

Math 16A
Kouba
Continuity

Def: Function $y=f(x)$ is continuous at $x=a$ if

- i.) $f(a)$ exists (finite number)
 - ii.) $\lim_{x \rightarrow a} f(x)$ exists (finite number)
- and
- iii.) $\lim_{x \rightarrow a} f(x) = f(a)$.

Fact: Sums, differences, products, quotients (denominator $\neq 0$), and compositions of continuous functions are continuous.

Fact: Every polynomial is continuous for all x -values.

Ex: Let $f(x) = 7x^5 - x^4 + 2x^3 - x + 20$; f is continuous for all values of x since it is a polynomial.

Ex: Let $f(x) = \frac{x^2 - 5x + 6}{2x^2 + x - 3}$; since $y = x^2 - 5x + 6$ (parabola)

and $y = 2x^2 + x - 3$ (parabola) are continuous for all x -values, it follows that $f(x) = \frac{x^2 - 5x + 6}{2x^2 + x - 3}$ (quotient)

is continuous for all x -values except where

$$y = 2x^2 + x - 3 = (2x + 3)(x - 1) = 0, \text{ that is, except at } x = 1 \text{ and } x = -3/2.$$

Ex: Let $f(x) = (3 + \sin x)^{50}$; since $g(x) = 3 + \sin x$ (well-known continuous function) and $h(x) = x^{50}$ (polynomial) are continuous for all values of x , it follows that their composition $f(x) = h(g(x)) = (3 + \sin x)^{50}$ is continuous for all values of x .