Section 1.6

3.) Since $Y=1$ (line) and $Y=X^2-4$ (parabola) are continuous for all x-values, it follows that $f(x)=\frac{1}{x^2-4}$ is continuous for all (quotient) x-values except where $x^2-4=(x-2)(x+2)=0$, that is, except at $x=2$ and $x=-2$.

5.) Since $Y=1$ (line) and $Y=4+x^2$ (parabola) are continuous for all x-values, it follows that $f(x)=\frac{1}{x^2+4}$ is continuous for all (quotient) x-values (since x^2+4 is never zero).

11.) Since $Y=x^2-1$ (parabola) and $Y=x$ (line) are continuous for all x-values, it follows that $f(x)=\frac{x^2-1}{x}$ is continuous for all x-values except $x=0$. (quotient)

14.) $y=x^3-8$ (polynomial) and $y=x-2$ (line) are continuous for all x-values, so it follows that $f(x)=\frac{x^3-8}{x-2}$ is continuous for all values of x except $x=2$.

21.) Since $Y = X-5$ (line) and $Y = X^2-9X+20$ (parabola) are continuous for all values of X, it follows that $f(X) = \dfrac{X-5}{X^2-9X+20}$ is (quotient) continuous for all values of X except where $X^2-9X+20 = (X-4)(X-5) = 0$, that is, except at $X=4$ and $X=5$.

25.) $f(x) = \begin{cases} -2X+3, & X<1 \\ x^2, & X \geq 1 \end{cases}$ We need only check for continuity at $X=1$ since $Y = -2X+3$ (line) and $Y = X^2$ (parabola) are continuous graphs :

i.) $f(1) = (1)^2 = 1$,

ii) $\lim\limits_{X \to 1^+} f(x) = \lim\limits_{X \to 1^+} (X^2) = (1)^2 = 1$ and

$\lim\limits_{X \to 1^-} f(x) = \lim\limits_{X \to 1^-} (-2X+3) = -2+3 = 1$ so $\lim\limits_{X \to 1} f(x) = 1$

and iii) i.) and ii) are equal.

Thus f is continuous at $X=1$ and all other X-values.

27.) $f(X) = \begin{cases} \frac{1}{2}X+1, & X \leq 2 \\ 3-X, & X>2 \end{cases}$ We need only check for continuity at $X=2$ since $Y = \frac{1}{2}X+1$ (line) and $Y = 3-X$ (line) are continuous graphs :

i.) $f(2) = \frac{1}{2}(2)+1 = 2$,

ii.) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3 - x) = 1$ and

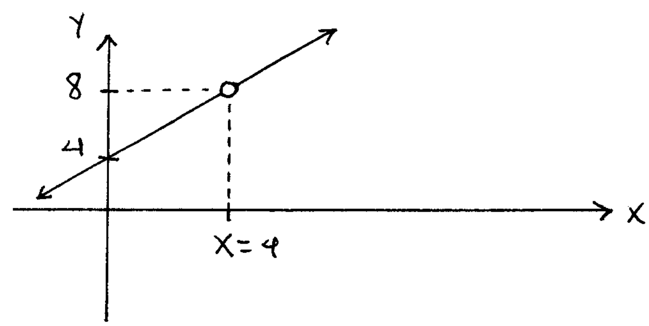
$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (\frac{1}{2}x + 1) = 2$, so $\lim_{x \to 2} f(x)$ does

not exist and f is NOT continuous at $x = 2$. Thus, f is continuous for all x-values except $x = 2$.

30.) $y = |4 - x|$ is continuous for all values of x (well known: V) and $y = 4 - x$ is continuous (line) for all values of x, so it follows that $f(x) = \frac{|4-x|}{4-x}$ (quotient) is continuous for all values of x except $x = 4$.

