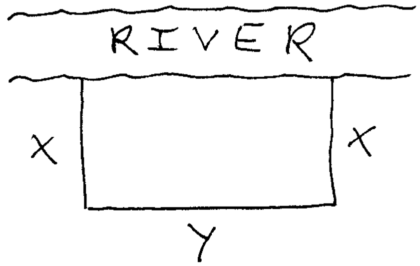
Section 3.4 (cont'd.)

14.)



$XY = 180{,}000 \rightarrow$

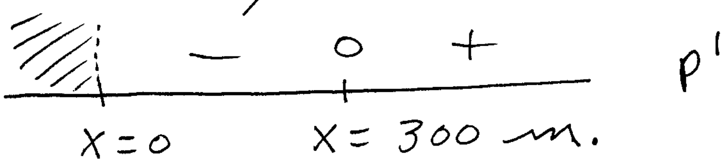
$y = \dfrac{180{,}000}{X}$;

minimize perimeter

$P = 2X + y = 2X + \dfrac{180{,}000}{X} \xrightarrow{D}$

$P' = 2 - \dfrac{180{,}000}{X^2} = 0 \rightarrow \dfrac{180{,}000}{X^2} = 2 \rightarrow$

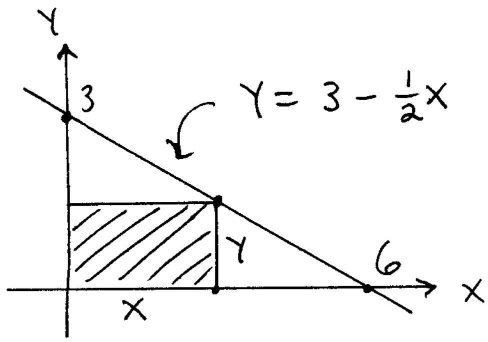
$X^2 = 90{,}000 \rightarrow X = 300$ m.



$y = 600$ m.

and abs. min. $P = 2(300) + 600 = 1200$ m.^{2}

24.)

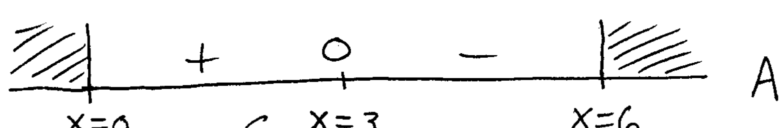


maximize area

$A = XY = X\left(3 - \frac{1}{2}X\right)$

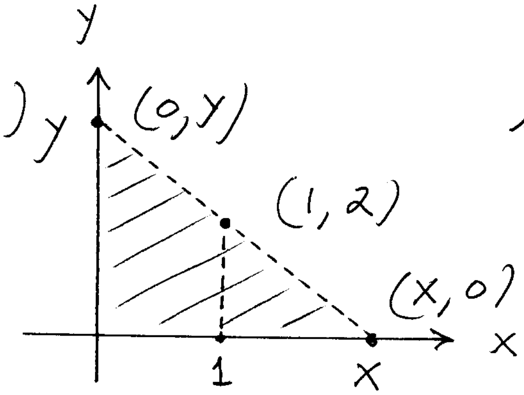
$= 3X - \frac{1}{2}X^2 \rightarrow$

$A' = 3 - X = 0$



abs. max. $\begin{cases} X = 3 \\ Y = 3/2 \\ A = 9/2 \end{cases}$

25.)



slopes: $\frac{y-2}{0-1} = \frac{2-0}{1-x} \rightarrow$

$y-2 = \frac{-2}{1-x} = \frac{2}{x-1} \rightarrow$

$y = 2 + \frac{2}{x-1} = \frac{2x}{x-1}$;

minimize area

$A = \frac{1}{2}xy = \frac{1}{2}x \cdot \frac{2x}{x-1} = \frac{x^2}{x-1} \xrightarrow{D}$

