Section 8.1

9.) $180^\circ = \pi$ so $30^\circ = \frac{\pi}{6}$

10.) $\frac{180^\circ}{150^\circ} = \frac{\pi}{x}$ so $x = \frac{5}{6}\pi$

13.) $\frac{180^\circ}{315^\circ} = \frac{\pi}{x}$ so $x = \frac{7}{4}\pi$

14.) $\frac{180^\circ}{120^\circ} = \frac{\pi}{x}$ so $x = \frac{2}{3}\pi$

15.) $-180^\circ = -\pi$ so $-30^\circ = -\frac{\pi}{6}$

16.) $\frac{-180^\circ}{-240^\circ} = \frac{-\pi}{x}$ so $x = \frac{-4}{3}\pi$

21.) $\pi = 180^\circ$ so $\frac{3}{2}\pi = 270^\circ$

22.) $\pi = 180^\circ$ so $\frac{7}{6}\pi = 210^\circ$

23.) $\pi = 180^\circ$ so $\frac{11}{6}\pi = 330^\circ$

24.) $\pi = 180^\circ$ so $\frac{7}{4}\pi = 315^\circ$

28.) $\pi = 180^\circ$ so $\frac{5}{2}\pi = 450^\circ$

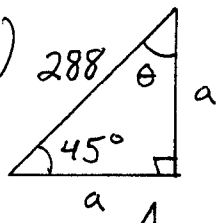
30.) $\pi = 180^\circ$ so $\frac{8}{3}\pi = 480^\circ$

31.) $\frac{-180^\circ}{-270^\circ} = \frac{-\pi}{x}$ so $x = -\frac{3}{2}\pi$

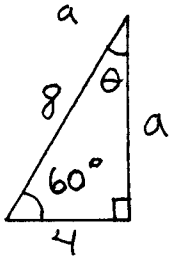
32.) $\pi = 180^\circ$ so $\frac{\pi}{9} = 20^\circ$

33.) $\frac{180^\circ}{144^\circ} = \frac{\pi}{x}$ so $x = \frac{4}{5}\pi$

34.) $\pi = 180^\circ$ so $-\frac{7}{12}\pi = -105^\circ$

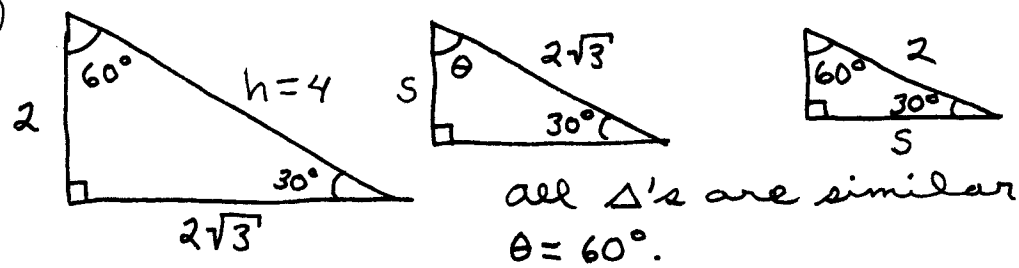
36.) 

$\theta = 45^\circ$ and $a^2 + a^2 = 288^2 \rightarrow$
$2a^2 = 82{,}944 \rightarrow a^2 = 41{,}472 \rightarrow$
$a = \sqrt{41{,}472} \approx 203.6$

37.) 

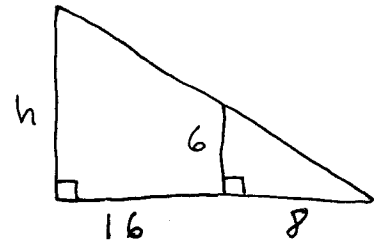
$\theta = 30^\circ$ and $4^2 + a^2 = 8^2 \rightarrow$
$a^2 = 48 \rightarrow a = \sqrt{48} = 4\sqrt{3}$

40.) By similar triangles $\frac{2}{2+1} = \frac{h}{1} \rightarrow h = \frac{2}{3}$

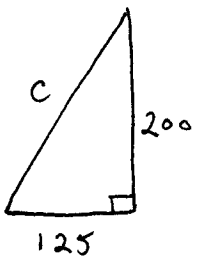
41.) 

all Δ's are similar so $\theta = 60^\circ$.