39.) $f(x) = \frac{x^2 - 16}{x - 4} = \frac{(x-4)(x+4)}{x-4}$ for $x \neq 4$;

that is, $f(x) = x + 4$ for $x \neq 4$. Function f has a removable discontinuity at $x = 4$:



i.) $f(4)$ is NOT defined, but

ii.) $\lim_{x \to 4} f(x) = \lim_{x \to 4} (x + 4) = 8$.

44.) $f(x) = \begin{cases} x^2-4 & \text{if } x \le 0 \\ 2x+4 & \text{if } x > 0 \end{cases}$;

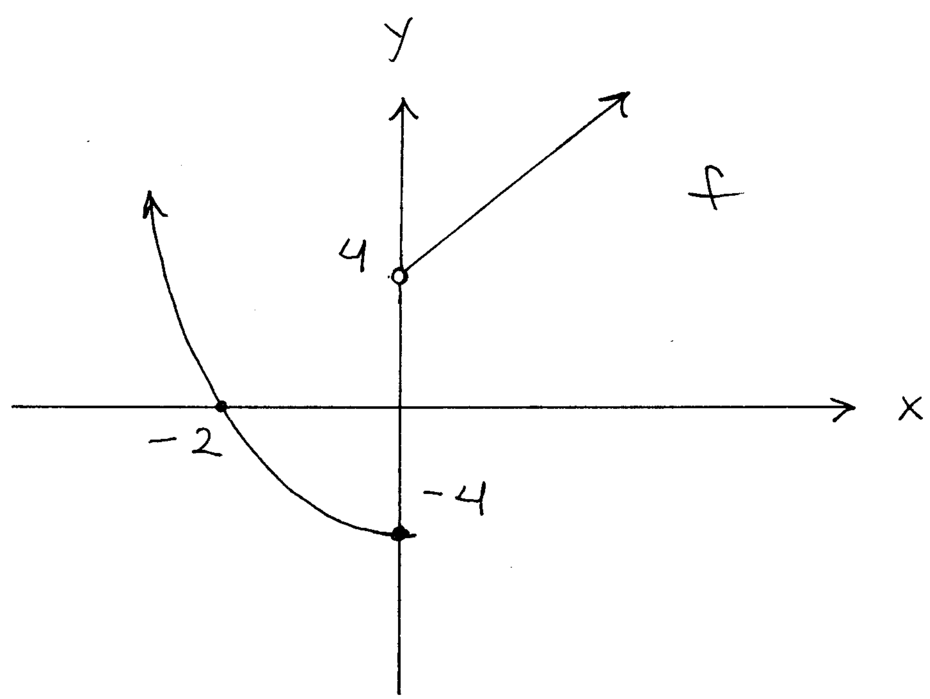
$y = x^2 - 4$ (parabola) is continuous for $x < 0$ and $y = 2x+4$ (line) is continuous for $x > 0$; <u>check at $x = 0$</u> :

i.) $f(0) = 0^2 - 4 = -4$,

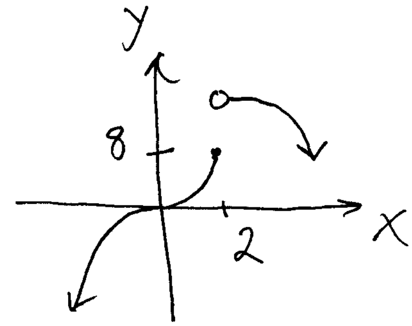
ii.) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (2x+4) = 4$ and

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x^2-4) = -4$, so

$\lim_{x \to 0} f(x)$ D.N.E. and f is NOT continuous at $x = 0$; thus, f is continuous for all values of x except $x = 0$.