$A' = \frac{(x-1)(2x) - x^2(1)}{(x-1)^2}$

$= \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2} = 0$

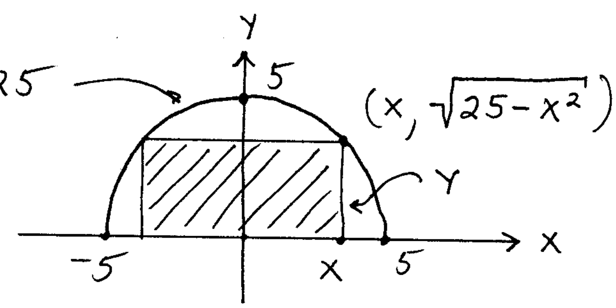
NO ⁞ − 0 +

$x=1$ $\quad$ $x=2$

$y = 4$

abs. min $A = 4$

26.)



maximize area

$A = (\text{base})(\text{height}) = (2x) \cdot y$

$= (2x) \cdot \sqrt{25-x^2} \rightarrow$

$A' = \cancel{2}x \cdot \frac{1}{\cancel{2}}(25-x^2)^{-1/2} \cdot (-2x) + (2) \cdot \sqrt{25-x^2}$

$= \frac{-2x^2}{\sqrt{25-x^2}} + \frac{2\sqrt{25-x^2}}{1} = \frac{-2x^2 + 2(25-x^2)}{\sqrt{25-x^2}}$

$= \frac{50-4x^2}{\sqrt{25-x^2}} = 0 \rightarrow \quad 50 - 4x^2 = 0 \rightarrow$

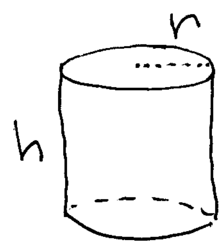
$$x^2 = \frac{50}{4} = \frac{25}{2} \rightarrow x = \pm\sqrt{\frac{25}{2}} \rightarrow x = +\frac{5}{\sqrt{2}}$$

$+ \quad 0 \quad -$ $\quad A'$

$x=0 \qquad x = \frac{5}{\sqrt{2}} \approx 3.54 \qquad x=5$

abs. max. $\left\{ \begin{array}{l} x = \frac{5}{\sqrt{2}} \approx 3.54 \\ y = \frac{5}{\sqrt{2}} \approx 3.54 \\ A = 25 \end{array} \right.$

28.)



Volume

$$\pi r^2 h = 21.6528 \text{ in.}^3 \rightarrow$$

$$h = \frac{21.6528}{\pi r^2} \approx \frac{6.8934}{r^2} \; ;$$

minimize surface area

$$S = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r \cdot \left(\frac{6.8934}{r^2}\right)$$

$$= 2\pi r^2 + 2\pi \cdot \left(\frac{6.8934}{r}\right) \xrightarrow{D}$$

$$S' = 4\pi r - 2\pi \cdot \frac{6.8934}{r^2}$$

$$= \frac{4\pi r^3 - 2\pi(6.8934)}{r^2} = 0 \rightarrow$$

$$4\pi r^3 - 2\pi(6.8934) = 0 \rightarrow r^3 = \frac{2\pi(6.8934)}{4\pi} \rightarrow$$

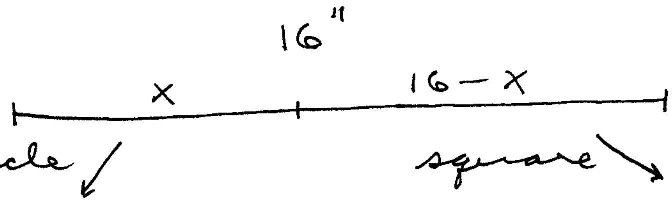
$r \approx 1.5105$ in.

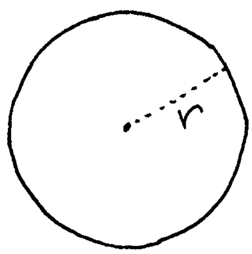
$- \quad 0 \quad +$ $\quad S'$

$r=0$

abs. min. $\left. \begin{array}{l} r \approx 1.5105 \text{ in.} \\ h \approx 3.02 \text{ in.} \\ S \approx 43.1149 \text{ in.}^3 \end{array} \right\}$

35.)

