Section 3.8

11.) $f(x) = x^3$, $\Delta x = 0.1$, $x: 1 \to 1.1$, $f'(x) = 3x^2$;
$\Delta y = f(1.1) - f(1) = (1.1)^3 - (1)^3 = 1.331 - 1 = 0.331$,
$dy = f'(1) \cdot \Delta x = 3(1)^2 (0.1) = 0.3$

12.) $f(x) = 1 - 2x^2$, $\Delta x = -0.1$, $x: 0 \to -0.1$, $f'(x) = -4x$;
$\Delta y = f(-0.1) - f(0) = 0.98 - 1 = -0.02$,
$dy = f'(0) \cdot \Delta x = (0)(-0.1) = 0$

17.) $x = 2$ and $f(x) = \frac{1}{x^2}$ so $f'(x) = \frac{-2}{x^3}$

a.) $\Delta x = 1$ so $dy = f'(2) \cdot \Delta x = \frac{-1}{4} \cdot 1 = \frac{-1}{4} = -.25$,
$\Delta y = f(3) - f(2) = \frac{1}{9} - \frac{1}{4} = \frac{-5}{36} \approx -.139$,
$\Delta y - dy = \frac{-5}{36} - \frac{-1}{4} = \frac{-5}{36} + \frac{9}{36} = \frac{4}{36} = \frac{1}{9} \approx .111$,
$\frac{dy}{\Delta y} = \frac{-1}{4} \div \frac{-5}{36} = \frac{1}{4} \cdot \frac{36}{5} = \frac{9}{5} = 1.8$;

b.) $\Delta x = .5$ so $dy = f'(2) \cdot \Delta x = \frac{-1}{4} \cdot (.5) = \frac{-1}{8} = -.125$,
$\Delta y = f(2.5) - f(2) = -.09$,
$\Delta y - dy = .035$, $\frac{dy}{\Delta y} = 1.389$;

c.) $\Delta x = .1$ so $dy = f'(2) \cdot \Delta x = \frac{-1}{4} \cdot (.1) = -.025$,
$\Delta y = f(2.1) - f(2) = -.023$,
$\Delta y - dy = .002$, $\frac{dy}{\Delta y} = 1.087$;

d.) $\Delta x = .01$ so $dy = f'(2) \cdot \Delta x = \frac{-1}{4} \cdot (.01) = -.0025$,

$\Delta Y = f(2.01) - f(2) = -.00248$,

$\Delta Y - dY = .00002$, $\dfrac{dY}{\Delta Y} = 1.008$;

e.) $\Delta x = .001$ so $dY = f'(2)\cdot \Delta x = \frac{-1}{4}\cdot(.001) = -.00025$,

$\Delta Y = f(2.001) - f(2) = -.0002498$,

$\Delta Y - dY = .0000001$, $\dfrac{dY}{\Delta Y} = 1.0008$

37.) a.) $A = x^2$ so $A' = 2x$ and

$dA = A' \cdot \Delta x = 2x \cdot \Delta x$,

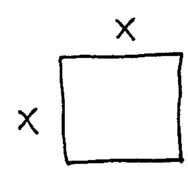
$\Delta A = A(x+\Delta x) - A(x) = (x+\Delta x)^2 - x^2$

$= \cancel{x^2} + 2x\cdot\Delta x + (\Delta x)^2 - \cancel{x^2} = 2(x\cdot\Delta x) + (\Delta x)^2$;

b.) $dA = 2(x\cdot\Delta x)$ is the region made up of the 2 rectangles which are x by Δx ;

c.) $\Delta A - dA = 2x\cdot\Delta x + (\Delta x)^2 - 2x\cdot\Delta x = (\Delta x)^2$ is the square which is Δx by Δx .

38.)



$A = x^2$, $A' = 2x$,

$x = 12$ in., $|\Delta x| \le \frac{1}{64}$ in.

a.) The absolute error for area is

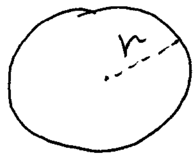
$|\Delta A| \approx |dA| = |A'\cdot\Delta x| = |2x\cdot\Delta x|$

$= 2x|\Delta x| \le 2(12)\cdot\frac{1}{64} = 3/8 \text{ in}^2$,

b.) The absolute percentage error for area is

$$\frac{|\Delta A|}{A} \approx \frac{|dA|}{A} = \frac{|A'\cdot\Delta x|}{A} = \frac{|2x\cdot\Delta x|}{x^2} = \frac{2|\Delta x|}{x}$$

$$\le \frac{2\left(\frac{1}{64}\right)}{12} = .0026 = .26\%$$

39.)  $A = \pi r^2$, $A' = 2\pi r$,
$r = 10$ in., $|\Delta r| \le \frac{1}{8}$ in.

