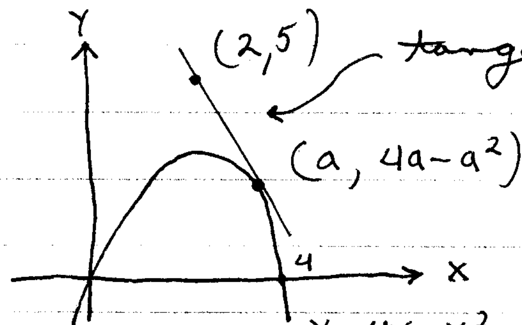
H.W. #8

(I.)



tangent line at $x=a$
(We must determine a)

$Y = 4X - X^2 \rightarrow Y' = 4 - 2X$

Slope of tangent line at $x = a$ is

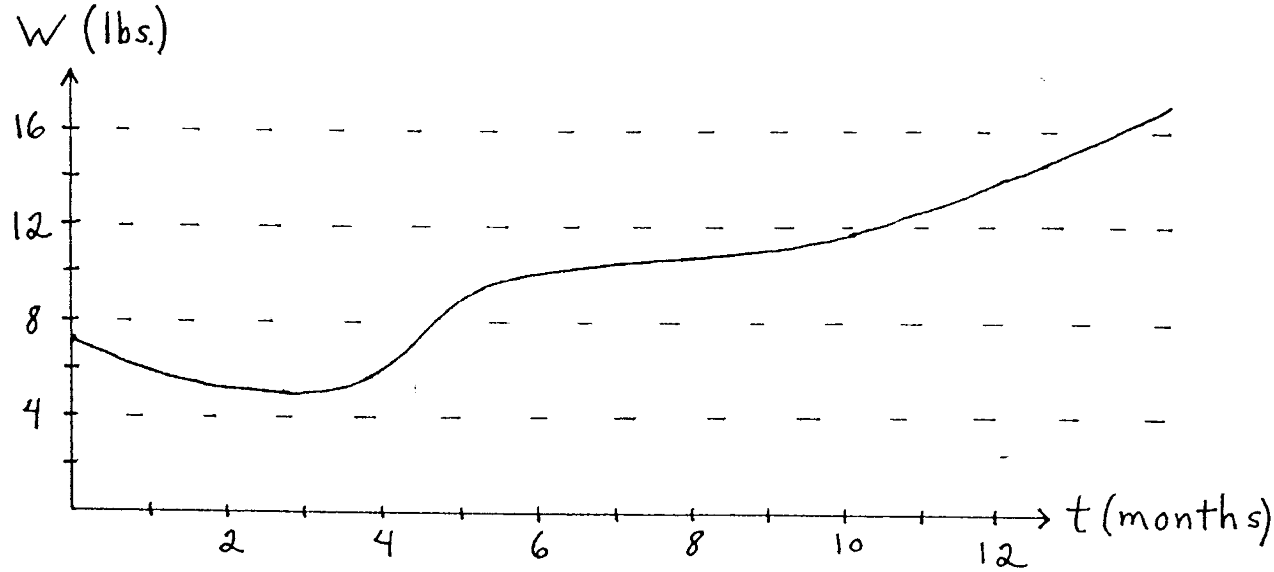
$$4 - 2a = \frac{5-(4a-a^2)}{2-a} \longrightarrow$$

derivative $\qquad$ $\frac{\text{rise}}{\text{run}}$

$8 - 4a - 4a + 2a^2 = 5 - 4a + a^2 \rightarrow a^2 - 4a + 3 = 0 \rightarrow$
$(a-1)(a-3) = 0 \rightarrow$

$a = 1 \rightarrow$ point $(1, 3)$ or
$a = 3 \rightarrow$ point $(3, 3)$.

2. The following chart represents the weight W (lbs.) of a newborn baby as a function of time t (months).



a. What is the baby's weight at birth ? after 3 months ? after 1 year ?

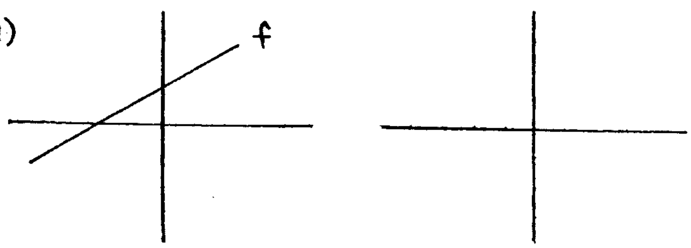
b. What is an estimate of the baby's growth rate (lbs./month) at birth ? after 3 months ? after 1 year ?

c. When is the baby growing at the fastest rate during its first year of life and what is an estimate for this rate ?