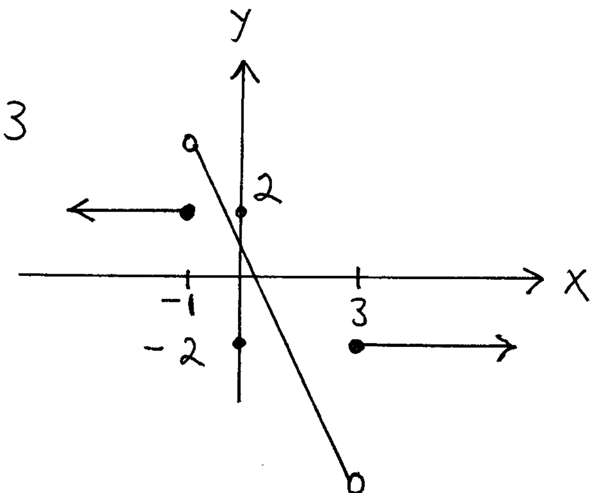


45.) $f(x) = \begin{cases} x^3 & \text{if } x \le 2 \\ ax^2 & \text{if } x > 2 \end{cases}$



$$\left.\begin{aligned} \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} (ax^2) = 4a \\ \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} (x^3) = 8 \end{aligned}\right\} \quad 4a = 8 \text{ so } a = 2.$$

46.) $f(x) = \begin{cases} 2 & \text{if } x \le -1 \\ ax+b & \text{if } -1 < x < 3 \\ -2 & \text{if } x \ge 3 \end{cases}$



$$\left.\begin{aligned} \lim_{x\to -1^+} f(x) &= \lim_{x\to -1^+} (ax+b) = b-a \\ \lim_{x\to -1^-} f(x) &= \lim_{x\to -1^-} (2) = 2 \end{aligned}\right\} \quad b-a = 2$$

$$\left.\begin{aligned} \lim_{x\to 3^+} f(x) &= \lim_{x\to 3^+} (-2) = -2 \\ \lim_{x\to 3^-} f(x) &= \lim_{x\to 3^-} (ax+b) = 3a+b \end{aligned}\right\} \quad 3a+b = -2 \qquad \text{so}$$

$$\left.\begin{aligned} b-a &= 2 \\ 3a+b &= -2 \end{aligned}\right\} \to \quad b = a+2 \quad \to \quad 3a + (a+2) = -2 \to 4a = -4 \to$$

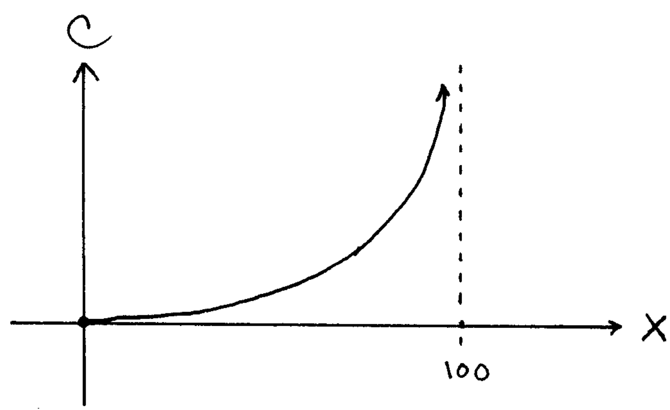
$a = -1$ and $b = 1$.

60.) $C = \frac{2x}{100-x}$, x percent

a.) Since x represents a measure of percent, negative values of x are impossible, and x is at least 0 and at most 100, but function C is not defined at $x=100$; thus the domain of C is

$$0 \le x < 100 .$$

b.)



Function C is continuous on $0 \le x < 100$ since it is the ratio of continuous functions on $0 \le x < 100$.

c.) If $x = 75$, then

$C = \frac{2(75)}{100-75} = 6$, so that the cost of removing 75% of the pollutants is \$6 million.

