Section 3.5

19.) Let X : # of radios in excess of 100; maximize total profit

$P =$ (profit per radio)(# of radios)

$= (30 - 0.1X)(100 + X) \xrightarrow{D}$

$P' = (30 - 0.1X)(1) + (-0.1)(100 + X)$

$= 30 - 0.1X - 10 - 0.1X$

$= 20 - 0.2X = 0 \longrightarrow X = 100$ radios

$$\begin{array}{ccc} + & 0 & - \\ \hline & X = 100 & \end{array} \quad P'$$

radios = 200

profit/radio = \$20

max. profit $P = \$4000$

20.) Let X: # of \$40 increases in rent; maximize total profit

$P =$ (profit per room)(# of rooms)

$= (580 - 45 + 40X)(50 - X)$

$= (535 + 40X)(50 - X) \xrightarrow{D}$

$P' = (535 + 40X)(-1) + (40)(50 - X)$

$= -535 - 40X + 2000 - 40X$

$= 1465 - 80X = 0 \longrightarrow X = 18.3125$

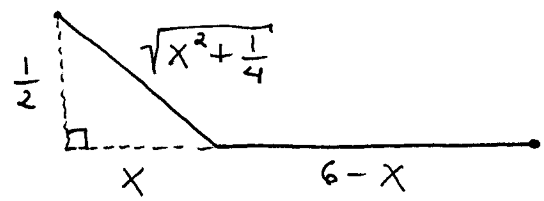
$$\begin{array}{ccc} + & 0 & - \\ \hline & X = 18.3125 & \end{array} \quad P'$$

$\uparrow$'s

profit/room = \$1267.50, # rooms ≈ 32

max. profit $P \approx \$40,560$

23.)



water : \$8/ft.

land : \$6/ft.

minimize total cost

$$C = 8(5280)\sqrt{x^2+\tfrac{1}{4}} + 6(5280)(6-x)$$

$$\rightarrow C' = 8(5280)\cdot\tfrac{1}{\cancel{2}}\left(x^2+\tfrac{1}{4}\right)^{-1/2}\cdot\cancel{2}x - 6(5280) = 0$$

$$\rightarrow \frac{8x}{\sqrt{x^2+\frac{1}{4}}} - 6 = 0 \longrightarrow 8x = 6\sqrt{x^2+\tfrac{1}{4}} \longrightarrow$$

$$64x^2 = 36\left(x^2+\tfrac{1}{4}\right) \rightarrow 64x^2 = 36x^2 + 9 \rightarrow$$

$$28x^2 = 9 \rightarrow x^2 = \frac{9}{28} \rightarrow x = \sqrt{\frac{9}{28}}$$

$$\underset{x=\sqrt{\frac{9}{28}}}{\overset{-\quad 0 \quad +}{\overline{\qquad\qquad\qquad}}}\; C'$$

$x = \sqrt{\frac{9}{28}}$ mi. and abs. min. cost

$C = \$204,050$

25.) $D = V\cdot T \rightarrow 110 = V\cdot T \rightarrow$ # of hours

$T = \dfrac{110}{V}$; minimize cost

$$C = C_{\text{fuel}} + C_{\text{driver}}$$

$$= (\text{cost per hour})(\#\text{ of hours}) + 10(\#\text{ of hours})$$

$$= \frac{V^2}{600}\cdot\frac{110}{V} + 10\cdot\frac{110}{V} \rightarrow$$

$$\boxed{C = \frac{11}{60}V + \frac{1100}{V}} \xrightarrow{D}$$

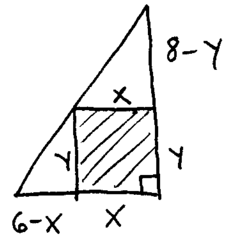
$$C' = \frac{11}{60} - \frac{1100}{V^2} = 0 \rightarrow \frac{11}{60} = \frac{1100}{V^2} \rightarrow$$

$$V^2 = 6000 \rightarrow V = \sqrt{6000} \approx 77.46 \text{ mph}.$$

$$\begin{array}{ccc} - & 0 & + \\ \hline & V = \sqrt{6000} \text{ mph} & \end{array} \quad C'$$

min cost $C \approx \$28.40$.

IV.) A.)



By similar Δ's

$$\frac{8}{6} = \frac{Y}{6-x} \rightarrow 48-8x = 6Y$$
$$\rightarrow Y = 8 - \tfrac{4}{3}x \,,$$

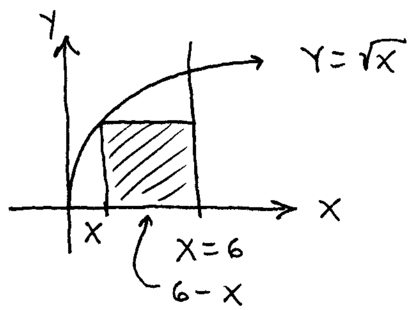
maximize area $A = xY = x\left(8 - \frac{4}{3}x\right) = 8x - \frac{4}{3}x^2 \rightarrow$

$$A' = 8 - \frac{8}{3}x = 0 \rightarrow x = 3$$

$$\underset{x=3}{\overset{+ \quad 0 \quad -}{\overline{\qquad\qquad\qquad}}} \; A'$$

$Y = 4$ and abs. max. $A = 12$.