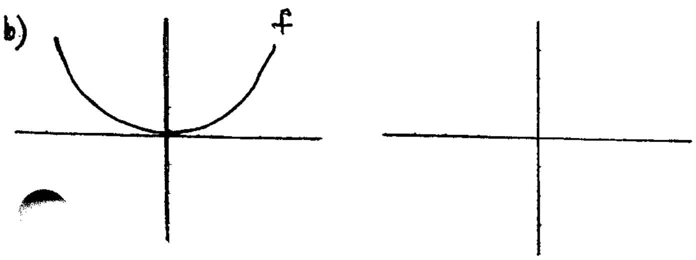
Math 16A
Kouba
Worksheet 4

1. Use the given graph of function f to sketch a graph of its derivative, f'.

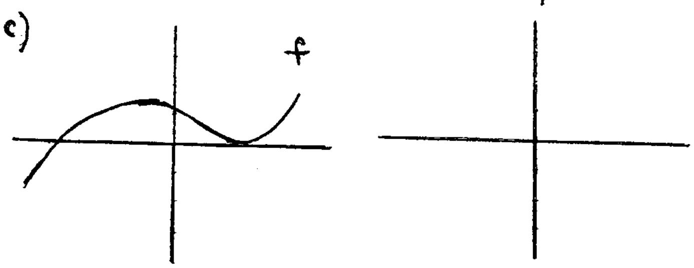
a)



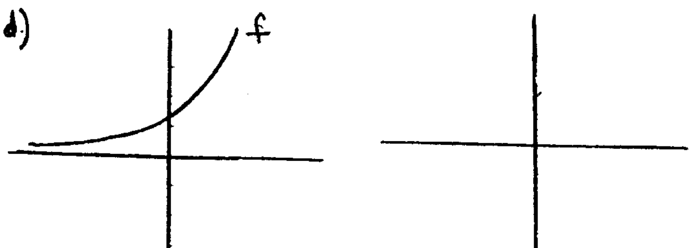
b)



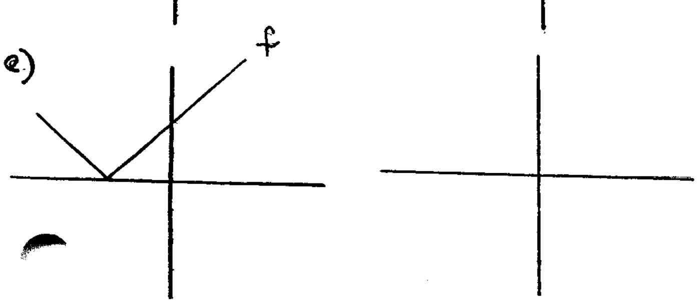
c)



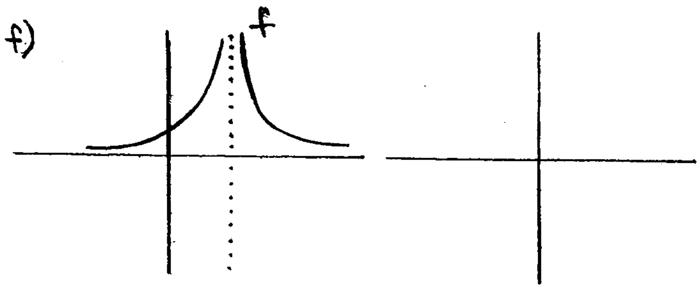
d)



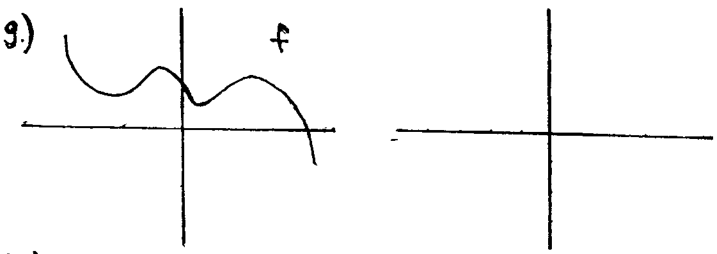
e)



