Section 2.1

5.) slope $= \frac{5}{5} = 1$

6.) slope $= \frac{4}{3}$

7.) slope $= 0$

8.) slope $= \frac{3/4}{3} = \frac{1}{4}$

9.) slope $= \frac{-1}{3}$

10.) slope $= \frac{-3}{1} = -3$

11.) 1997: slope $\approx \frac{2000-1000}{9-5} = \$250/\text{yr.}$ (million)

2000: slope $\approx \frac{2500-1600}{11-7} = \$225/\text{yr.}$ (million)

2002: slope $\approx \$0/\text{yr.}$

12.) 1998 : slope $\approx \frac{2000-750}{10-7} \approx \$417/\text{yr.}$ (million)

2001 : slope $\approx \frac{1700-2000}{13-9} = -\$75/\text{yr.}$ (million)

2003 : slope $\approx \frac{1000-100}{11-9} = \$450/\text{yr.}$ (million)

14.) Running velocity is slope of distance graph $\frac{ds}{dt} = \frac{\Delta s}{\Delta t} = \frac{\text{rise}}{\text{run}}$ $\left(\frac{\text{meters}}{\text{min.}}\right)$

a.) $s = f(t)$ faster since $f'(t_1) > g'(t_1)$

b.) $s = g(t)$ faster since $g'(t_2) > f'(t_2)$

c.) $s = g(t)$ faster since $g'(t_3) > f'(t_3)$
(and g passes f at this moment)

d.) g beats f

$$\boxed{\text{DEF: } f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

15.) $f(x) = 3 \rightarrow f'(x) = \lim_{\Delta x \to 0} \frac{3-3}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{0}{\Delta x} = \lim_{\Delta x \to 0} 0 = 0$$

18.) $f(x) = \frac{1}{2}x + 5 \rightarrow f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{2}(x+\Delta x)+5 - \left(\frac{1}{2}x+5\right)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\frac{1}{2}x} + \frac{1}{2}\cdot\Delta x + \cancel{5} - \cancel{\frac{1}{2}x} - \cancel{5}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{2}\frac{\cancel{\Delta x}}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} \frac{1}{2} = \frac{1}{2}$$

22.) $f(x) = \sqrt{x+2} \longrightarrow$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{(x+\Delta x)+2} - \sqrt{x+2}}{\Delta x} \cdot \frac{\sqrt{(x+\Delta x)+2} + \sqrt{x+2}}{\sqrt{(x+\Delta x)+2} + \sqrt{x+2}}$$

$$= \lim_{\Delta x \to 0} \frac{(\cancel{x}+\Delta x)+\cancel{2} - (\cancel{x+2})}{\Delta x\cdot\left(\sqrt{x+\Delta x+2} + \sqrt{x+2}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}\cdot\left(\sqrt{x+\Delta x+2} + \sqrt{x+2}\right)} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x+2} + \sqrt{x+2}}$$

$$= \frac{1}{\sqrt{x+2} + \sqrt{x+2}} = \frac{1}{2\sqrt{x+2}}$$

25.) $f(x) = \frac{1}{x+2} \longrightarrow f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x+2} - \frac{1}{x+2}}{\frac{\Delta x}{1}}$

$$= \lim_{\Delta x \to 0} \frac{\cancel{(x+2)} - \cancel{(x}+\Delta x\cancel{+2)}}{(x+\Delta x+2)(x+2)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}}{(x+\Delta x+2)(x+2)\cdot\cancel{\Delta x}} = \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x+2)(x+2)}$$

$$= \frac{-1}{(x+2)(x+2)} = \frac{-1}{(x+2)^2}$$

32.) $f(x) = x^2 + 2x + 1 \longrightarrow$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + 2(x+\Delta x) + 1 - (x^2+2x+1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\cdot\Delta x + (\Delta x)^2 + \cancel{2x} + 2\cdot\Delta x + \cancel{1} - \cancel{x^2} - \cancel{2x} - \cancel{1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\cdot(2x + \Delta x + 2)}{\cancel{\Delta x}} = 2x + 0 + 2$$

$$= 2x + 2 \rightarrow$$

$\underline{f'(x) = 2x+2}$; slope of tangent line is

$f'(-3) = 2(-3) + 2 = -4$.

34.) $f(x) = x^3 + 2x \rightarrow f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 + 2(x+\Delta x) - (x^3+2x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^3} + 3x^2\cdot\Delta x + 3x\cdot(\Delta x)^2 + (\Delta x)^3 + \cancel{2x} + 2\cdot\Delta x - \cancel{x^3} - \cancel{2x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\left(3x^2 + 3x\cdot\Delta x + (\Delta x)^2 + 2\right)}{\cancel{\Delta x}}$$

$= 3x^2 + 0 + 0 + 2 = 3x^2 + 2 \rightarrow \underline{f'(x) = 3x^2 + 2}$;

slope of tangent line is

$f'(1) = 3(1)^2 + 2 = 5$

43.) $f(x) = \frac{1}{x} \rightarrow f'(x) = \lim_{\Delta x \to 0} \dfrac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x} - (\cancel{x}+\Delta x)}{(x+\Delta x)\cdot x} \cdot \frac{1}{\Delta x} = \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}^{\,1}}{(x+\Delta x)x\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x)x} = \frac{-1}{x^2} \rightarrow \underline{f'(x) = \frac{-1}{x^2}} \; ;$$

slope of tangent line is $m = f'(1) = \frac{-1}{(1)^2} = -1$ so equation of line is $y-1 = (-1)(x-1) \rightarrow$ $\underline{y = 2-x}$.

49.) $y = |x+3|$ is differentiable everywhere except $x = -3$ (corner).

50) $y = |x^2-9| = |(x-3)(x+3)|$ is differentiable everywhere except $x=3$ and $x=-3$ (corners).

51.) $y = (x-3)^{2/3}$ is differentiable everywhere except $x=3$ (corner).

52.) $y = x^{2/5}$ is differentiable everywhere except $x=0$ (corner).

<u>Proof</u>: $f'(0) = \lim_{\Delta x \to 0} \dfrac{f(0+\Delta x) - f(0)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{f(\Delta x) - 0}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(\Delta x)^{2/5}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{(\Delta x)^{3/5}} = \text{"}\frac{1}{0^{\pm}}\text{"} = \pm \infty$$

so $f'(0)$ DNE and f is not differentiable at $x=0$.

53.) $y = \sqrt{x-1}$ is differentiable for $x > 1$ but not $x = 1$ (vertical tangent line).

54.) $y = \frac{x^2}{x^2-4}$ is differentiable everywhere except $x = 2$ and $x = -2$ (points of discontinuity).