B.)



maximize area

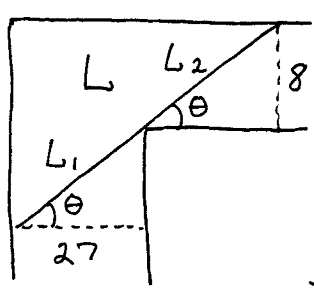
$$A = (6-x)Y$$
$$= (6-x)\sqrt{x} = 6\sqrt{x} - x^{3/2} \rightarrow$$
$$A' = 6\frac{1}{2\sqrt{x}} - \frac{3}{2}\sqrt{x} = \frac{3}{\sqrt{x}} - \frac{3\sqrt{x}}{2}$$

$$\rightarrow A' = \frac{6-3x}{2\sqrt{x}} = 0 \rightarrow x = 2$$

$$\underset{x=2}{\overset{+ \quad 0 \quad -}{\overline{\qquad\qquad\qquad}}} \; A'$$

so dimensions are 4 by $\sqrt{2}$ and abs. max. $A = 4\sqrt{2}$.

V.)



$$\cos\theta = \frac{27}{L_1} \rightarrow L_1 = 27\sec\theta \,,$$

$$\sin\theta = \frac{8}{L_2} \rightarrow L_2 = 8\csc\theta \,,$$

minimize length

$$L = L_1 + L_2 = 27\sec\theta + 8\csc\theta \rightarrow$$
$$L' = 27\sec\theta\tan\theta - 8\csc\theta\cot\theta$$
$$= 27 \cdot \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} - 8 \cdot \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}$$

$$= \frac{27\sin^3\theta - 8\cos^3\theta}{\cos^2\theta\ \sin^2\theta} = 0 \rightarrow$$

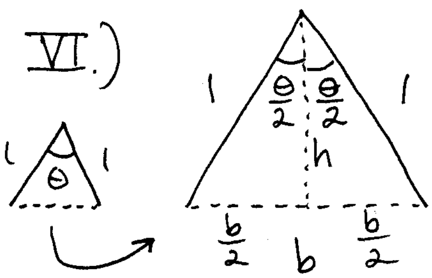
$$27\sin^3\theta - 8\cos^3\theta = 0 \rightarrow \tan^3\theta = \frac{8}{27} \rightarrow$$

$$\tan\theta = \left(\frac{8}{27}\right)^{1/3} = \frac{2}{3} \rightarrow \theta \approx 33.7^\circ$$

$- \quad 0 \quad +$ $\quad L'$
$\theta = 33.7^\circ$

and abs. min.
$L \approx 46.87$ ft.

VI.)



$$\cos\frac{\theta}{2} = \frac{h}{1} \text{ and } \sin\frac{\theta}{2} = \frac{\frac{b}{2}}{1} \rightarrow$$

$$h = \cos\frac{\theta}{2} \text{ and } b = 2\sin\frac{\theta}{2},$$

maximize area

$$A = \frac{1}{2}bh = \frac{1}{2}\left(2\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}\right) = \frac{1}{2}\sin 2\left(\frac{\theta}{2}\right) = \frac{1}{2}\sin\theta$$

$$\rightarrow A' = \frac{1}{2}\cos\theta = 0 \rightarrow \theta = \frac{\pi}{2} = 90^\circ$$

$+ \quad 0 \quad -$ $\quad A'$
$\theta = \frac{\pi}{2}$

and abs. max.
$A = \frac{1}{2}$ ft.^{2}.

VII.) Let x be the number of \$5 increases, then maximize revenue

$$R = (\#\text{ of rooms})(\text{charge per room})$$
$$= (100-4x)(50+5x)$$
$$= 5000 + 300x - 20x^2 \quad \xrightarrow{D}$$
$$R' = 300 - 40x = 0 \longrightarrow \quad x = \frac{30\cancel{0}}{4\cancel{0}} = 7\tfrac{1}{2}$$

$$\begin{array}{ccc} + & 0 & - \end{array} \quad R'$$
$x = 7\tfrac{1}{2}$ \$5 increases

max revenue $R = \$6125$,

\# rooms = 70, charge per room = \$87.50

VIII.) x: demand $\quad$ p: price

$x = \frac{c}{p^2}$ and $p = \$20$, $x = 125$ boxes so

$$125 = \frac{c}{400} \longrightarrow c = 50{,}000 \quad \text{so}$$

$$x = \frac{50{,}000}{p^2} \quad \text{or} \quad \text{price } p = \sqrt{\frac{50{,}000}{x}} \ ;$$

cost $C = 750 + 5x$ so profit

$$P_r = (\text{revenue}) - (\text{cost})$$
$$= px - (750 + 5x)$$
$$= \sqrt{\frac{50{,}000}{x}} \cdot x - 750 - 5x = \sqrt{50{,}000} \cdot \sqrt{x} - 750 - 5x \ ;$$
$$P_r' = \sqrt{50{,}000} \cdot \frac{1}{2\sqrt{x}} - 5 = 0 \rightarrow \cdots \rightarrow x = 500 \text{ boxes}$$

$$\begin{array}{ccc} + & 0 & - \end{array} \quad P_r'$$
$x = 500$ boxes

$p = \$10$

and max. profit is

$P_r = \$1750$