Section 3.6

1.) $\lim_{x\to 0^+} \frac{x^2+1}{x^2} = \text{"}\frac{1}{0^+}\text{"} = +\infty$, $\lim_{x\to 0^-} \frac{x^2+1}{x^2} = \text{"}\frac{1}{0^+}\text{"} = +\infty$,

$\lim_{x\to +\infty} \frac{x^2+1}{x^2} = \lim_{x\to +\infty}\left(1+\frac{1}{x^2}\right) = 1+0 = 1$ and $\lim_{x\to -\infty} \frac{x^2+1}{x^2}$

$= \lim_{x\to -\infty}\left(1+\frac{1}{x^2}\right) = 1+0 = 1$, so y-axis is vertical asymptote and $y=1$ is horizontal asymptote.

3.) $\lim_{x\to 2^+} \frac{x^2-2}{x^2-x-2} = \lim_{x\to 2^+} \frac{x^2-2}{(x-2)(x+1)} = \text{"}\frac{2}{0^+}\text{"} = +\infty$,

$\lim_{x\to 2^-} \frac{x^2-2}{(x-2)(x+1)} = \text{"}\frac{2}{0^-}\text{"} = -\infty$, $\lim_{x\to -1^+} \frac{x^2-2}{(x-2)(x+1)} = \text{"}\frac{-1}{0^-}\text{"} = +\infty$,

$\lim_{x\to -1^-} \frac{x^2-2}{(x-2)(x+1)} = \text{"}\frac{-1}{0^+}\text{"} = -\infty$ and

$\lim_{x\to \pm\infty} \frac{x^2-2}{x^2-x-2} = \lim_{x\to \pm\infty} \frac{1-\frac{2}{x^2}}{1-\frac{1}{x}-\frac{2}{x^2}} = \frac{1}{1} = 1$ so

lines $x=2$ and $x=-1$ are vertical asymptotes and $y=1$ is horizontal asymptote.

4.) $\lim_{x\to 1^+} \frac{2+x}{1-x} = \text{"}\frac{3}{0^-}\text{"} = -\infty$, $\lim_{x\to 1^-} \frac{2+x}{1-x} = \text{"}\frac{3}{0^+}\text{"} = +\infty$;

$\lim_{x\to +\infty} \frac{2+x}{1-x} = \lim_{x\to +\infty} \frac{\frac{2}{x}+1}{\frac{1}{x}-1} = \frac{0+1}{0-1} = -1$ and

$\lim_{x\to -\infty} \frac{2+x}{1-x} = \lim_{x\to -\infty} \frac{\frac{2}{x}+1}{\frac{1}{x}-1} = \frac{0+1}{0-1} = -1$ so

vertical asymptote is line $x=1$ and horizontal asymptote is line $y=-1$.

6.) There are no vertical asymptotes (since division by zero is not possible);

$$\lim_{x \to \pm\infty} \frac{-4x}{x^2+4} = \lim_{x \to \pm\infty} \frac{\frac{-4}{x}}{1+\frac{4}{x^2}} = \frac{0}{1} = 0$$

so $Y=0$ is a horizontal asymptote.

9.) $$\lim_{x \to \pm\infty} \frac{3x^2}{x^2+2} = \lim_{x \to \pm\infty} \frac{3}{1+\frac{2}{x^2}} = \frac{3}{1+0} = 3$$

and $x=0, Y=0$, so (f)

10.) $$\lim_{x \to +\infty} \frac{2x}{\sqrt{x^2+2}} = \lim_{x \to +\infty} \frac{2x}{\sqrt{x^2(1+\frac{2}{x^2})}} = \lim_{x \to +\infty} \frac{2x}{\sqrt{x^2}\sqrt{1+2/x^2}}$$

$$= \lim_{x \to +\infty} \frac{2x}{|x|\sqrt{1+2/x^2}} = \lim_{x \to +\infty} \frac{2\cancel{x}}{\cancel{x}\sqrt{1+2/x^2}} = \frac{2}{\sqrt{1+0}} = 2 ;$$

