Section 2.2

1.) a.) $Y = X^2 \xrightarrow{D} Y' = 2X$ at $X = 1$ slope
$m = 2(1) = 2$.

b.) $Y = X^{1/2} \xrightarrow{D} Y' = \frac{1}{2}X^{-1/2}$ at $X = 1$ slope
$m = \frac{1}{2}(1)^{-1/2} = 1/2$.

4.) a.) $Y = X^{-1/2} \xrightarrow{D} Y' = \frac{-1}{2}X^{-3/2}$ at $X = 1$ slope
$m = \frac{-1}{2}(1)^{-3/2} = -1/2$.

b.) $Y = X^{-2} \xrightarrow{D} Y' = -2X^{-3}$ at $X = 1$ slope
$m = -2(1)^{-3} = -2$.

5.) $Y = 3 \xrightarrow{D} Y' = 0$

6.) $f(x) = -2 \xrightarrow{D} f'(x) = 0$

7.) $f(x) = 4X + 1 \xrightarrow{D} f'(x) = 4 + 0 = 4$

8.) $g(x) = 3X - 1 \xrightarrow{D} g'(x) = 3 - 0 = 3$

9.) $g(x) = X^2 + 4X - 1 \xrightarrow{D} g'(x) = 2X + 4 - 0 = 2X + 4$

10.) $Y = t^2 + 2t - 3 \xrightarrow{D} Y' = 2t + 2 - 0 = 2t + 2$

11.) $f(t) = -3t^2 + 2t - 4 \xrightarrow{D} f'(t) = -6t + 2$

12.) $Y = X^3 - 9X^2 + 2 \xrightarrow{D} Y' = 3X^2 - 18X$

13.) $s(t) = t^3 - 2t + 4 \xrightarrow{D} s'(t) = 3t^2 - 2$

14.) $f(x) = 2x^3 - x^2 + 3x - 1 \xrightarrow{D} f'(x) = 6x^2 - 2x + 3$

15.) $Y = 4 \cdot t^{4/3} \xrightarrow{D} Y' = 4 \cdot \frac{4}{3} t^{1/3}$

16.) $h(x) = x^{5/2} \xrightarrow{D} h'(x) = \frac{5}{2} x^{3/2}$

17.) $f(x) = 4 \cdot x^{1/2} \xrightarrow{D} f'(x) = 4 \cdot \frac{1}{2} x^{-1/2} = 2x^{-1/2}$

18.) $g(x) = 4 \cdot x^{1/3} + 2 \xrightarrow{D} g'(x) = 4 \cdot \frac{1}{3} x^{-2/3}$

19.) $Y = 4x^{-2} + 2x^2 \xrightarrow{D} Y' = -8x^{-3} + 4x$

20.) $s = 4t^{-1} + 1 \xrightarrow{D} s' = -4t^{-2}$

23.) $Y = \frac{1}{(4x)^3} = \frac{1}{4^3 x^3} = \frac{1}{64} x^{-3} \xrightarrow{D}$

$Y' = \frac{1}{64} \cdot -3x^{-4} = \frac{-3}{64} x^{-4}$

26.) $y = \frac{4x}{x^{-3}} = 4x \cdot x^3 = 4x^4 \xrightarrow{D} Y' = 4 \cdot 4x^3 = 16x^3$

30.) $Y = 3x\left(x^2 - \frac{2}{x}\right) = 3x^3 - 6 \xrightarrow{D} Y' = 9x^2$ so

at $x = 2$, $Y' = 36$.

31.) $Y = (2x+1)^2 = 4x^2 + 4x + 1 \xrightarrow{D} Y' = 8x + 4$

so at $x=0$, $y'=4$.

33.) $f(x)=x^2-4x^{-1}-3x^{-2} \xrightarrow{D} f'(x)=2x+4x^{-2}+6x^{-3}$

34.) $f(x)=x^2-3x-3x^{-2}+5x^{-3} \rightarrow f'(x)=2x-3+6x^{-3}-15x^{-4}$

35.) $f(x)=x^2-2x-2x^{-4} \rightarrow f'(x)=2x-2+8x^{-5}$

36.) $f(x)=x^2+4x+x^{-1} \rightarrow f'(x)=2x+4-x^{-2}$

40.) $f(x)=(3x^2-5x)(x^2+2)=3x^4-5x^3+6x^2-10x$
$\xrightarrow{D} f'(x)=12x^3-15x^2+12x-10$

44.) $f(x)=\dfrac{-6x^3+3x^2-2x+1}{x}=-6x^2+3x-2+x^{-1}$
$\xrightarrow{D} f'(x)=-12x+3-x^{-2}$

45.) $f(x)=x^{4/5}+x \xrightarrow{D} f'(x)=\frac{4}{5}x^{-1/5}+1$

48.) $y=x^3+x \xrightarrow{D} y'=3x^2+1$ at $(-1,-2) \rightarrow$
$m=3(-1)^2+1=4 \rightarrow y=mx+b \rightarrow$
$y=4x+b \rightarrow -2=4(-1)+b \rightarrow b=2 \rightarrow$
$\boxed{y=4x+2}$.