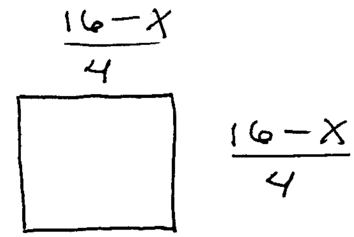




perimeter x

so $2\pi r = x$

$\rightarrow r = \dfrac{x}{2\pi}$



and

and $A_{\odot} = \pi r^2 =$

$= \pi \left(\dfrac{x}{2\pi}\right)^2$

$= \dfrac{\pi}{4\pi^2} x^2$

$= \dfrac{x^2}{4\pi}$;

$A_{\square} = \left(\dfrac{16-x}{4}\right)^2$

$= \dfrac{1}{16}(x-16)^2$;

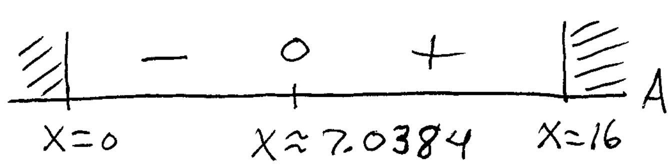
minimize total area

$A = A_{\odot} + A_{\square} = \dfrac{x^2}{4\pi} + \dfrac{1}{16}(x-16)^2 \xrightarrow{D}$

$A' = \dfrac{2}{4\pi} x + \dfrac{1}{16} \cdot 2(x-16)$

$= \dfrac{1}{2\pi} x + \dfrac{1}{8} x - 2 = \left(\dfrac{1}{2\pi} + \dfrac{1}{8}\right) x - 2 = 0 \rightarrow$

$x = \dfrac{2}{\frac{1}{2\pi} + \frac{1}{8}} \approx 7.0384$



$\Big\{$ abs. min. $A \approx 8.9616$

circle : $r \approx 1.1202$

square : 2.2404×2.2404

39.) Let x be # of additional weeks; maximize value

$$V = (\text{\# of bushels})(\$ \text{ per bushel})$$

$$= (120 + 4x)(4 - 0.1x) \xrightarrow{D}$$

$$V' = (120 + 4x)(-0.1) + (4)(4 - 0.1x)$$

$$= -12 - 0.4x + 16 - 0.4x$$

$$= 4 - 0.8x = 0 \rightarrow 4 = 0.8x \rightarrow$$

$x = 5$ weeks

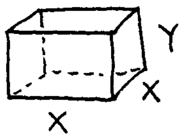
	+	0	−	V'
$x=0$		$x = 5$ wks.		

140 bushels

\$3.50 per bushel

abs. max. $V = \$490$

II.)



volume $x^2 Y = 80 \rightarrow Y = \frac{80}{x^2}$,

minimize cost

$$C = 5(x^2) + 2(4xY) = 5x^2 + 8x\left(\frac{80}{x^2}\right) = 5x^2 + \frac{640}{x} \rightarrow$$

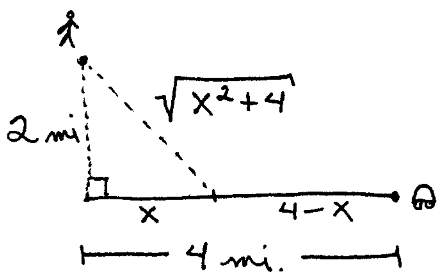
$$C' = 10x - \frac{640}{x^2} = \frac{10x^3 - 640}{x^2} = \frac{10(x^3 - 64)}{x^2} = 0$$

$$\begin{array}{ccc} - & 0 & + \end{array} \quad C'$$

$x = 4$ ft. , $Y = 5$ ft. and

abs. min. $C = \$240$

III.)



woods: 3 mph

road: 5 mph

$T = \frac{D}{R}$

minimize time

$$T = \frac{\sqrt{x^2+4}}{3} + \frac{4-x}{5} \rightarrow T' = \frac{1}{3} \cdot \frac{1}{2}(x^2+4)^{-\frac{1}{2}} \cdot 2x - \frac{1}{5}$$

$$= \frac{x}{3\sqrt{x^2+4}} - \frac{1}{5} = 0 \rightarrow 5x = 3\sqrt{x^2+4} \longrightarrow$$

$$25x^2 = 9(x^2+4) \rightarrow 16x^2 = 36 \rightarrow x = 3/2$$

$$\begin{array}{ccc} - & 0 & + \end{array} \quad T'$$

$x = 3/2$ mi. and abo. min. $T = \frac{4}{3}$ hr.