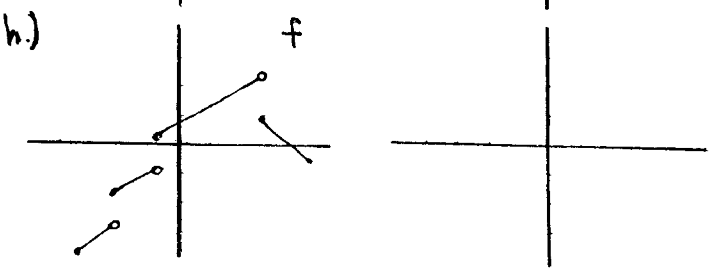
f)



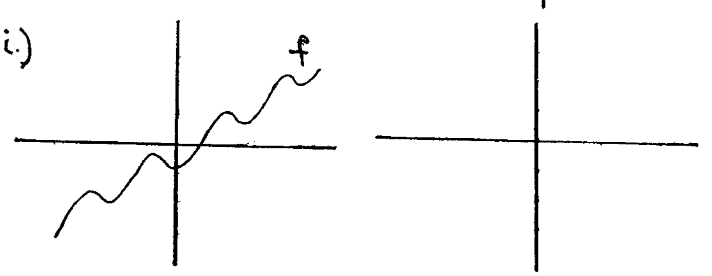
g.)



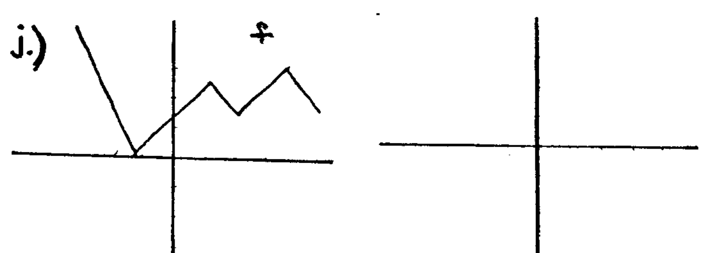
h.)



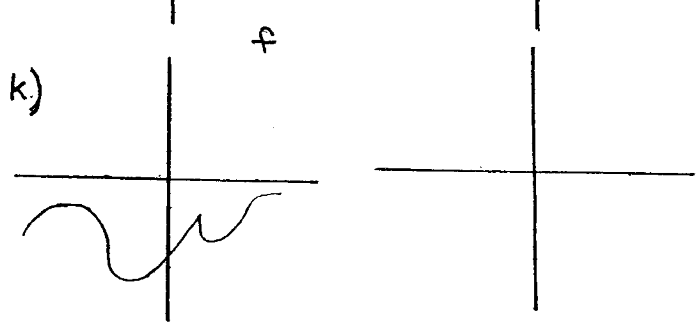
i.)



j.)



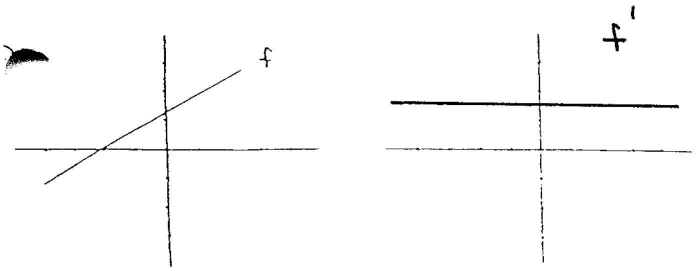
k.)



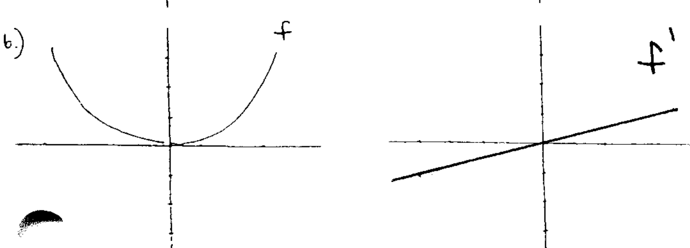
Math 16A
Kouba
Worksheet 4

1. Use the given graph of function f to sketch a graph of its derivative, f'.

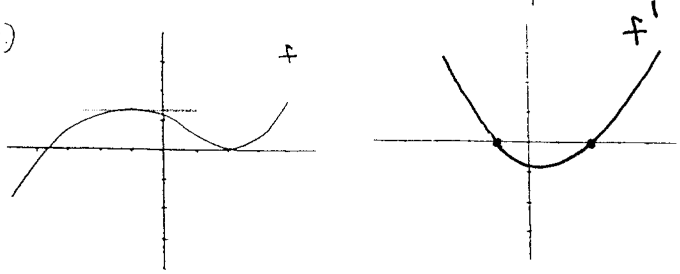
a)