$$\lim_{x \to -\infty} \frac{2x}{\sqrt{x^2+2}} = \cdots = \lim_{x \to -\infty} \frac{2x}{|x|\sqrt{1+2/x^2}} = \lim_{x \to -\infty} \frac{2\cancel{x}}{-\cancel{x}\sqrt{1+2/x^2}}$$

$$= \frac{-2}{\sqrt{1+0}} = -2 \qquad (b)$$

11.) $$\lim_{x \to \pm\infty} \frac{x}{x^2+2} = \lim_{x \to \pm\infty} \frac{1}{x+\frac{2}{x}} = \text{"}\frac{1}{\infty+0}\text{"} = 0$$

and $x=0, Y=0$, so (c)

12.) $$\lim_{x \to \pm\infty} \left(2+\frac{x^2}{x^4+1}\right) = \lim_{x \to \pm\infty} \left(2+\frac{1}{x^2+\frac{1}{x^2}}\right) = \text{"}2+\frac{1}{\infty+0}\text{"} = 2$$

and $x=0, Y=2$, so (a)

13.) $$\lim_{x \to \pm\infty} \left(5-\frac{1}{x^2+1}\right) = 5-0 = 5$$ and $x=0, Y=4$, so (e)

14.) $\lim_{x \to \pm\infty} \frac{2x^2 - 3x + 5}{x^2 + 1} = \lim_{x \to \pm\infty} \frac{2 - \frac{3}{x} + \frac{5}{x^2}}{1 + \frac{1}{x^2}} = \frac{2 - 0 + 0}{1 + 0} = 2$

and $x = 0$, $y = 5$, so (d)

15.) $\lim_{x \to -2^-} \frac{1}{(x+2)^2} = \text{"}\frac{1}{0^+}\text{"} = +\infty$

16.) $\lim_{x \to -2^-} \frac{1}{x+2} = \text{"}\frac{1}{0^-}\text{"} = -\infty$

19.) $\lim_{x \to 4^-} \frac{x^2}{x^2 - 16} = \text{"}\frac{16}{0^-}\text{"} = -\infty$

22.) $\lim_{x \to 0^-} \left(x^2 - \frac{1}{x}\right) = 0 - (-\infty) = +\infty$

23.) $\lim_{x \to \infty} \frac{2x - 1}{3x + 2} = \lim_{x \to \infty} \frac{2 - 1/x}{3 + 2/x} = \frac{2 - 0}{3 + 0} = \frac{2}{3}$

24.) $\lim_{x \to \infty} \frac{5x^3 + 1}{10x^3 - 3x^2 + 7} = \lim_{x \to \infty} \frac{5 + 1/x^3}{10 - \frac{3}{x} + \frac{7}{x^3}}$

$= \frac{5 + 0}{10 - 0 + 0} = \frac{5}{10} = \frac{1}{2}$

26.) $\lim_{x \to \infty} \frac{2x^{10} - 1}{10x^{11} - 3} = \lim_{x \to \infty} \frac{\frac{2}{x} - \frac{1}{x^{11}}}{10 - \frac{3}{x^{11}}} = \frac{0 - 0}{10 - 0} = 0$

27.) $\lim_{x \to -\infty} \frac{5x^2}{x + 3} = \lim_{x \to -\infty} \frac{5x}{1 + \frac{3}{x}} = \text{"}\frac{-\infty}{1 + 0}\text{"} = -\infty$

28.) $\lim_{x \to +\infty} \frac{x^3 - 2x^2 + 3x + 1}{x^2 - 3x + 2} = \lim_{x \to +\infty} \frac{x - 2 + \frac{3}{x} + \frac{1}{x^2}}{1 - \frac{3}{x} + \frac{2}{x^2}} = +\infty$

29.) $\lim_{x \to \infty} \left(2x - \frac{1}{x^2}\right) = \text{"}+\infty - 0\text{"} = \infty$