Math 16A
Kouba
Worksheet 2

1.) Determine the constants a (and b) so that each of the following functions is continuous for all values of x.

a.) $f(x) = \begin{cases} \dfrac{x^2 - 7x + 6}{x - 6} & , \quad x \neq 6 \\ a & , \quad x = 6 \end{cases}$

b.) $f(x) = \begin{cases} a^2 x - a & , \quad x \geq 1 \\ 2 & , \quad x < 1 \end{cases}$

c.) $f(x) = \begin{cases} \dfrac{a + x}{a + 1} & , \quad x < 0 \\ a x^3 + 3 & , \quad x \geq 0 \end{cases}$

d.) $f(x) = \begin{cases} 3 & , \quad x \leq 1 \\ a x^2 + b & , \quad 1 < x \leq 2 \\ 5 & , \quad x > 2 \end{cases}$

e.) $f(x) = \begin{cases} a x - b & , \quad x \leq -1 \\ 2x + 3a + b & , \quad -1 < x \leq 1 \\ 4 & , \quad x > 1 \end{cases}$

2. An example of continuity as a measure of fairness-- A small city proposes the following tax scheme for its residents in order to upgrade public parks. The annual income and proposed amount of tax appear in the table below.

a. Sketch a graph of the amount of tax paid as a function of annual income x.

b. In your opinion, is this tax scheme a fair one ? (Use the graph in part a.) Explain.

Annual Income	Tax
\$0 - \$20,000	0.3% of income
\$20,001 - \$40,000	the larger of \$60 and 0.2% of income
\$40,001 - \$60,000	a flat fee of \$20 plus 0.18% of income
\$60,001 and higher	the larger of \$128 and 0.16% of income

Worksheet 2

1.) a.) $\lim_{x\to 6} f(x) = \lim_{x\to 6} \frac{x^2-7x+6}{x-6} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 6} \frac{\cancel{(x-6)}(x-1)}{\cancel{x-6}} = 5$

so choose $a=5$.

b.) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (a^2x-a) = a^2-a$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2) = 2$

$a^2-a=2 \rightarrow$

$a^2-a-2=0 \rightarrow (a-2)(a+1)=0 \rightarrow a=2$ or $a=-1$.

c.) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (ax^3+3) = 3$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \left(\frac{a+x}{a+1}\right) = \frac{a}{a+1}$

$3 = \frac{a}{a+1} \rightarrow$

$3a+3=a \rightarrow 2a=-3 \rightarrow a=-3/2$.

d.) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (ax^2+b) = a+b$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (3) = 3$

$a+b=3$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (5) = 5$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (ax^2+b) = 4a+b$

$4a+b=5$

$a+b=3 \rightarrow b=3-a$

$4a+b=5 \rightarrow 4a+(3-a)=5 \rightarrow$

$3a = 2 \rightarrow$ $a = 2/3$, $b = 7/3$.

e.) $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (2x + 3a + b) = -2 + 3a + b$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (ax - b) = -a - b$

$-2 + 3a + b = -a - b \rightarrow$

$4a + 2b = 2 \rightarrow$ $\boxed{2a + b = 1}$;

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (4) = 4$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x + 3a + b) = 2 + 3a + b$