a.) The absolute error for area is

$$|\Delta A| \approx |dA| = |A' \cdot \Delta r| = |2\pi r \cdot \Delta r|$$
$$= 2\pi r |\Delta r| \le 2\pi(10)\left(\tfrac{1}{8}\right) = \frac{5\pi}{2} \text{ in.}^2 \approx 7.8 \text{ in.}^2$$

b.) The absolute percentage error for area is

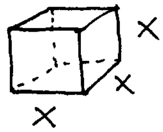
$$\frac{|\Delta A|}{A} \approx \frac{|dA|}{A} = \frac{|A' \cdot \Delta r|}{A} = \frac{|2\pi r \cdot \Delta r|}{\pi r^2} = \frac{2\,|\Delta r|}{r}$$
$$\le \frac{2\left(\frac{1}{8}\right)}{10} = 0.025 = 2.5\%$$

42.) $C = \dfrac{3t}{27+t^3}$,

$$C' = \frac{(27+t^3)(3) - (3t)(3t^2)}{(27+t^3)^2} = \frac{81-6t^3}{(27+t^3)^2},$$

$t: 1 \to \frac{3}{2}$ so $\Delta t = \frac{1}{2}$ and change in concentration is

$$\Delta C \approx dC = C' \cdot \Delta t = \frac{81-6t^3}{(27+t^3)^2} \cdot \left(\tfrac{1}{2}\right) \quad (\text{let } t = 1)$$
$$= \frac{75}{1568} \approx .0478 \text{ mg./ml.}$$

40.) 

$x = 12$ in., $|\Delta x| \le .03$ in.

a.) $V = x^3$, $V' = 3x^2$ so maximum absolute error is $|\Delta V| \approx |dV| = |V' \cdot \Delta x| = |3x^2 \cdot \Delta x|$

$$= 3x^2 |\Delta x| \le 3(12)^2(.03) = 12.96 \text{ in}^3;$$

maximum absolute percentage error is

$$\frac{|\Delta V|}{V} \approx \frac{|dV|}{V} = \frac{|V' \cdot \Delta x|}{x^3} \le \frac{12.96}{(12)^3} = .75\%$$

b.) $S = 6x^2$, $S' = 12x$ so maximum absolute error is $|\Delta S| = |dS| = |S' \cdot \Delta x| = |12x \cdot \Delta x|$

$$= 12x|\Delta x| \le 12(12)(.03) = 4.32 \text{ in}^2;$$

maximum absolute percentage error is

$$\frac{|\Delta S|}{S} \approx \frac{|dS|}{S} = \frac{|S' \cdot \Delta x|}{6x^2} \le \frac{4.32}{6(12)^2} = .5\%$$

41.) $V = \frac{4}{3}\pi r^3$, $r = 6$ in., $|\Delta r| \le .02$ in.,
$V' = 4\pi r^2$; maximum absolute error is

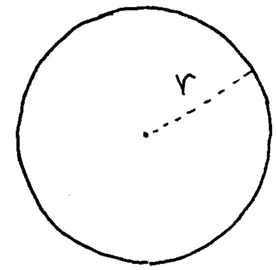
$$|\Delta V| \approx |dV| = |V' \cdot \Delta r| = |4\pi r^2 \cdot \Delta r| = 4\pi r^2 \cdot |\Delta r|$$

$$\le 4\pi(6)^2(.02) = 2.88\pi = 9.05 \text{ in}^3;$$

maximum absolute relative error is

$$\frac{|\Delta V|}{V} \approx \frac{|dV|}{V} = \frac{|V' \cdot \Delta r|}{\frac{4}{3}\pi r^3} \le \frac{2.88\cancel{\pi}}{\frac{4}{3}\cancel{\pi}(6)^3} = .01$$

Worksheet 8



1.) $\frac{|\Delta r|}{r} \le 3\%$ then

a.) diameter $D = 2r$ so maximum absolute percentage error is

$$\frac{|\Delta D|}{D} \approx \frac{|dD|}{D} = \frac{|D' \cdot \Delta r|}{D} = \frac{|2 \cdot \Delta r|}{2r} = \frac{|\Delta r|}{r} \le 3\%$$

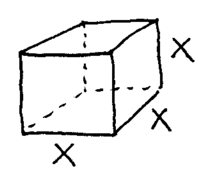
b.) circumference $C = 2\pi r$ so maximum absolute percentage error is

$$\frac{|\Delta C|}{C} \approx \frac{|dC|}{C} = \frac{|C' \cdot \Delta r|}{C} = \frac{|2\pi \cdot \Delta r|}{2\pi r} = \frac{|\Delta r|}{r} \le 3\%$$

c.) area $A = \pi r^2$ so maximum absolute percentage error is

$$\frac{|\Delta A|}{A} \approx \frac{|dA|}{A} = \frac{|A' \cdot \Delta r|}{A} = \frac{|2\pi r \cdot \Delta r|}{\pi r^2} = 2\frac{|\Delta r|}{r} \le 6\%$$