31.) $\lim_{x \to -\infty} \left(\frac{2x}{x-1} + \frac{3x}{x+1} \right) = \lim_{x \to -\infty} \left(\frac{2}{1-\frac{1}{x}} + \frac{3}{1+\frac{1}{x}} \right)$

$= \frac{2}{1} + \frac{3}{1} = 5$

32.) $\lim_{x \to +\infty} \left(\frac{2x^2}{x-1} + \frac{3x}{x+1} \right) = \lim_{x \to +\infty} \frac{2x^2(x+1) + 3x(x-1)}{(x-1)(x+1)}$

$= \lim_{x \to +\infty} \frac{2x^3 + 5x^2 - 3x}{x^2 - 1} = \lim_{x \to +\infty} \frac{2x + 5 - \frac{3}{x}}{1 - \frac{1}{x^2}} = +\infty$

37.) $f(x) = \frac{2x}{\sqrt{x^2+4}}$

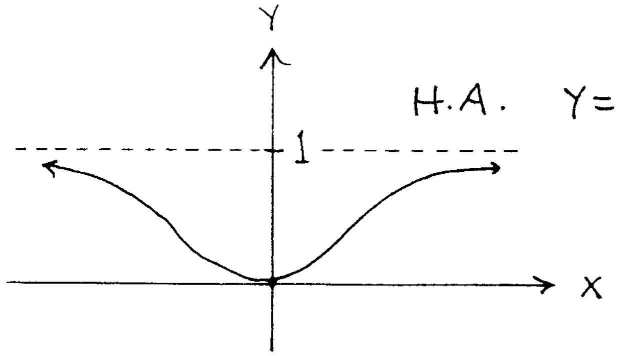
x	-10^6	-10^4	-10^2	10^0	10^2	10^4	10^6
$f(x)$	-2	-2	-1.999	0.894	1.999	2	2

guess : $\lim_{x \to +\infty} f(x) = +2$

and $\lim_{x \to -\infty} f(x) = -2$.

41.) $f(x) = \dfrac{x^2}{x^2+9} \longrightarrow x=0,\ y=0$ and

$\lim\limits_{x\to\pm\infty} \dfrac{x^2}{x^2+9} = \lim\limits_{x\to\pm\infty} \dfrac{1}{1+9/x^2} = \dfrac{1}{1+0} = 1$ so



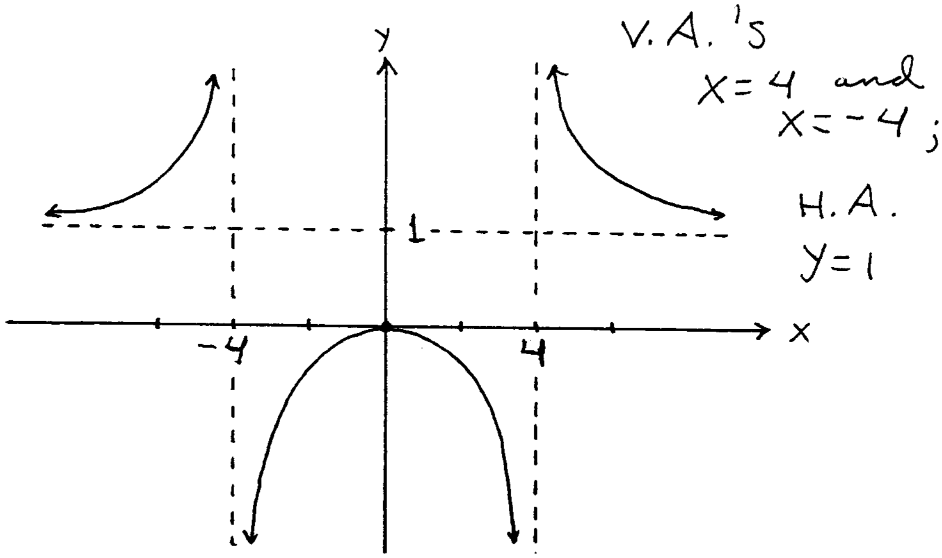
43.) $g(x) = \dfrac{x^2}{x^2-16} \longrightarrow x=0,\ y=0$ and $x=4, x=-4$

are not in domain :