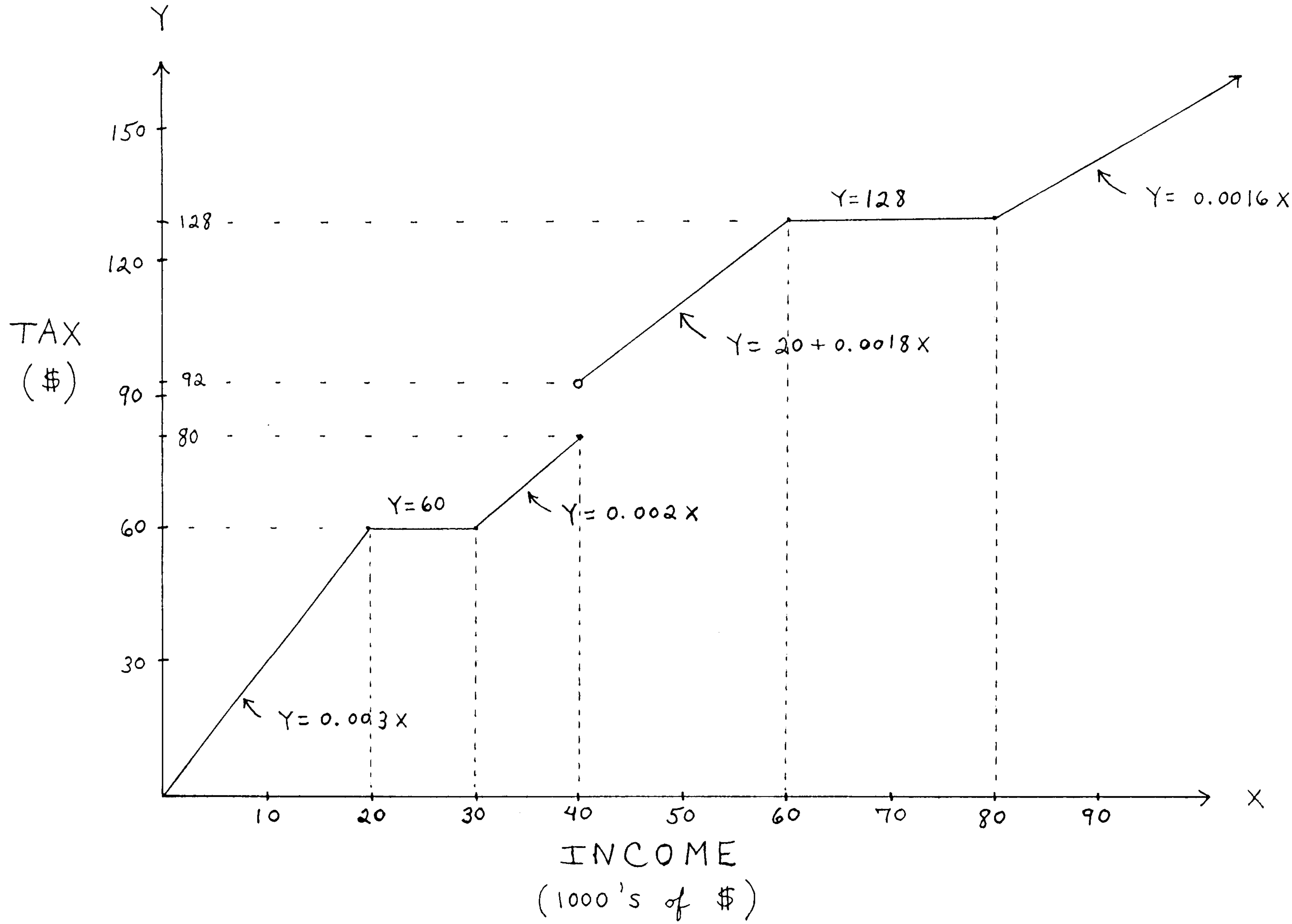
$2 + 3a + b = 4 \rightarrow$ $\boxed{3a + b = 2}$;

$b = 1 - 2a$ so $3a + (1 - 2a) = 2 \rightarrow$
$a = 1$ and $b = -1$.

2.) a.) See next page.

b.) This tax scheme is unfair for 2 reasons. The flat portions of the scheme are unfair to those with incomes at the beginning of the flat portion. For example, a person making \$20,000 pays the same tax, \$60, as one making \$30,000. The point of discontinuity is also unfair. A person making \$40,000 pays \$80 in taxes and a person making \$40,001 pays \$92 in taxes!

An Example of Continuity as Fairness



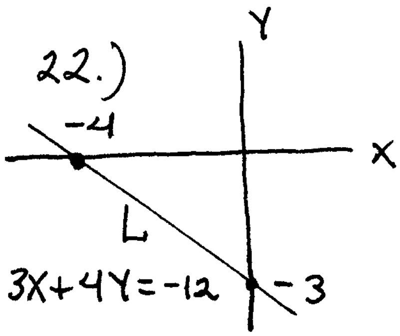
Worksheet 1

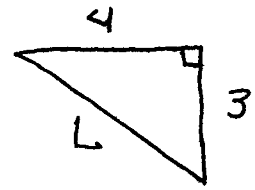
21.) Find equation of line passing through pts. $(5.2, 27.8)$ and $(5.3, 29.2)$:

slope $m = \dfrac{29.2 - 27.8}{5.3 - 5.2} = 14$ so line is

$Y - 27.8 = 14\,(X - 5.2)$ OR $Y = 14X - 45$

22.)