2.) $\frac{|\Delta x|}{x} \le 5\%$ then



a.) surface area $S = 6x^2$ so maximum absolute percentage error is

$$\frac{|\Delta S|}{S} \approx \frac{|dS|}{S} = \frac{|S' \cdot \Delta x|}{S} = \frac{|12x \cdot \Delta x|}{6x^2} = 2\frac{|\Delta x|}{x} \le 10\%$$

b.) volume $V = x^3$ so maximum absolute percentage error is

$$\frac{|\Delta V|}{V} \approx \frac{|dV|}{V} = \frac{|V' \cdot \Delta x|}{V} = \frac{|3x^2 \cdot \Delta x|}{x^3} = 3\frac{|\Delta x|}{x} \le 15\%$$

3.) a.) Let $f(x) = \sqrt{x}$, $x: 100 \longrightarrow 103$, $\Delta x = 3$

$f'(x) = \frac{1}{2\sqrt{x}}$; $\Delta y = f(103) - f(100) = \sqrt{103} - 10$,

$dy = f'(100) \cdot \Delta x = \left(\frac{1}{20}\right)(3) = 0.15$, and $\Delta y \approx dy$ so

$\sqrt{103} - 10 \approx 0.15 \longrightarrow \sqrt{103} \approx 10.15$

(by calculator: $\sqrt{103} \approx 10.149$)

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b.) Let $f(x) = \sqrt{x}$, $x: 25 \longrightarrow 23$, $\Delta x = -2$

$f'(x) = \frac{1}{2\sqrt{x}}$; $\Delta y = f(23) - f(25) = \sqrt{23} - 5$,

$dy = f'(25) \cdot \Delta x = \left(\frac{1}{10}\right)(-2) = -0.2$, and $\Delta y \approx dy$ so

$\sqrt{23} - 5 \approx -0.2 \rightarrow \sqrt{23} \approx 4.8$

(by calculator: $\sqrt{23} \approx 4.796$)

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c.) Let $f(x) = x^{1/3}$, $x: 27 \rightarrow 28$, $\Delta x = 1$

$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}}$; $\Delta y = f(28) - f(27) = 28^{1/3} - 3$,

$dy = f'(27) \cdot \Delta x = \frac{1}{3 \cdot 27^{2/3}} \cdot (1) = \frac{1}{3(9)} = \frac{1}{27}$, and $\Delta y \approx dy$ so

$28^{1/3} - 3 \approx \frac{1}{27} \rightarrow 28^{1/3} \approx 3\frac{1}{27}$ (≈ 3.037)

(by calculator: $28^{1/3} \approx 3.036$)

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d.) Let $f(x) = x^{1/4}$, $x: 81 \rightarrow 79$, $\Delta x = -2$

$f'(x) = \frac{1}{4} x^{-3/4} = \frac{1}{4x^{3/4}}$; $\Delta y = f(79) - f(81) = 79^{1/4} - 3$,

$dy = f'(81) \cdot \Delta x = \frac{1}{4 \cdot 81^{3/4}} \cdot (-2) = \frac{1}{4(27)}(-2) = \frac{-1}{54}$, and

$\Delta Y \approx dY \rightarrow 79^{1/4} - 3 \approx \frac{-1}{54} \rightarrow 79^{\frac{1}{4}} \approx 2\frac{53}{54}$ (≈ 2.981)

(by calculator : $79^{1/4} \approx 2.981$)

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e.) Let $f(x) = \tan x$, $x: \frac{\pi}{4} \rightarrow \frac{\pi}{4} + 0.2$, $\Delta x = 0.2$

$f'(x) = \sec^2 x$; $\Delta Y = f(\frac{\pi}{4}+0.2) - f(\frac{\pi}{4})$

$= \tan(\frac{\pi}{4}+0.2) - \tan\frac{\pi}{4}$

$= \tan(\frac{\pi}{4}+0.2) - 1$,

$dY = f'(\frac{\pi}{4}) \cdot \Delta x = \sec^2(\frac{\pi}{4}) \cdot (0.2)$

$= \frac{1}{\cos^2(\frac{\pi}{4})} \cdot (0.2)$

$= \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} \cdot (0.2)$

$= \frac{1}{\left(\frac{1}{2}\right)} \cdot (0.2) = 0.4$,

and $\Delta Y \approx dY \rightarrow \tan(\frac{\pi}{4} + 0.2) - 1 \approx 0.4$

$\rightarrow \tan(\frac{\pi}{4}+0.2) \approx 1.4$

(by calculator : $\tan(\frac{\pi}{4}+0.2) \approx 1.51$)