$\lim\limits_{x\to4^+} \dfrac{x^2}{x^2-16} = \text{"}\dfrac{16}{0^+}\text{"} = +\infty$, $\lim\limits_{x\to4^-} \dfrac{x^2}{x^2-16} = \text{"}\dfrac{16}{0^-}\text{"} = -\infty$,

$\lim\limits_{x\to-4^+} \dfrac{x^2}{x^2-16} = \text{"}\dfrac{16}{0^-}\text{"} = -\infty$, $\lim\limits_{x\to-4^-} \dfrac{x^2}{x^2-16} = \text{"}\dfrac{16}{0^+}\text{"} = +\infty$,

$\lim\limits_{x\to\pm\infty} \dfrac{x^2}{x^2-16} = \lim\limits_{x\to\pm\infty} \dfrac{1}{1-\frac{16}{x^2}} = \dfrac{1}{1-0} = 1$ so

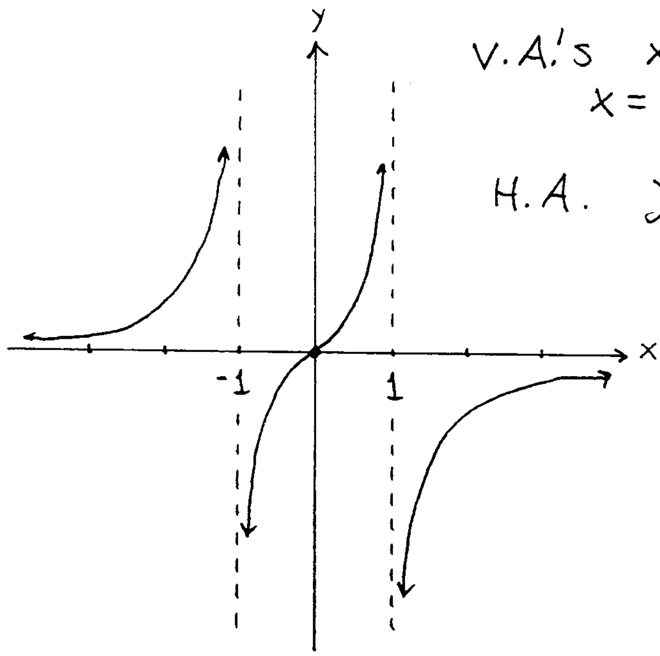


48.) $Y = \frac{2X}{1-X^2} \rightarrow X=0, Y=0$ and $X=1, X=-1$ are not in domain :

$\lim_{X\to 1^+} \frac{2X}{1-X^2} = \text{"}\frac{2}{0^-}\text{"} = -\infty$, $\lim_{X\to 1^-} \frac{2X}{1-X^2} = \text{"}\frac{2}{0^+}\text{"} = +\infty$,

$\lim_{X\to -1^+} \frac{2X}{1-X^2} = \text{"}\frac{-2}{0}\text{"} = -\infty$, $\lim_{X\to -1^-} \frac{2X}{1-X^2} = \text{"}\frac{-2}{0^-}\text{"} = +\infty$,

$\lim_{X\to \pm\infty} \frac{2X}{1-X^2} = \lim_{X\to\pm\infty} \frac{\frac{2}{X}}{\frac{1}{X^2}-1} = \frac{0}{0-1} = 0$ so



V.A.'s $x=1$ and $x=-1$;

