$r=0$ $\quad r = 4$ in. and $h = 4$ in. and

abs. min. $S = 48\pi$ in.^{2}