49.) $f(x)=x^{1/3}+x^{1/5} \xrightarrow{D} f'(x)=\frac{1}{3}x^{-2/3}+\frac{1}{5}x^{-4/5}$ at
$(1,2) \rightarrow m=\frac{1}{3}(1)^{-2/3}+\frac{1}{5}(1)^{-4/5}=\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$
$\rightarrow y=mx+b \rightarrow y=\frac{8}{15}x+b \rightarrow 2=\frac{8}{15}(1)+b$
$\rightarrow b=\frac{22}{15} \rightarrow \boxed{y=\frac{8}{15}x+\frac{22}{15}}$.

51.) $y = -x^4 + 3x^2 - 1 \xrightarrow{D}$

$y' = -4x^3 + 6x = 2x(3 - 2x^2) \xrightarrow{=0}$ $\boxed{x = 0}$ or

$3 - 2x^2 = 0 \rightarrow x^2 = 3/2 \rightarrow$ $\boxed{x = \pm\sqrt{3/2}}$

53.) $y = \frac{1}{2}x^2 + 5x \xrightarrow{D}$

$y' = x + 5 = 0 \rightarrow$ $\boxed{x = -5}$.

61.) x : number of candy bars

cost : $250 + 0.6x$ (\$)

revenue : $1.0x = x$ (\$)

so profit = revenue − cost →

$P = x - (250 + 0.6x)$

$= 0.4x - 250$ →

$P' = 0.4 = \$0.40$,

the profit on each candy car .

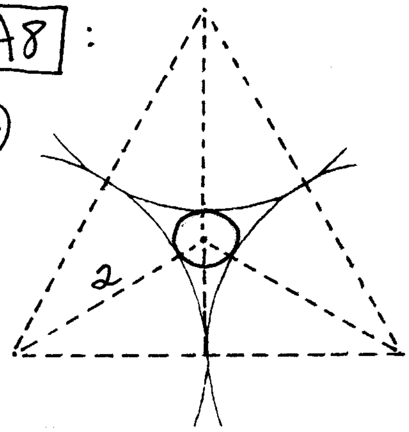
65.) FALSE : Let $f(x) = x^2$ and $g(x) = x^2 + 1$; then $f'(x) = 2x = g'(x)$, but $f(x) \neq g(x)$.

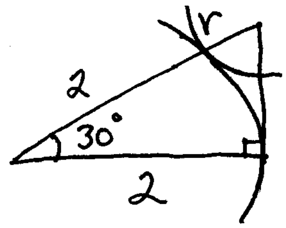
66.) TRUE : $f(x) = g(x) + c \rightarrow Df(x) = D(g(x) + c)$ $\rightarrow f'(x) = g'(x) + 0 \rightarrow f'(x) = g'(x)$.

Let r be radius of small circle.

SA8 :

a.)



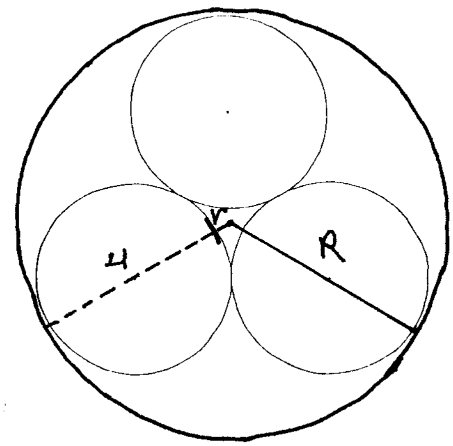


$$\cos 30^\circ = \frac{2}{r+2}$$

$$\rightarrow r+2 = \frac{2}{\frac{\sqrt{3}}{2}}$$

$$\rightarrow r = \frac{4}{\sqrt{3}} - 2 \approx 0.31$$

b.) Let R be radius of large circle. Then $R = 4 + r = 2 + \frac{4}{\sqrt{3}} \approx 4.31$



SA15 : Initially 50 l. of solution with 5 g. salt; in comes 2 l./min. with $\frac{1}{2}$ g. salt/l. :

a.) $t = 10$ min. $\rightarrow$ $50 + 2(10) = 70$ l. of solution

b.) $t = 10$ min. $\rightarrow$ $5 + 1(10) = 15$ g. of salt

c.) $t = 10$ min. $\rightarrow$ concentration is

$$\frac{15 \text{ g.}}{70 \text{ l.}} = \frac{3}{14} \text{ g./l.} \approx 0.214 \text{ g./l.}$$

d.) x minutes $\rightarrow$ concentration is

$$\frac{5 + 1(x) \text{ g.}}{50 + 2(x) \text{ l.}} = \frac{1}{3} \text{ g./l.} \rightarrow \frac{5+x}{50+2x} = \frac{1}{3} \rightarrow$$

$$15 + 3x = 50 + 2x \rightarrow x = 35 \text{ min.}$$