H.A. $y=0$

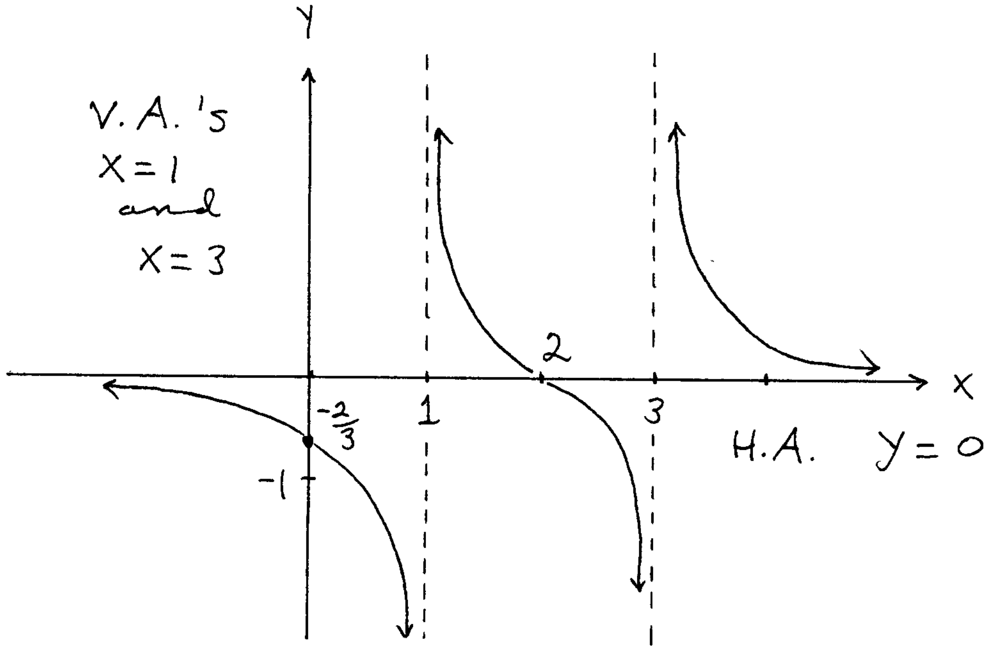
52.) $Y = \frac{X-2}{X^2-4X+3} = \frac{X-2}{(X-1)(X-3)} \rightarrow X=2, Y=0$ and $X=0, Y=-2/3$ and $X=1, X=3$ are not in domain :

$\lim_{X\to 1^+} \frac{X-2}{(X-1)(X-3)} = \text{"}\frac{-1}{0^-}\text{"} = +\infty$, $\lim_{X\to 1^-} \frac{X-2}{(X-1)(X-3)} = \text{"}\frac{-1}{0^+}\text{"} = -\infty$,

$\lim_{X\to 3^+} \frac{X-2}{(X-1)(X-3)} = \text{"}\frac{1}{0^+}\text{"} = +\infty$, $\lim_{X\to 3^-} \frac{X-2}{(X-1)(X-3)} = \text{"}\frac{1}{0^-}\text{"} = -\infty$,

$\lim_{X\to\pm\infty} \frac{X-2}{X^2-4X+3} = \lim_{X\to\pm\infty} \frac{\frac{1}{X}-\frac{2}{X^2}}{1-\frac{4}{X}+\frac{3}{X^2}} = \frac{0}{1} = 0$ so

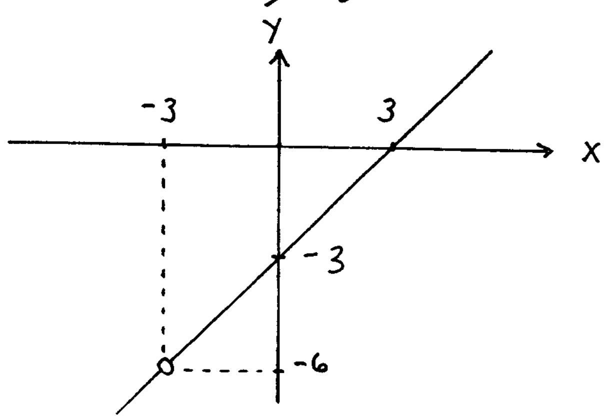
V.A.'s
$x = 1$
and
$x = 3$



H.A. $y = 0$

54.) $g(x) = \frac{x^2 - 9}{x + 3} \rightarrow x = 0,\ y = -3$ and $x = -3$ is not in domain ; if $y = 0$, then $x = 3$ and