$2^2 + (2\sqrt{3})^2 = h^2 \rightarrow 4 + 12 = h^2 \rightarrow h = 4$; by similar Δ's $\rightarrow \frac{2}{4} = \frac{s}{2\sqrt{3}} \rightarrow s = \sqrt{3}$.

47.) 

$\frac{h}{16+8} = \frac{6}{8} \rightarrow h = 18$ ft.

48.) 

$c^2 = 125^2 + 200^2 = 55{,}625$

$\rightarrow \quad c = 235.8$ ft.

50.) $\theta = \left(\frac{25}{60}\right)(2\pi) = \frac{5}{6}\pi$ radians so distance $s = r\theta = \left(3\frac{1}{2}\right)\left(\frac{5}{6}\pi\right) = \frac{35}{12}\pi$ inches

53.) a.) 1 revolution $= 2\pi$ radians so

$\frac{3142 \text{ rad.}}{\text{min.}} \times \frac{1 \text{ rev.}}{2\pi \text{ rad.}} \approx 500.1 \text{ rev./min.}$

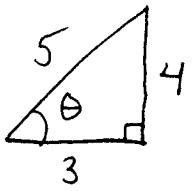
b.) $10{,}000 \text{ rev.} \times \frac{1 \text{ min.}}{500.1 \text{ rev.}} \approx 20 \text{ min.}$

54.) Circumference of disk is $C = 2\pi r = 2\pi(1.68) = 3.36\pi$ inches. A point on the circumference travels 3.36π inches in one revolution. Thus speed

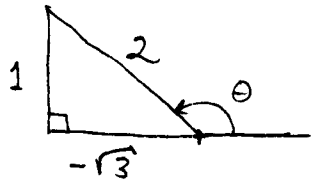
$\frac{360 \text{ rev.}}{\text{min.}} \times \frac{3.36\pi \text{ in.}}{\text{rev.}}$

≈ 3800.1 in./min.

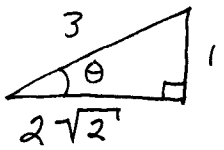
Section 8.2

1.) 

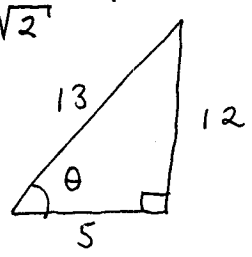
$\cos\theta = 3/5$, $\sin\theta = 4/5$, $\tan\theta = 4/3$,
$\sec\theta = 5/3$, $\csc\theta = 5/4$, $\cot\theta = 3/4$

5.) 

$\cos\theta = \frac{-\sqrt{3}}{2}$, $\sin\theta = \frac{1}{2}$, $\tan\theta = \frac{-1}{\sqrt{3}}$,
$\sec\theta = \frac{-2}{\sqrt{3}}$, $\csc\theta = 2$, $\cot = -\sqrt{3}$.

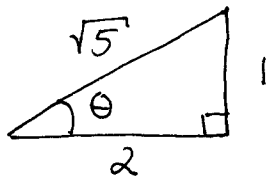
8.) 

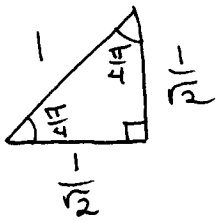
$\tan\theta = \frac{1}{2\sqrt{2}}$

10.) 

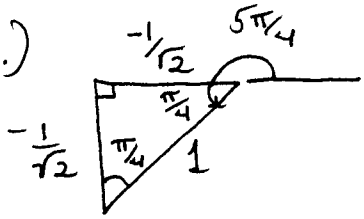
$\cot\theta = \frac{5}{12}$

12.) $\sin\theta = \frac{1}{\sqrt{5}}$



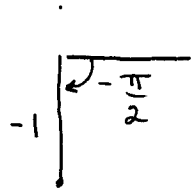
26.) a.) 

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$,
$\tan\frac{\pi}{4} = 1$

b.) 

$\cos\frac{5\pi}{4} = -\sqrt{2}/2$, $\sin\frac{5\pi}{4} = -\sqrt{2}/2$,
$\tan\frac{5\pi}{4} = 1$

28.) a.)



$\cos -\frac{\pi}{2} = 0$, $\sin -\frac{\pi}{2} = -1$,

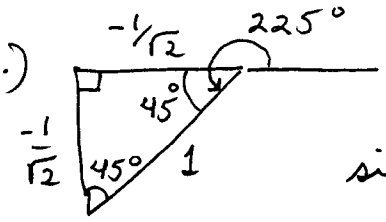
$\tan -\frac{\pi}{2}$ is undefined

b.)