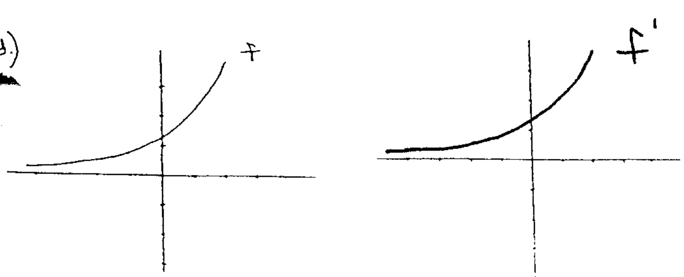
b)



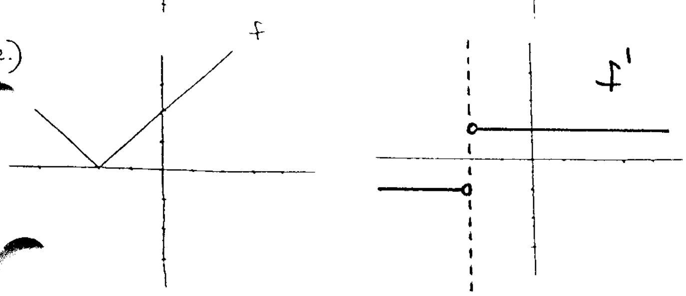
c)



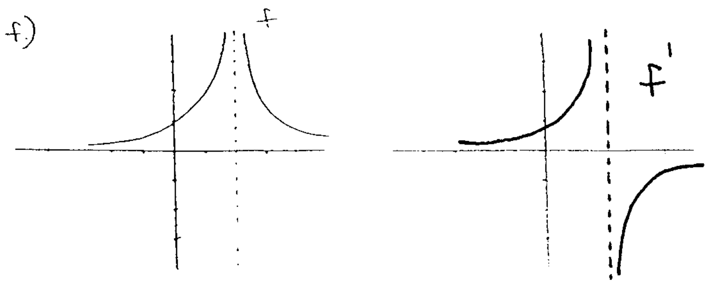
d)



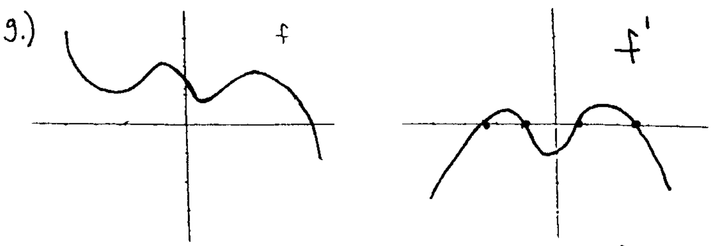
e)



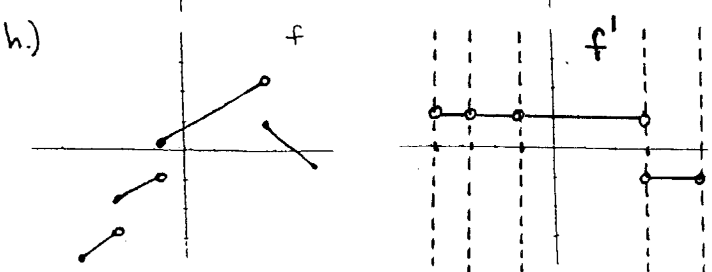
f)



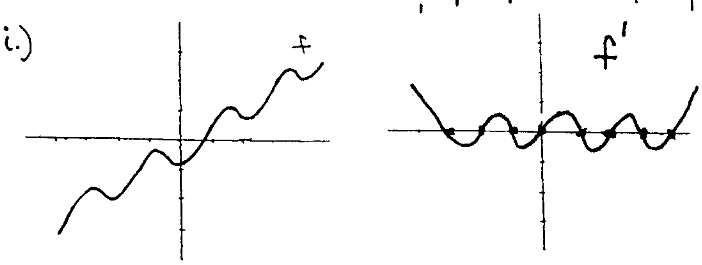
g.)



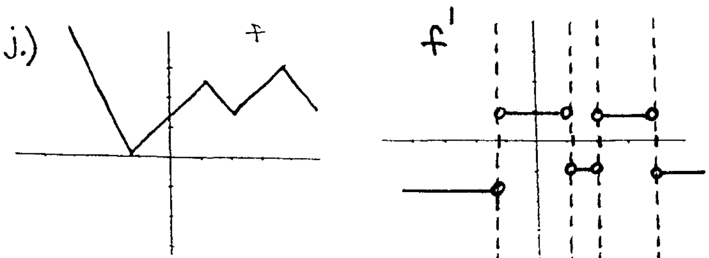
h.)