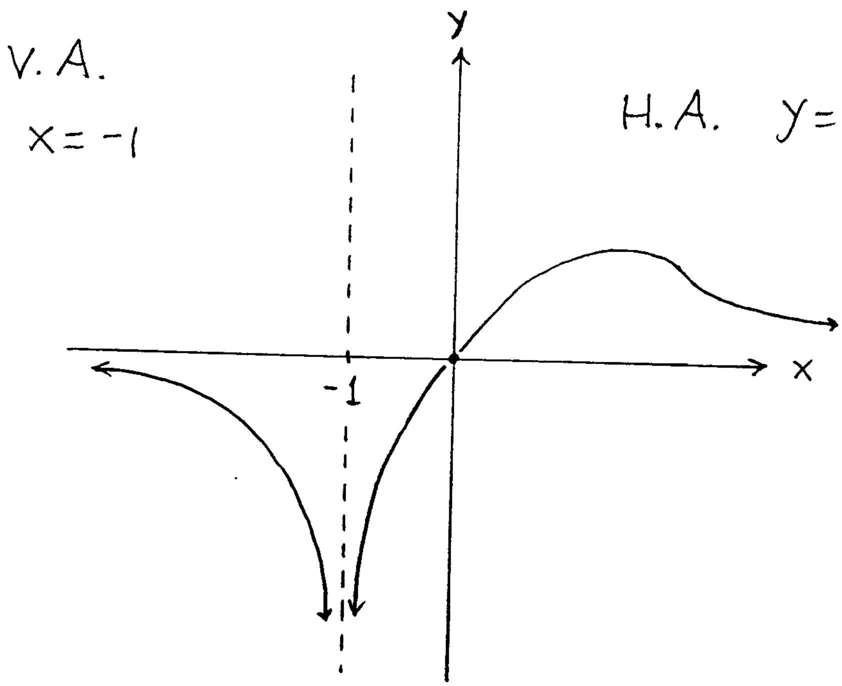
$\lim_{x \to -3} g(x) = \lim_{x \to -3} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}} = -6$ so



56.) $Y = \dfrac{x}{(x+1)^2} \rightarrow x = 0,\ Y = 0$ and $x = -1$ is not in domain :

$$\lim_{x \to -1} \frac{x}{(x+1)^2} = \text{"}\frac{-1}{0^+}\text{"} = -\infty$$ and

$$\lim_{x \to \pm\infty} \frac{x}{x^2+2x+1} = \lim_{x \to \pm\infty} \frac{\frac{1}{x}}{1+\frac{2}{x}+\frac{1}{x^2}} = \frac{0}{1} = 0$$ so



59.) $C = \dfrac{528\,p}{100-p}$ for $0 \le p < 100$:

a.) $p = 25 \rightarrow C = \$176$ (million)

b.) $p = 50 \rightarrow C = \$528$ (million)

c.) $p = 75 \rightarrow C = \$1584$ (million)

d.) $\lim\limits_{p \to 100^-} C = \lim\limits_{p \to 100^-} \dfrac{528\,p}{100-p} = \text{"}\dfrac{52{,}800}{0^+}\text{"} = +\infty$

(millions of \$)

63.) $N = \dfrac{30 + 40t}{1 + 0.1\,t}$, $t \ge 0$:

a.) $t = 5$ yr. $\rightarrow N \approx 153$ elk

$t = 10$ yr. $\rightarrow N = 215$ elk

$t = 25$ yr. $\rightarrow N = 294$ elk

b.) The limiting size of the herd is

$$\lim_{t \to +\infty} N = \lim_{t \to +\infty} \frac{30 + 40t}{1 + 0.1\,t}$$

$$= \lim_{t \to +\infty} \frac{\frac{30}{t} + 40}{\frac{1}{t} + 0.1}$$

$$= \frac{40}{0.1} = 400 \text{ deer}$$

64.) Let x be number of items with total cost $C = 34.5x + 15{,}000$ and total revenue $R = 69.9x$:

a.) Total profit $P = R - C = 69.9x - (34.5x + 15{,}000)$
$\rightarrow$ $P = 35.4x - 15{,}000$. Then profit per item or average profit is

$$\overline{P} = \frac{P}{x} = \frac{R-C}{x} = \frac{35.4x - 15{,}000}{x} \rightarrow$$

$$\overline{P} = 35.4 - \frac{15{,}000}{x} .$$

b.) $x = 1000 \rightarrow \overline{P} = \$20.40$

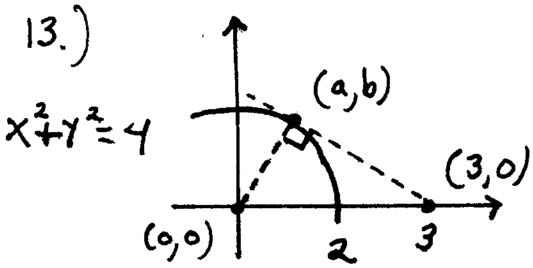
$x = 10{,}000 \rightarrow \overline{P} = \$33.90$

$x = 100{,}000 \rightarrow \overline{P} = \$35.25$

c.) $\lim_{x \to +\infty} \overline{P} = \lim_{x \to +\infty} \left(35.4 - \frac{15{,}000}{x}\right) = \$35.40$

Worksheet 1

13.)



Slopes of dotted lines are negative reciprocals:

$$\frac{b-0}{a-3} = \frac{-1}{\frac{b-0}{a-0}} \rightarrow$$

$\frac{b}{a-3} = \frac{-a}{b} \rightarrow \boxed{b^2 = -a^2+3a}$; and (a,b) is on circle

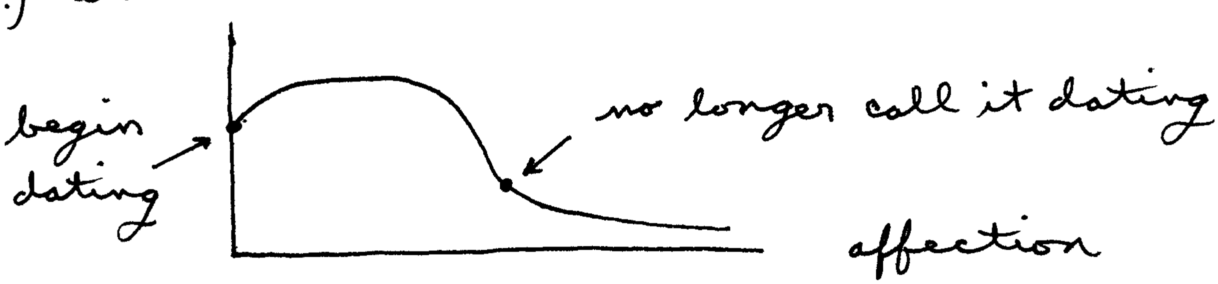
so $\boxed{a^2+b^2=4} \rightarrow a^2+(-a^2+3a)=4 \rightarrow 3a=4 \rightarrow$

$a=\frac{4}{3} \rightarrow \left(\frac{4}{3}\right)^2+b^2=4 \rightarrow b^2=\frac{20}{9} \rightarrow b=\pm\frac{\sqrt{20}}{3}$

so points of tangency are $\left(\frac{4}{3}, \frac{\sqrt{20}}{3}\right)$ and $\left(\frac{4}{3}, \frac{-\sqrt{20}}{3}\right)$.

17.) An increase in the number of workers increases productivity up to a point. After a certain point, however, too many workers interfere with each other and productivity begins to drop.

18.) entertainment



20.) a.) domain: $x \le 0$, $2 \le p < 6$

b.) range: $r \ge 0$

c.) If $r > 5$, then one value of p; if $0 \le r < 2$, then one value of p.

26.) a.) domain of f^{-1}: 3, -7, 19, 4, 178, 2, 1

b.)

x	3	-7	19	4	178	2	1
$f^{-1}(x)$	1	2	3	4	5	6	7