$\cos \frac{\pi}{2} = 0$, $\sin \frac{\pi}{2} = 1$,

$\tan \frac{\pi}{2}$ is undefined

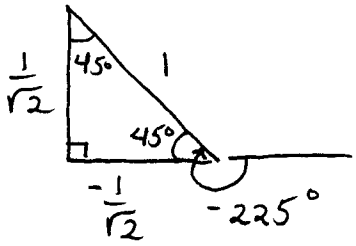
29.) a.)



$\cos 225° = -1/\sqrt{2}$,

$\sin 225° = \frac{-1}{\sqrt{2}}$, $\tan 225° = 1$

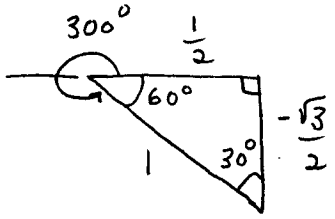
b.)



$\cos(-225°) = -1/\sqrt{2}$,

$\sin(-225°) = \frac{1}{\sqrt{2}}$, $\tan(-225°) = -1$

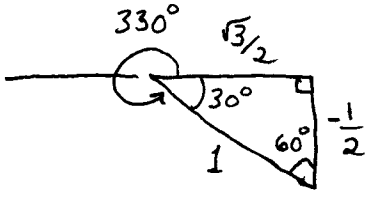
30.) a.)



$\cos 300° = \frac{1}{2}$, $\sin 300° = -\sqrt{3}/2$,

$\tan 300° = -\sqrt{3}$

b.)



$\cos 330° = \sqrt{3}/2$, $\sin 330° = -\frac{1}{2}$,

$\tan 330° = -1/\sqrt{3}$

47.) $2\sin^2\theta = 1 \rightarrow \sin^2\theta = \frac{1}{2} \rightarrow$

$\sin\theta = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2} \rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

50.) $2\cos^2\theta - \cos\theta = 1 \rightarrow 2(\cos\theta)^2 - (\cos\theta) - 1 = 0 \rightarrow$

$(2(\cos\theta) + 1)((\cos\theta) - 1) = 0$

$\cos\theta = \frac{-1}{2}$ $\qquad$ $\cos\theta = 1 \rightarrow \theta = 0°$ or $360°$

$\hookrightarrow \theta = 120°$ or $240°$.

51.) $\sin 2\theta - \cos\theta = 0 \rightarrow 2\sin\theta\cos\theta - \cos\theta = 0$

$\rightarrow \cos\theta\,(2\sin\theta - 1) = 0$

$\cos\theta = 0$ $\qquad$ $\sin\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

52.) $\cos 2\theta + 3\cos\theta + 2 = 0 \rightarrow$

$(2\cos^2\theta - 1) + 3\cos\theta + 2 = 0 \rightarrow$

$2\cos^2\theta + 3\cos\theta + 1 = (2\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{-1}{2}$ $\qquad$ $\cos\theta = -1 \rightarrow$

$\theta = \frac{2}{3}\pi, \frac{4}{3}\pi, \pi$.

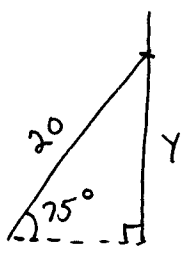
55.) $\cos^2\theta + \sin\theta = 1 \rightarrow (1 - \sin^2\theta) + \sin\theta = 1 \rightarrow$

$\sin\theta - \sin^2\theta = 0 \rightarrow \sin\theta \cdot (1 - \sin\theta) = 0$

$\sin\theta = 0$ $\qquad$ $\sin\theta = 1 \rightarrow$

$\theta = 0°, 180°, 360°$ or $\theta = 90°$.

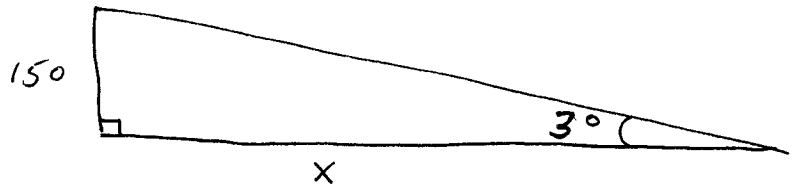
63.)