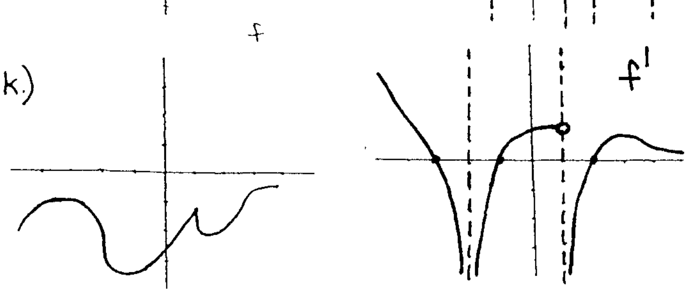
i.)



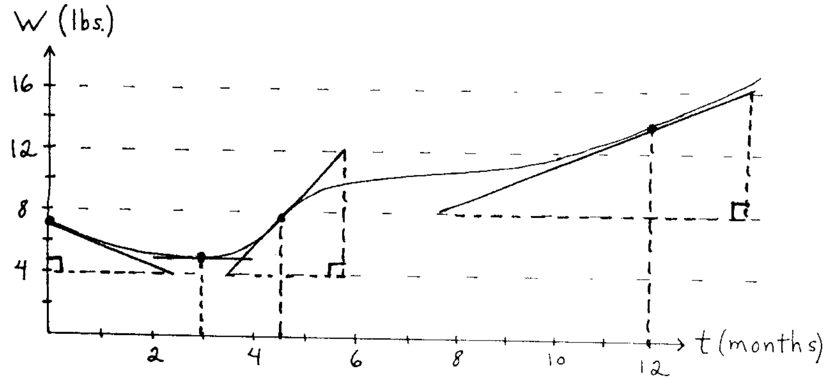
j.)



k.)



2.)

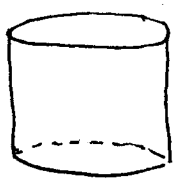


a.) $t=0 \to W = 7$ lbs. , $t = 3$ mo. $\to W = 5$ lbs. ,
$t = 1$ yr. $\to W = 14$ lbs.

b.) growth rate : slope of tangent line
$t=0 \to$ slope $= \frac{-3}{2.5} = -1.2$ lbs./mo. ,
$t = 3$ mo. $\to$ slope $= 0$ lbs./mo. ,
$t = 1$ yr. $\to$ slope $= \frac{8}{6.5} = 1.23$ lbs./mo.

c.) The baby is growing at the fastest rate when $t = 4\frac{1}{2}$ months. The growth rate is
$\frac{8}{2.5} = 3.2$ lbs./mo.

$\boxed{\text{SA4}}$:

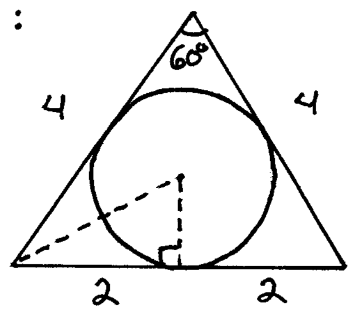


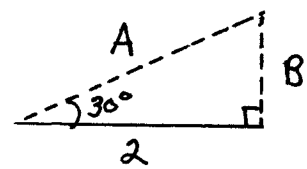
$C = 2\pi r \to 25 = 2\pi r \to$
$r = \frac{25}{2\pi}$ m. so volume

$V = \pi r^2 h = \pi \left(\frac{25}{2\pi}\right)^2 (15) = 746.04$ m^3 (ice)
(conversion : 1 $ft.^3 = 0.028$ m^3)
$V = 26{,}644.243$ $ft.^3$ (ice)
(density ice $= 0.917$ density water so volume water $= 0.917$ volume ice)
$V = 24{,}432.771$ $ft.^3$ (water) ;

1 acre = 43,560 ft. so depth of water is $\dfrac{24,432.771 \text{ ft}^3}{43,560 \text{ ft}} = 0.56$ ft. $= 6.7$ inches

$\boxed{\text{SA7}}$:



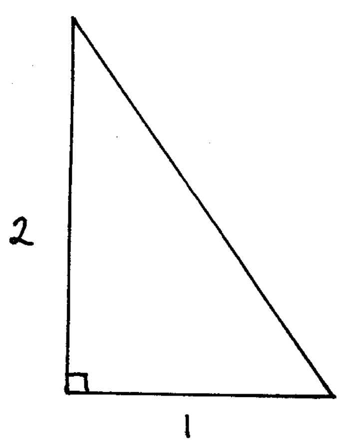


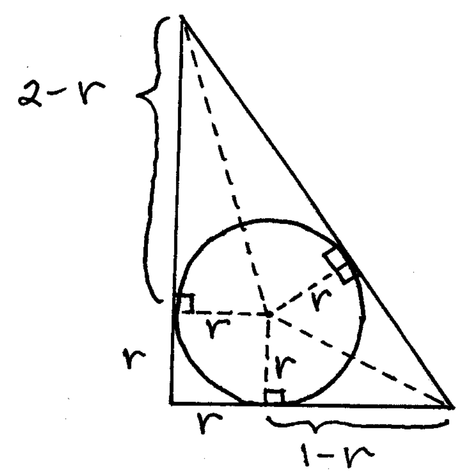
a.) $\cos 30° = \dfrac{2}{A} \rightarrow$

$A = \dfrac{2}{\cos 30°} = \dfrac{2}{\frac{\sqrt{3}}{2}} = \dfrac{4}{\sqrt{3}}$ mi.

b.) $\sin 30° = \dfrac{B}{A} \rightarrow B = A \sin 30° = \dfrac{4}{\sqrt{3}} \cdot \dfrac{1}{2} = \dfrac{2}{\sqrt{3}}$ mi.

$\boxed{\text{SA9}}$:





Area large Δ = Areas of 4 small Δ's + Area $\square$

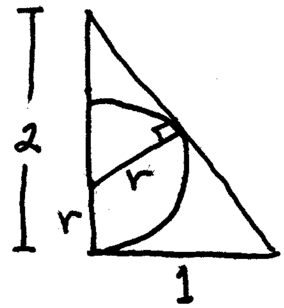
$\frac{1}{2}(1)(2) = \frac{1}{2}(1-r)r + \frac{1}{2}(1-r)r + \frac{1}{2}(2-r)r + \frac{1}{2}(2-r)r + r^2$

$\rightarrow \quad 1 = \frac{1}{2}(r-r^2) + \frac{1}{2}(r-r^2) + \frac{1}{2}(2r-r^2) + \frac{1}{2}(2r-r^2) + r^2$

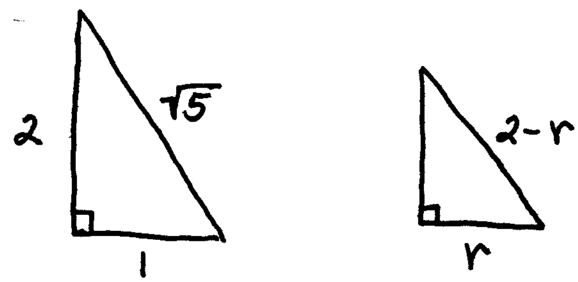
$\rightarrow \quad 1 = r - r^2 + 2r - \cancel{r^2} + \cancel{r^2} \rightarrow r^2 - 3r + 1 = 0 \rightarrow$

$r = \dfrac{3 \pm \sqrt{9-4}}{2} = \dfrac{3-\sqrt{5}}{2} \approx 0.38$

$\boxed{SA10}$:



Use similar triangles:



$$\frac{\sqrt{5}}{1} = \frac{2-r}{r} \rightarrow \sqrt{5}\,r = 2-r \rightarrow r+\sqrt{5}\,r = 2 \rightarrow$$

$$(1+\sqrt{5})r = 2 \rightarrow r = \frac{2}{1+\sqrt{5}}$$

$\boxed{SA11}$: Old K.E. $= \frac{1}{2}mv^2$

a.) New K.E. $= \frac{1}{2}(2m)(3v)^2 = 18\left(\frac{1}{2}mv^2\right)$
$= 18$ (Old K.E.)

b.) New K.E. $= \frac{1}{2}(1.4m)(0.75v)^2$
$= 0.7875\left(\frac{1}{2}mv^2\right) = 0.7875$ (Old K.E.)

so K.E. decreases by $0.2125 = 21.25\%$.

c.) New K.E. $= \frac{1}{2}(0.25m)(2v)^2 = \frac{1}{2}mv^2 =$ Old K.E.

so K.E. remains unchanged.