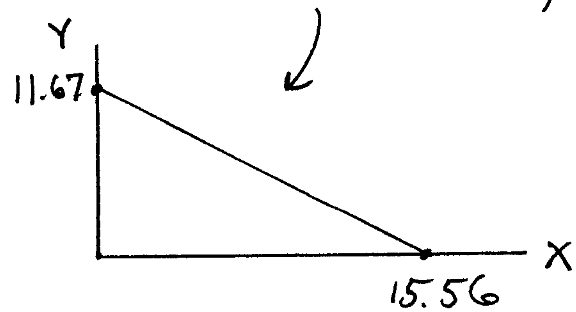


$3^2 + 4^2 = L^2 \rightarrow$

$L = 5$

24.) a.) Let X : # of Mexican peppers
Y : # of Indian peppers

$900X + 1200Y = 14{,}000$

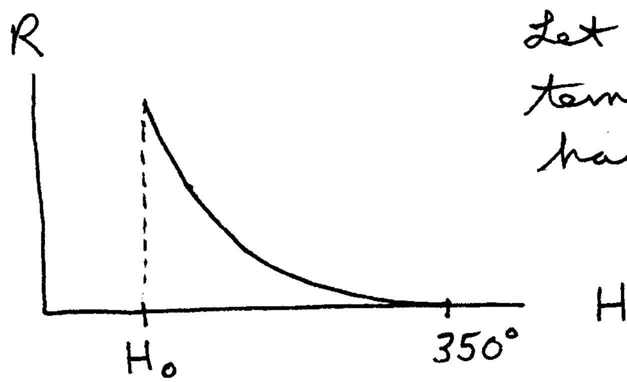


b.) Solve for Y :

$Y = -0.75X + 11.67$

25.) a.) $R = k\,(350 - H)$

b.)



Let H_0 : initial temperature of ham

27.) a.) $C = 100 + 2q = f(q) \rightarrow$

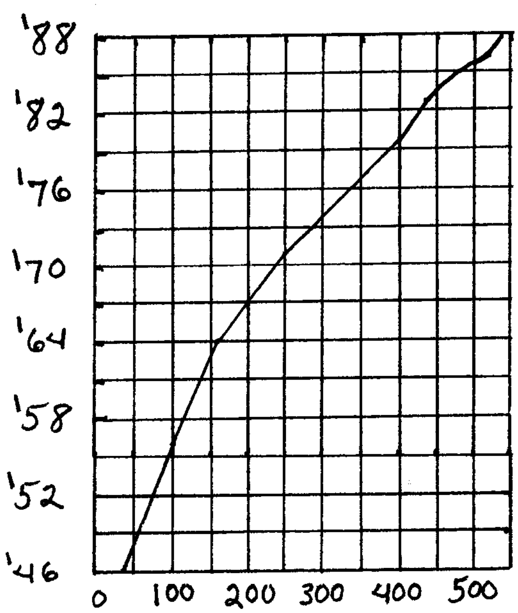
$$q = \frac{1}{2}(C - 100) = f^{-1}(C)$$

b.) The inverse function inputs total cost and outputs the number of articles produced.

28.) a.) Function f is invertible since it satisfies the horizontal line test.

b.) $f^{-1}(400) = 1980$

c.) Graph of f^{-1} $\longrightarrow$



29.) a.) supply curve

b.) demand curve

Manufacturer: As the selling price p of an item increases, the manufacturer will eagerly produce more items q (a.)

Consumer: As the selling price p of an item increases, the consumer will respond by buying fewer items q (b.)