$\sin 75° = \frac{Y}{20} \rightarrow$

$Y = 20 \cdot \sin 75° \approx 19.32$ ft

65.)



$\tan 3^\circ = \frac{150}{x} \rightarrow \quad x = \frac{150}{\tan 3^\circ} \approx 2862$ ft.

Section 8.3

9.) $y = -2 \sin 10x \rightarrow$ amp. $= |-2| = 2$,

$0 \le 10x \le 2\pi \rightarrow 0 \le x \le \frac{\pi}{5} \rightarrow$ period $= \frac{\pi}{5}$.

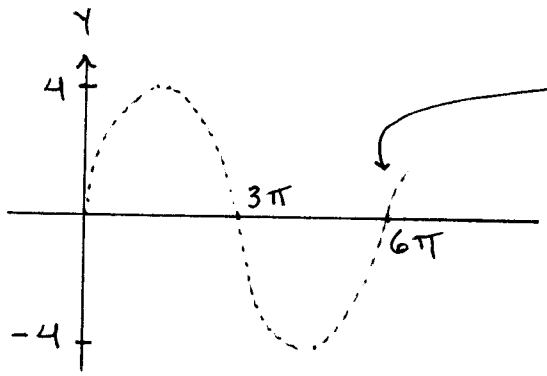
14.) $y = \frac{2}{3} \cos \frac{\pi}{10} x \rightarrow$ amp. $= \frac{2}{3}$,

$0 \le \frac{\pi}{10} x \le 2\pi \rightarrow 0 \le x \le \frac{2\pi}{\frac{\pi}{10}} = 20 \rightarrow$ period $= 20$.

16.) $y = 7 \tan 2\pi x \rightarrow 0 \le 2\pi x \le \pi \rightarrow$

$0 \le x \le \frac{1}{2} \rightarrow$ period $= \frac{1}{2}$.

28.)



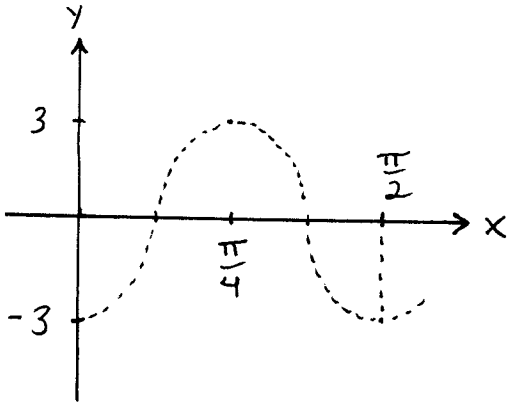
$y = 4 \sin \frac{x}{3} \rightarrow$ amp. $= 4$

$0 \le \frac{x}{3} \le 2\pi \rightarrow$

$0 \le x \le 6\pi \rightarrow$

period $= 6\pi$

32.)



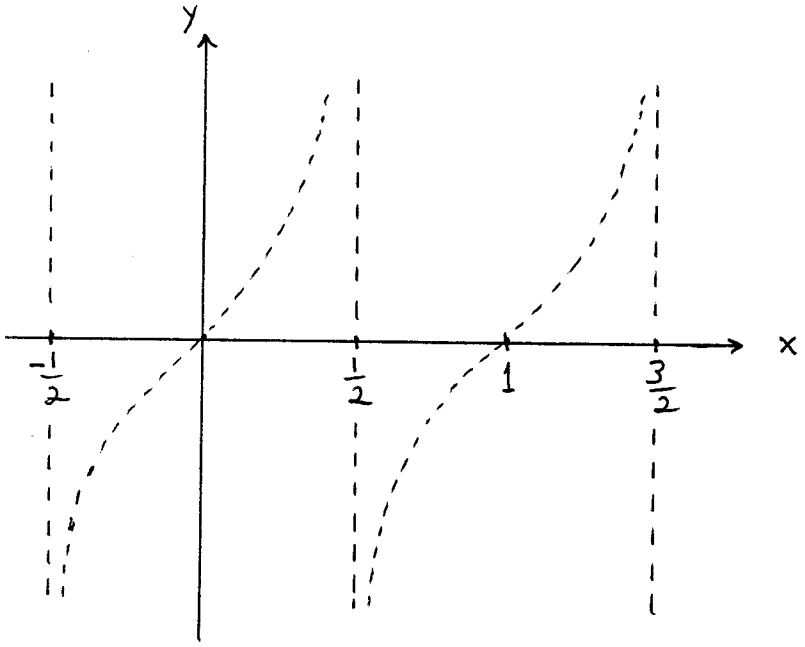
$Y = -3\cos 4X \rightarrow$ amp. $= 3$,

$0 \le 4X \le 2\pi \rightarrow$

$0 \le X \le \frac{\pi}{2} \rightarrow$

period $= \frac{\pi}{2}$

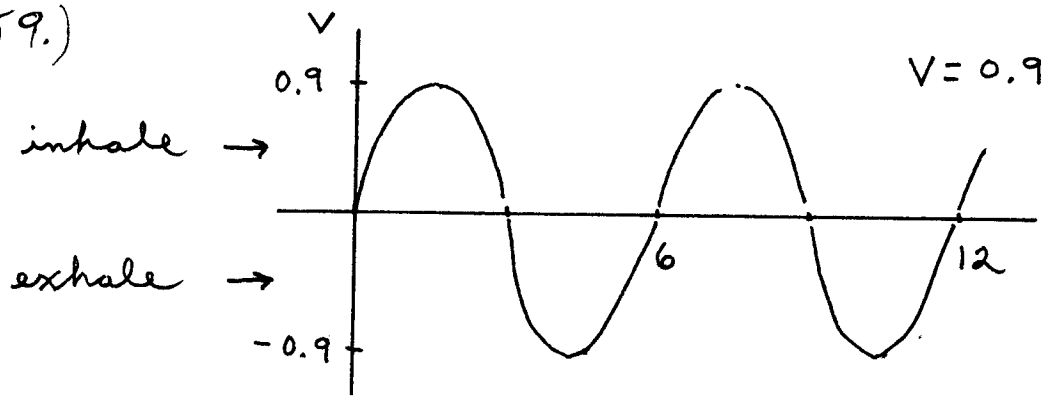
40.)



$Y = 3\tan(\pi X) \rightarrow \quad 0 \le \pi X \le \pi \rightarrow \quad 0 \le X \le 1$

$\rightarrow$ period $= 1$

59.)



$V = 0.9 \sin \frac{\pi}{3} t$

a.) $0 \le \frac{\pi}{3}t \le 2\pi \rightarrow 0 \le t \le \frac{2\pi}{\frac{\pi}{3}} = 6$ so

time for 1 cycle is 6 sec. (period)

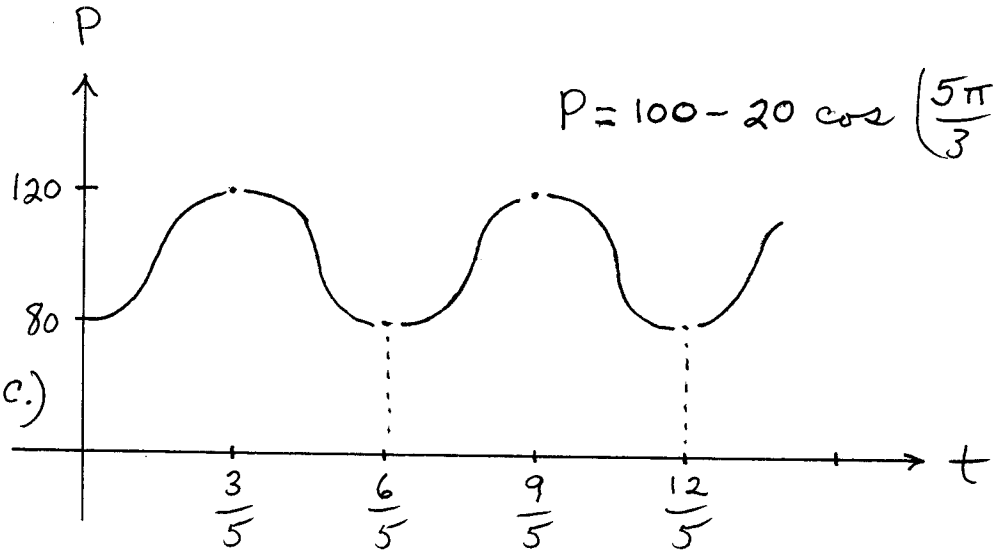
b.) $\frac{1 \text{ cycle}}{6 \text{ sec.}} = \frac{1 \text{ cycle}}{6 \text{ sec.}} \cdot \frac{60 \text{ sec.}}{1 \text{ min.}} = \frac{10 \text{ cycles}}{\text{min.}}$ (frequency)

c.) See sketch above.

62.)

$0 \le \frac{5\pi}{3}t \le 2\pi$

$0 \le t \le \frac{6}{5}$



$P = 100 - 20\cos\left(\frac{5\pi}{3}t\right)$

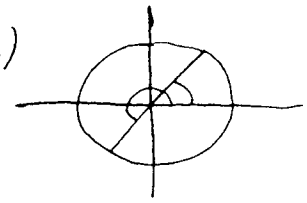
a.) period : $\frac{6}{5}$ seconds (one heartbeat)

b.) $\frac{1 \text{ heartbeat}}{6/5 \text{ sec.}} = \frac{\frac{5}{6} \text{ heartbeats}}{1 \text{ sec.}}$

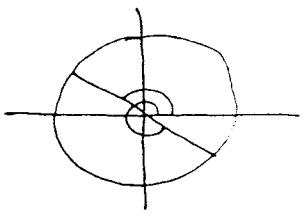
$= 50$ heartbeats / 60 sec.

$= 50$ heartbeats / min.

Worksheet 3

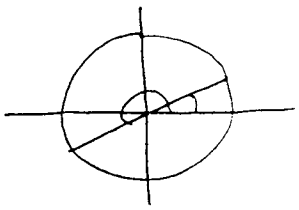
1.) a.) 

$\tan\theta = 1 \rightarrow$ $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$.

b.) 

$\cot\theta = \frac{\cos\theta}{\sin\theta} = -\sqrt{3} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}$

$\rightarrow$ $\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

c.) 

$\tan 3\theta = \frac{\sin 3\theta}{\cos 3\theta} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\frac{-1}{2}}{\frac{-\sqrt{3}}{2}}$

$\rightarrow$ $3\theta = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}, \frac{25\pi}{6}, \frac{31\pi}{6}, \frac{37\pi}{6}, \ldots$

$\rightarrow$ $\theta = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$.

d.) $\sin\theta \cdot \cos 2\theta = 0 \rightarrow$

$\sin\theta = 0 \rightarrow$ $\theta = 0, \pi$ and

$\cos 2\theta = 0 \rightarrow$ $2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \ldots$

$\rightarrow$ $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

e.) $2\sin\theta\cos\theta = \frac{1}{2} \rightarrow \sin 2\theta = \frac{1}{2} \rightarrow$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \ldots \rightarrow$

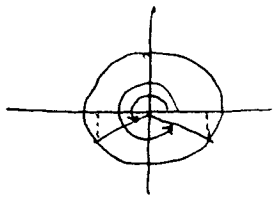
$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$.

f.) $\sin^2\theta - \frac{1}{2}\sin\theta - \frac{1}{2} = 0 \rightarrow$

$(\sin\theta - 1)(\sin\theta + \frac{1}{2}) = 0 \rightarrow$

$\sin\theta = 1$ or $\sin\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{2}$ $\qquad$ $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$



2.) a.) $\sin 22.5^\circ \cdot \cos 22.5^\circ = \frac{1}{2}(2\sin 22.5^\circ \cos 22.5^\circ)$

$= \frac{1}{2}\sin 2(22.5^\circ) = \frac{1}{2}\sin 45^\circ = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$

b.) $\cos 2\theta = 1 - 2\sin^2\theta \rightarrow \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta) \rightarrow$

$\sin^2\frac{\pi}{8} = \frac{1}{2}(1 - \cos 2\cdot\frac{\pi}{8}) = \frac{1}{2}(1 - \cos\frac{\pi}{4}) = \frac{1}{2}(1 - \frac{\sqrt{2}}{2}) = \frac{2-\sqrt{2}}{4}$

c.) $\cos 2\theta = 2\cos^2\theta - 1 \rightarrow \cos^2\theta = \frac{1}{2}(1 + \cos 2\theta) \rightarrow$

$\cos^2\frac{\pi}{12} = \frac{1}{2}(1 + \cos 2\cdot\frac{\pi}{12}) = \frac{1}{2}(1 + \cos\frac{\pi}{6}) = \frac{1}{2}(1 + \frac{\sqrt{3}}{2}) = \frac{2+\sqrt{3}}{4}$

d.) $\sin 15^\circ = \sqrt{\sin^2 15^\circ} = \sqrt{\frac{1}{2}(1 - \cos 30^\circ)} = \frac{\sqrt{2-\sqrt{3}}}{2}$

e.) $\cos 15^\circ = \sqrt{\cos^2 15^\circ} = \sqrt{\frac{1}{2}(1 + \cos 30^\circ)} = \frac{\sqrt{2+\sqrt{3}}}{2}$

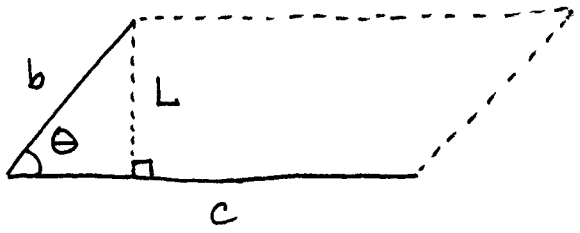
3.) a.) $V = (3)(4)(5)\sin\frac{\pi}{2}\cdot\sin\frac{\pi}{2} = 60$

b.) $V = (3)(4)(5)\sin\frac{\pi}{4}\cdot\sin\frac{5\pi}{6} = 60\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = 15\sqrt{2}$

c.) $V = (1.5)(2.6)(3.2)\sin\frac{3\pi}{4}\cdot\sin\frac{\pi}{3} = 12.48\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \approx 7.64$

4.) Volume = Area of base × Height

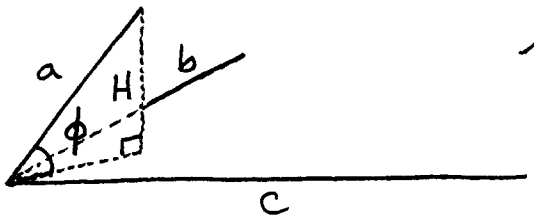
base



$\sin\theta = \frac{L}{b} \rightarrow$

$L = b\sin\theta$ then

Area of base $= cL = cb\sin\theta$;



height, H

$\sin\phi = \frac{H}{a} \rightarrow$

$H = a\sin\phi$;

then,

Volume = Area of base × Height

$= cb\sin\theta \cdot a\sin\phi$

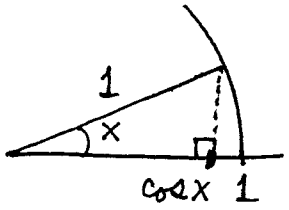
$= abc\sin\theta\sin\phi$

Trig Supplement

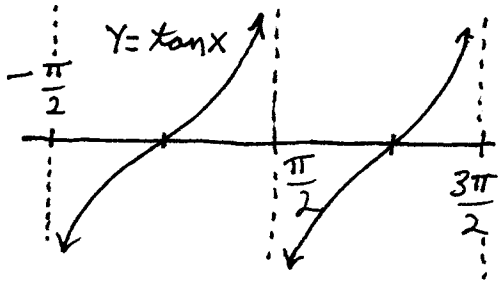
ST 1.) a.) $\lim_{x\to 0^+} \frac{4\cos x + 5}{3\sin x + 2} = \frac{4(\cos 0)+5}{3(\sin 0)+2}$

$= \frac{4(1)+5}{3(0)+2} = \frac{9}{2}$

b.) $\lim_{x\to 0^+} \frac{3}{1-\cos x} = \frac{\text{"}3\text{"}}{0^+} = +\infty$



c.) $\lim_{x\to \frac{\pi}{2}^-} \tan x = +\infty$



d.) $\lim_{x\to \frac{\pi}{2}^+} \tan x = -\infty$

e.) $\lim_{x\to 0} \frac{\sin 2x}{\sin x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2\sin x\cos x}{\sin x}$

$= 2(\cos 0) = 2(1) = 2$

f.) $\lim_{x\to 0} \frac{\cos 2x - 1}{\cos x - 1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{(2\cos^2 x - 1) - 1}{\cos x - 1}$

$= \lim_{x\to 0} \frac{2\cos^2 x - 2}{\cos x - 1} = \lim_{x\to 0} \frac{2(\cos^2 x - 1)}{\cos x - 1}$

$= \lim_{x\to 0} \frac{2(\cos x - 1)(\cos x + 1)}{\cos x - 1}$

$= 2((\cos 0)+1) = 2(1+1) = 4.$